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## **The Nonreciprocity of Microwave Transmission along a Bianisotropic-Ferrite Metastructure**

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**Abstract**—Propagation of microwaves along a plane layered structure containing a bianisotropic and a magnetized ferrite layer is theoretically investigated. The dispersion equation that allows taking into account an air gap or an active layer (SPASER) located between these layers is derived. This equation is numerically solved with allowance for the dispersion characteristics of the ferrite and bianisotropic metamaterial, and, with the help of the obtained solution, the spectra of the slowing factor and nonreciprocity parameter of the wave transmission in the structure are found. The influence of the magnitude and direction of the external magnetic field and of the gap thickness on the nonreciprocity parameter is considered. The validity of the conclusion (drawn from experiments) that it is possible to provide for the nonreciprocal transmission of signals at frequencies substantially exceeding the ferromagnetic resonance frequency attainable in the presence of the available magnetic field is confirmed.

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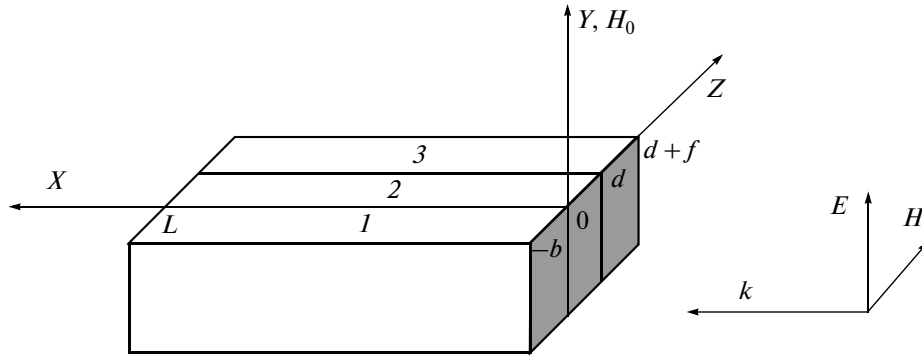
### INTRODUCTION

Extensive development of composite media has stimulated the consideration of artificial dielectrics characterized by effective permittivity, artificial magnetics with effective permeability, chiral media with the chirality tensor, and bianisotropic media with all of these parameters [1, 2]. The investigation of metastructures that are combinations of artificial media with both each other and artificial materials is of interest. In this study, the propagation of microwaves in a metastructure containing a bianisotropic plane layer and a ferrite plane layer is analyzed.

The necessity of investigating the propagation of microwaves in bianisotropic-ferrite metastructures is additionally stimulated by the experimental discovery of the nonreciprocal propagation of microwaves in lattice of resonance elements—magnetized ferrite structures that are placed in a waveguide [3, 4] (where the revealed nonreciprocity hundredfold exceeds that in the absence of a lattice) and in free space [4]. The nonreciprocity of the microwave propagation in a ferrite placed in free space in the absence of a lattice is not observed. In [4], it is supposed that the observed nonreciprocal effects can be attributed to the lattice-induced formation of surface waves (or plasmon polaritons) with the rotating magnetic field and different ferrite absorption factors of the waves such that the direction of the magnetic field rotation coincides with and is opposite to the direction of spin precession in the ferrite. In [5, 6], surface waves formed by a bianisotropic layer (without a ferrite) in a rectangular

waveguide and in free space are studied. Here, we consider waves in a bianisotropic layer—dielectric—ferrite plate structure (Fig. 1)

Bianisotropic media, as numerous other artificial media, are often realized in the form of wire media or regular structures consisting of small conducting resonance elements ( $\Omega$ -shaped particles, planar double split rings (PDSRs), short rods or strips, etc.) [7]. Therefore, with the help of a bianisotropic layer, we simulate the influence of a lattice of resonance elements just as a magnetic metamaterial layer simulates a PDSR lattice in [8]. Assume that effective permittivity  $\boldsymbol{\varepsilon}$  and permeability  $\boldsymbol{\mu}$  of the bianisotropic medium are diagonal tensors. The elements  $\varepsilon_{ij} = \varepsilon_j$ , as well as  $\mu_{ij} = \mu_j$ , are nonzero and, generally, are different for different  $j$ . Generally, the nonzero elements of the chirality tensor are  $\kappa_{yz} = \kappa_{zy}^T = \kappa$ . Permittivity of the ferrite  $\varepsilon_f$  is a scalar, and the elements of its permeability tensor are  $\mu$  and  $\pm i\mu_a$  as usual [9]. The expressions for these quantities used in the calculation are presented below. Imaginary parts  $\mu$  and  $\mu_a$  describe the absorption of waves by the ferrite, and the real parts describe the effect of the ferrite (having a finite thickness) on the structure and the wave propagation constant. In the case when the permittivity of the dielectric layer  $\varepsilon_d$  is 1, this layer simulates the air gap between the ferrite and lattice. The dispersion equation derived below can also be applied to study structures containing an active layer (SPASER [10]). In the latter case, the intrinsic permittivity of the active layer with a positive



**Fig. 1.** Considered metastructure: ( $H_0$ ) the external magnetostatic field; ( $k$ ,  $E$ , and  $H$ ) the wave, electric, and magnetic vectors of the wave; (1) bianisotropic layer; (2) dielectric; and (3) ferrite.

(for fields of the form  $E \exp(i\omega t)$ ) imaginary part is used as  $\varepsilon_d$ .

Note that waves in a ferrite–dielectric plate were studied earlier, and these studies were necessitated by the development of resonance strip isolators [11]. The situation we consider here differs from that described in [11] primarily by the presence of the dispersion of the refractive index. This dispersion is due to the resonances of elements forming the bianisotropic medium. The second difference is the possibility of attaining the refractive index values substantially exceeding those of natural dielectrics.

## 1. FIELD DISTRIBUTIONS

Consider harmonic waves of frequency  $\omega$  that propagate along the  $X$  axis

$$\begin{aligned} \vec{E} &= \vec{E} \exp[i(\omega t - k_0 n_x x)] + \text{c.c.}; \\ \vec{H} &= \vec{H} \exp[i(\omega t - k_0 n_x x)] + \text{c.c.} \end{aligned} \quad (1)$$

Here,  $k_0 = \omega \sqrt{\varepsilon_0 \mu_0}$  is the wave number in free space and  $n_x = n' + in''$  is the effective refractive index or the slowing factor. The electric field of waves is polarized in the same way as it is in the experiments on the observation of the nonreciprocal wave transmission [3, 4] in parallel to external magnetic field  $H_0$ , i.e., along the  $Y$  axis. By analogy with the study of problems with a ferrite plate in [11], we restrict the consideration to waves whose amplitudes are independent of the coordinate measured in the direction of magnetostatic field  $H_0$ .

The investigation of nonreciprocity necessitates the dispersion equation for our metastructure and its solutions obtained with allowance for the dispersion properties of the materials of the layers forming the structure.

Proceeding from the Maxwell equations and taking into account the requirement of the absence of fields

at infinity, we find the field amplitudes depending on transverse coordinate  $z$ :

$$\begin{aligned} E_y &= A^+ \varepsilon_0^{-1/2} \exp(k_0 p z); \\ H_x &= -ip \mu_0^{-1/2} A^+ \exp(k_0 p z); \\ H_z &= n_x \mu_0^{-1/2} A^+ \exp(k_0 p z) \end{aligned} \quad (2)$$

in the vacuum (or air) region for  $z < -b$ ;

$$\begin{aligned} E_y &= \varepsilon_0^{-1/2} (A_c^{\text{bm}} \cos(k_0 q z) + A_s^{\text{bm}} \sin(k_0 q z)); \\ H_x &= iq \mu_0^{-1/2} \mu_x^{-1} (A_c^{\text{bm}} \sin(k_0 q z) - A_s^{\text{bm}} \cos(k_0 q z)); \\ H_z &= (n_x - i\kappa) \mu_0^{-1/2} \mu_z^{-1} (A_c^{\text{bm}} \cos(k_0 q z) + A_s^{\text{bm}} \sin(k_0 q z)) \end{aligned} \quad (3)$$

in the bianisotropic material layer for  $-b < z < 0$ ;

$$\begin{aligned} E_y &= \varepsilon_0^{-1/2} [A_{\text{cosh}}^{\text{d}} \cosh(k_0 s z) + A_{\text{sinh}}^{\text{d}} \sinh(k_0 s z)]; \\ H_x &= -is \mu_0^{-1/2} [A_{\text{cosh}}^{\text{d}} \sinh(k_0 s z) + A_{\text{sinh}}^{\text{d}} \cosh(k_0 s z)]; \\ H_z &= n_x \mu_0^{-1/2} [A_{\text{cosh}}^{\text{d}} \cosh(k_0 s z) + A_{\text{sinh}}^{\text{d}} \sinh(k_0 s z)] \end{aligned} \quad (4)$$

in the dielectric layer for  $0 < z < d$ ;

$$\begin{aligned} E_y &= \varepsilon_0^{-1/2} (A_c^{\text{f}} \cos(k_0 r z) + A_s^{\text{f}} \sin(k_0 r z)); \\ H_x &= i \mu_0^{-1/2} (\mu^2 - \mu_a^2)^{-1} [(\mu_a n_x A_c^{\text{f}} - \mu_r A_s^{\text{f}}) \cos(k_0 r z) \\ &\quad + (\mu_a n_x A_s^{\text{f}} + \mu_r A_c^{\text{f}}) \sin(k_0 r z)]; \\ H_z &= \mu_0^{-1/2} (\mu^2 - \mu_a^2)^{-1} [(\mu n_x A_c^{\text{f}} - \mu_a r A_s^{\text{f}}) \cos(k_0 r z) \\ &\quad + (\mu n_x A_s^{\text{f}} + \mu_a r A_c^{\text{f}}) \sin(k_0 r z)] \end{aligned} \quad (5)$$

in the ferrite layer for  $d < z < d + f$ ; and

$$\begin{aligned} E_y &= A^- \varepsilon_0^{-1/2} \exp(-k_0 p z); \\ H_x &= ip \mu_0^{-1/2} A^- \exp(-k_0 p z); \\ H_z &= n_x \mu_0^{-1/2} A^- \exp(-k_0 p z). \end{aligned} \quad (6)$$

in free space for  $z > d + f$ .

Formulas (2)–(6) contain parameters  $p$ ,  $q$ ,  $r$ , and  $s$  determined from the relationships

$$p^2 = n_x^2 - 1, \quad (7)$$

$$q^2 \mu_z = \mu_x (n_{\text{bm}}^2 - n_x^2), \quad (8)$$

$$r^2 = \varepsilon_f \mu_{\perp} - n_x^2, \quad (9)$$

$$s^2 = n_x^2 - \varepsilon_d. \quad (10)$$

Here,  $\mu_x$ , and  $\mu_z$  are the elements of the permeability of the bianisotropic material,  $n_{\text{bm}}$  is its effective refractive index,

$$n_{\text{bm}}^2 = \varepsilon_y \mu_z - \kappa^2, \quad (11)$$

$\mu_{\perp} = \mu - \mu_a^2/\mu$  is the effective the permeability of the ferrite for the transverse electromagnetic wave.

## 2. THE DISPERSION EQUATION

The continuity conditions for the tangential components of the electric and magnetic fields on the vacuum–bianisotropic material (at  $z = -b$ ), bianisotropic material–dielectric (at  $z = 0$ ), dielectric–ferrite (at  $z = d$ ), and ferrite–vacuum (at  $z = d + f$ ) interfaces yield the following equations coupling amplitudes  $A^{\pm}$ ,  $A_{s,c}^{\text{bm}}$ ,  $A_{\text{sinh},\text{cosh}}^{\text{d}}$ , and  $A_{s,c}^{\text{f}}$  in the planes with coordinates  $z$  indicated below:

$$A^+ \exp(-k_0 p b) - A_c^{\text{bm}} \cos(k_0 q b) + A_s^{\text{bm}} \sin(k_0 q b) = 0; \quad (12)$$

$$A^+ p \mu_x \exp(-k_0 p b) - A_c^{\text{bm}} q \sin(k_0 q b) - A_s^{\text{bm}} q \cos(k_0 q b) = 0$$

for  $z = -b$ ;

$$A_c^{\text{bm}} - A_{\text{cosh}}^{\text{d}} = 0; \quad (14)$$

$$-q A_s^{\text{bm}} + s \mu_x A_{\text{sinh}}^{\text{d}} = 0 \quad (15)$$

for  $z = 0$ ;

$$A_{\text{cosh}}^{\text{d}} \cosh(k_0 s d) + A_{\text{sinh}}^{\text{d}} \sinh(k_0 s d) - A_c^{\text{f}} \cos(k_0 r d) - A_s^{\text{f}} \sin(k_0 r d) = 0; \quad (16)$$

$$A_{\text{cosh}}^{\text{d}} s (\mu^2 - \mu_a^2) \sinh(k_0 s d) + A_{\text{sinh}}^{\text{d}} s (\mu^2 - \mu_a^2) \cosh(k_0 s d) + A_c^{\text{f}} [\mu_a n_x \cos(k_0 r d) + \mu r \sin(k_0 r d)] + A_s^{\text{f}} [\mu_a n_x \sin(k_0 r d) - \mu r \cos(k_0 r d)] = 0$$

for  $z = d$ ; and

$$A_c^{\text{f}} \cos[k_0 r (d + f)] + A_s^{\text{f}} \sin[k_0 r (d + f)] - A^- \exp[-k_0 p (d + f)] = 0, \quad (18)$$

$$A_c^{\text{f}} \{ \mu_a n_x \cos[k_0 r (d + f)] + \mu r \sin[k_0 r (d + f)] \} + A_s^{\text{f}} \{ \mu_a n_x \sin[k_0 r (d + f)] - \mu r \cos[k_0 r (d + f)] \} - A^- p (\mu^2 - \mu_a^2) \exp[-k_0 p (d + f)] = 0$$

for  $z = d + f$ .

The set of relationships (12)–(19) form a system of simultaneous equations when its determinant  $D$  is zero, i.e., the dispersion equation

$$D(n_x(\omega)) = |d_{\alpha\beta}| = 0 \quad (20)$$

is satisfied. The nonzero elements of this eighth-order determinant are

$$\begin{aligned} d_{11} &= 1, \quad d_{12} = \cos(k_0 q b), \quad d_{13} = \sin(k_0 q b), \\ d_{21} &= p \mu_x, \quad d_{22} = q \sin(k_0 q b), \quad d_{23} = -q \cos(k_0 q b), \\ d_{32} &= 1, \quad d_{34} = 1, \\ d_{43} &= -q, \quad d_{45} = s \mu_x, \\ d_{54} &= \cosh(k_0 s d), \quad d_{55} = \sinh(k_0 s d), \\ d_{56} &= -\cos(k_0 r d), \quad d_{57} = -\sin(k_0 r d), \\ d_{64} &= s (\mu^2 - \mu_a^2) \sinh(k_0 s d), \\ d_{65} &= s (\mu^2 - \mu_a^2) \cosh(k_0 s d), \\ d_{66} &= \mu_a n_x \cos(k_0 r d) + \mu r \sin(k_0 r d), \\ d_{67} &= \mu_a n_x \sin(k_0 r d) - \mu r \cos(k_0 r d), \\ d_{76} &= \cos[k_0 r (d + f)], \\ d_{77} &= \sin[k_0 r (d + f)], \quad d_{78} = 1, \\ d_{86} &= \mu_a n_x \cos[k_0 r (d + f)] + \mu r \sin[k_0 r (d + f)], \\ d_{87} &= \mu_a n_x \sin[k_0 r (d + f)] - \mu r \cos[k_0 r (d + f)], \\ d_{88} &= p (\mu^2 - \mu_a^2). \end{aligned} \quad (21)$$

Below, the obtained dispersion equation is used for investigating the dependences of the nonreciprocity of propagation of microwaves in the considered metastructure on their frequency, the direction and magnitude of the external magnetic field, and the distance between the ferrite and bianisotropic layers.

## 3. THE NONRECIPROCITY OF WAVE PROPAGATION

Let us specify the terms. The bianisotropic material may prove to be a so-called negative (double negative) medium at the frequencies such that its effective permittivity and permeability are simultaneously negative [12]. Waves in such media (and in metastructures containing them) can be forward and backward. For forward waves, the direction of phase shift coincides with the direction energy (power) flux propagation (and, hence, attenuation). For backward waves, these directions are opposite. Therefore, for forward (backward) waves,  $n'$  and  $n''$  have opposite (identical) signs [13]. In the studies dealing with the nonreciprocity of wave propagation, another meaning of forward and backward waves is considered: it is assumed that a forward wave propagates from the source, and the backward wave moves toward it, i.e., moves in the direction opposite to the direction of the forward wave propagation [11]. In such situations, it is reasonable to speak

about transmitted and counterpropagating waves rather than forward and backward waves. It is necessary to indicate where a wave is transmitted from and specify the direction of its propagation. It is important that the directions of propagation of the energies (and attenuation of the amplitudes) of the transmitted and counterpropagating waves are opposite: the signs of  $n''$  are opposite for these waves. The directions of the phase shift are of no importance. Below, we use notation  $n'_+$  and  $n''_+$  for the wave whose energy is transported (and decreases) in the direction of the  $X$  axis and notation  $n'_-$  and  $n''_-$  for the wave whose energy is transported in the opposite direction of the  $X$  axis. It follows from (1) that  $n'_+ < 0$  and  $n''_+ > 0$ . For the metastructure shown in Fig. 1, we consider two terminal planes: the near plane with  $x = 0$  and the far plane with  $x = L$ . For the near terminal, the wave with  $n''_+ < 0$  is a transmitted one, and the wave with  $n'_+ > 0$  is a counterpropagating one. For the far terminal, the wave with  $n''_- > 0$  is a transmitted one, and the wave with  $n'_- < 0$  is a counterpropagating one. The energy of the wave that is transmitted from the near terminal and covers a path of length  $L$  in the interior of the metastructure is attenuated by the factor  $T_+^{-1} = \exp(-2k_0 n''_+ L)$ , and the energy of the wave transmitted from the far terminal is attenuated by the factor  $T_-^{-1} = \exp(2k_0 n''_- L)$ . The ratio of these attenuation factors

$$T_+/T_- = \exp(2k_0(n''_+ + n''_-)L) = \exp \delta L,$$

can serve as a measure of the nonreciprocity of wave propagation. When  $n''_+ = -n''_-$ , the propagation is reciprocal. The quantity

$$\delta = 2k_0(n''_+ + n''_-) \quad (22)$$

is the specific transmission nonreciprocity parameter per unit path length.

#### 4. THE SPECTRA OF THE SLOWING FACTOR AND SPECIFIC NONRECIPROcity PARAMETER OF WAVE PROPAGATION

The frequency dependences of the slowing factor  $n_x = n' + in''$  and the nonreciprocity parameter of wave propagation along the metastructure are determined by the ferromagnetic resonance (FMR) of the ferrite and the resonance of bianisotropic material elements (RBME, the resonance of the lattice in the experiments). The dependences of dielectric and ferrite permittivities  $\varepsilon_d$  and  $\varepsilon_f$ , respectively, on frequency are practically unnoticeable against these resonances.

The FMR-induced dispersion of the components of the ferrite permeability tensor is described by the formulas

$$\begin{aligned} \mu &= \mu' - i\mu'' = 1 \\ &+ \frac{\omega_H \omega_M \tau^2 \left( (\omega_H^2 - \omega^2) \tau^2 + 1 \right) - i \omega_M T \left( (\omega_H^2 + \omega^2) \tau^2 + 1 \right)}{\left( (\omega_H^2 - \omega^2) \tau^2 - 1 \right)^2 + 4 \omega_H^2 \tau^2}, \\ \mu_a &= \mu'_a - i\mu''_a \\ &= \frac{\omega \omega_M \tau^2 \left[ \left( (\omega_H^2 - \omega^2) \tau^2 - 1 \right) - 2i \omega_H \tau \right]}{\left( (\omega_H^2 - \omega^2) \tau^2 - 1 \right)^2 + 4 \omega_H^2 \tau^2} \operatorname{sgn} H_0, \end{aligned} \quad (23)$$

where  $\omega_H = \gamma |H_0|$  is the FMR frequency in free ferrite, the frequency  $\omega_M = 4\pi\gamma M_0$  is determined by saturation magnetization  $M_0$  of the ferrite,  $\gamma$  is the gyromagnetic ratio, and  $\tau$  is the transverse relaxation time.

The characteristics of the bianisotropic material (its refractive index  $n_{\text{bm}}$  and permeability elements  $\mu_x$  and  $\mu_z$ ) are involved in parameter  $q$  determined by relationship (9) and, therefore, enter the dispersion equation.

As an example, let us consider a bianisotropic material consisting of PDSRs with a negligible loss. According to [14], the parameters of this material are as follows:

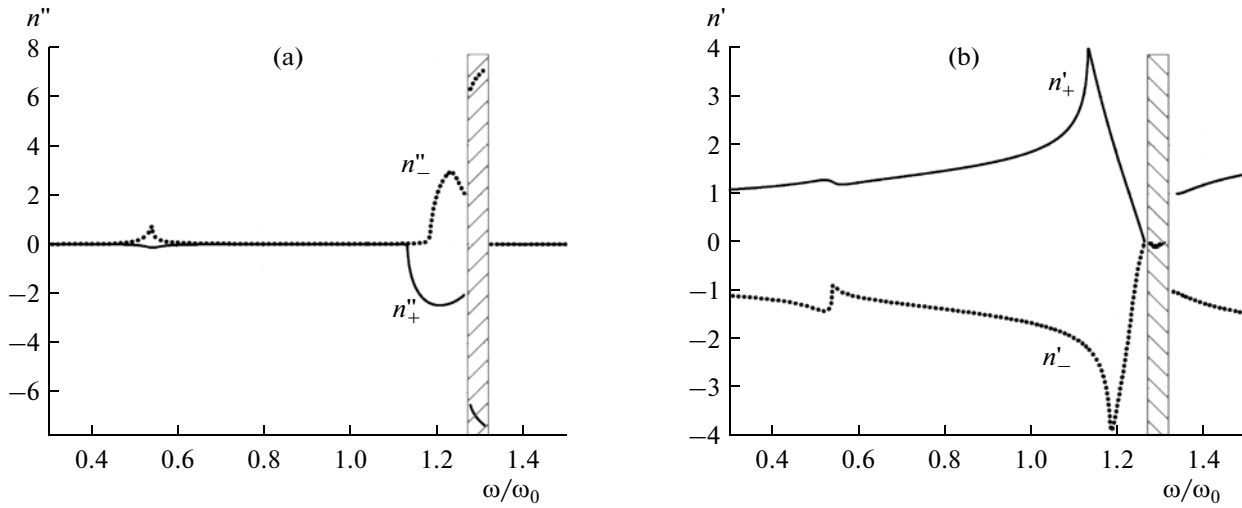
$$n_{\text{bm}}^2 = \varepsilon_y(0) \left( 1 - \omega^2 / \omega_n^2 \right) / \left( 1 - \omega^2 / \omega_b^2 \right), \quad (24)$$

$$\mu_z = \left( 1 - \omega^2 / \omega_\mu^2 \right) / \left( 1 - \omega^2 / \omega_b^2 \right), \quad \mu_x = 1. \quad (25)$$

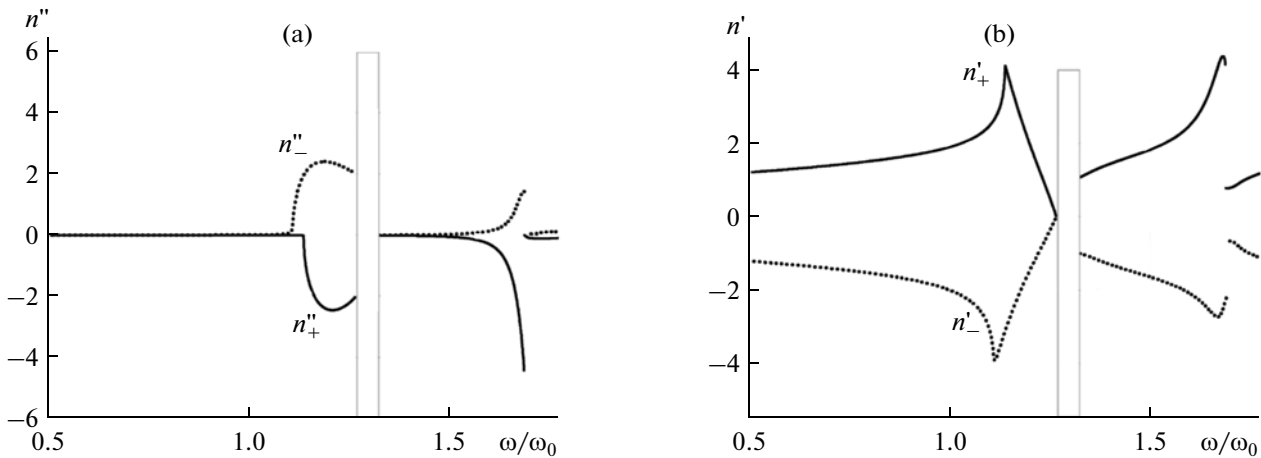
Here,  $\varepsilon_y(0)$  is the low-frequency limit of the  $y$  element of the permittivity tensor;  $\omega_n$ ,  $\omega_\mu$ , and  $\omega_b$  are the frequencies of the reversals of the signs of effective refractive index  $n_{\text{bm}}$  and corresponding permeability  $\mu_z$ .

Figure 2 depicts the frequency dependences (spectra) of the imaginary and real parts of the wave slowing factors in this structure. The results are obtained from the solution of the dispersion equation with allowance for dependences (23)–(25) for the positive direction of magnetostatic field  $H_0$ . Quantity  $H_0$  corresponds to the FMR frequency  $\omega_H = 0.3\omega_0$ . In the calculation, the following parameters of the bianisotropic layer are used:  $\omega_n = 1.52\omega_0$ ,  $\omega_\mu = 1.27\omega_0$ ,  $\omega_b = 0.74\omega_0$ , the RBME frequency  $\omega_0 = 2\pi \times 6$  GHz,  $n_0^2 = 26$ , and  $b = 0.24$  mm (as in the calculation for Fig. 2 from [6]). The ferrite parameters are  $f = 1$  mm,  $\omega_M = 0.7\omega_0$ ,  $\tau^{-1} = 0.026\omega_0$ , and  $\varepsilon_f = 16$ .

The curves from Figs. 2–5 refer to the lowest mode that has the minimum positive value of  $\operatorname{Re} p$  and the maximum transverse dimension. In the experiment, these modes are usually best matched with the incident wave. The stepwise changes in the curves are related with transitions from one mode to another (due to changes in  $\operatorname{Re} p$  and the mode width) or with



**Fig. 2.** Frequency dependences of the (a) imaginary and (b) real parts of slowing factor  $n$  for  $H_0 > 0$ ,  $\omega_H/\omega_0 = 0.3$ , and  $d = 0.15$  mm. The solid curves refer to the waves transmitted along the  $X$ , and the dashed curves refer to the oppositely directed waves. The rectangle marks the region where the backward wave exists.



**Fig. 3.** Frequency dependences of the (a) imaginary and (b) real parts of slowing factor  $n$  for  $H_0 > 0$ ,  $\omega_H/\omega_0 = 1.4$ , and  $d = 0.15$  mm. The solid curves refer to the waves transmitted along the  $X$ , and the dashed curves refer to the oppositely directed waves. The rectangle marks the region where propagating waves are not found.

intersection of the boundary with the region where there are no propagating modes.

Figure 3 shows the spectra of the imaginary and real parts of the wave slowing factors for the positive (aligned with the  $Y$  axis) magnetostatic field corresponding to the FMR frequency  $\omega_H = 1.4\omega_0$ .

The same calculation is performed for the inverted magnetostatic field.

Note certain features of the calculated spectra.

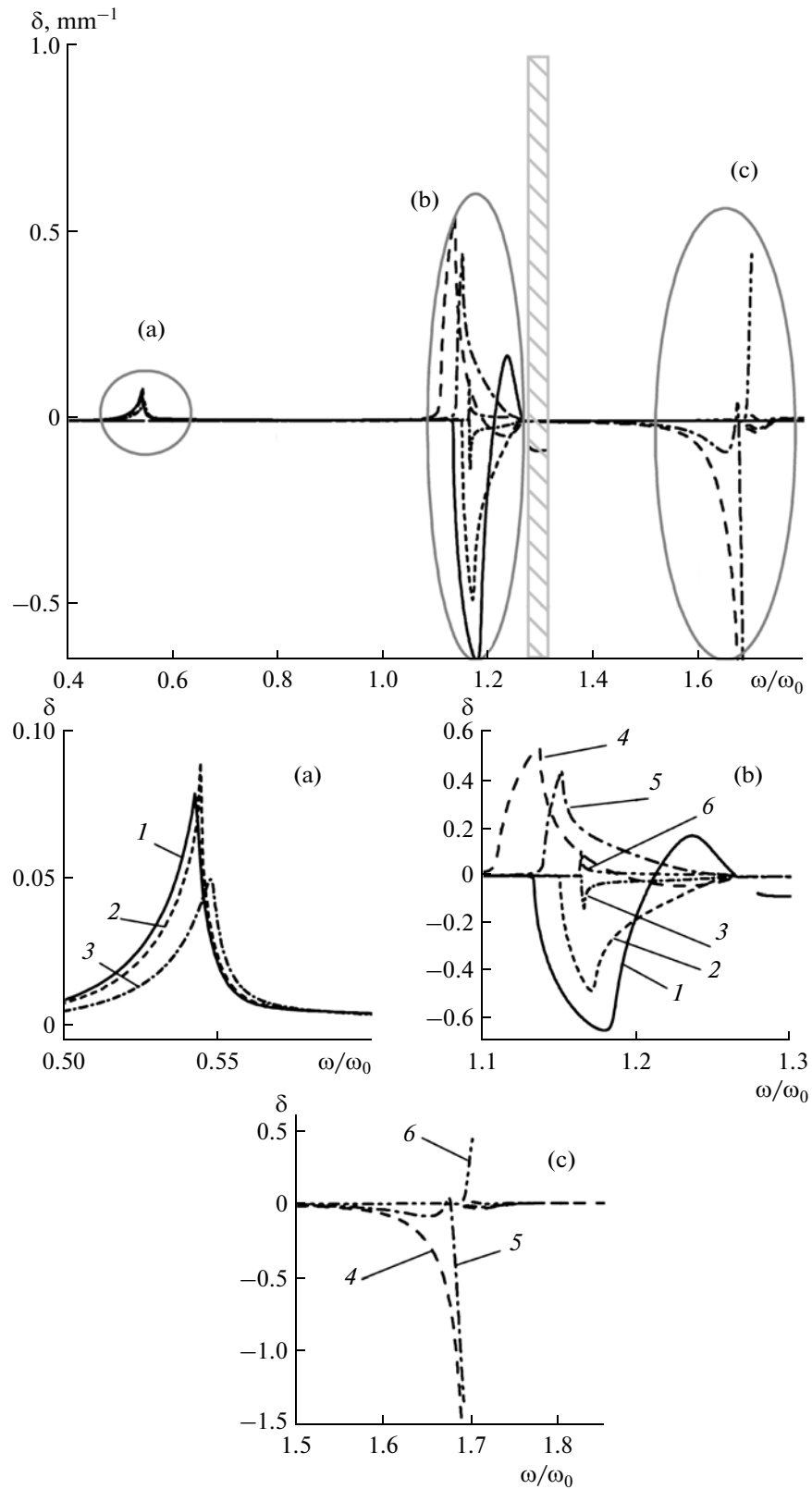
(i) In each of the figures like Figs. 2a and 3a, we can separate two ranges where quantity  $|n''|$  substantially exceeds its values in the remaining segment of the spectrum. One range is related with the FMR; however, it does not coincide with FMR frequency  $\omega_H$  but is tuned as this frequency changes. The other range is

adjacent to the resonance frequency of the wire structure elements. As the external field intensity changes, this range shifts within a bounded interval around the RBME frequency.

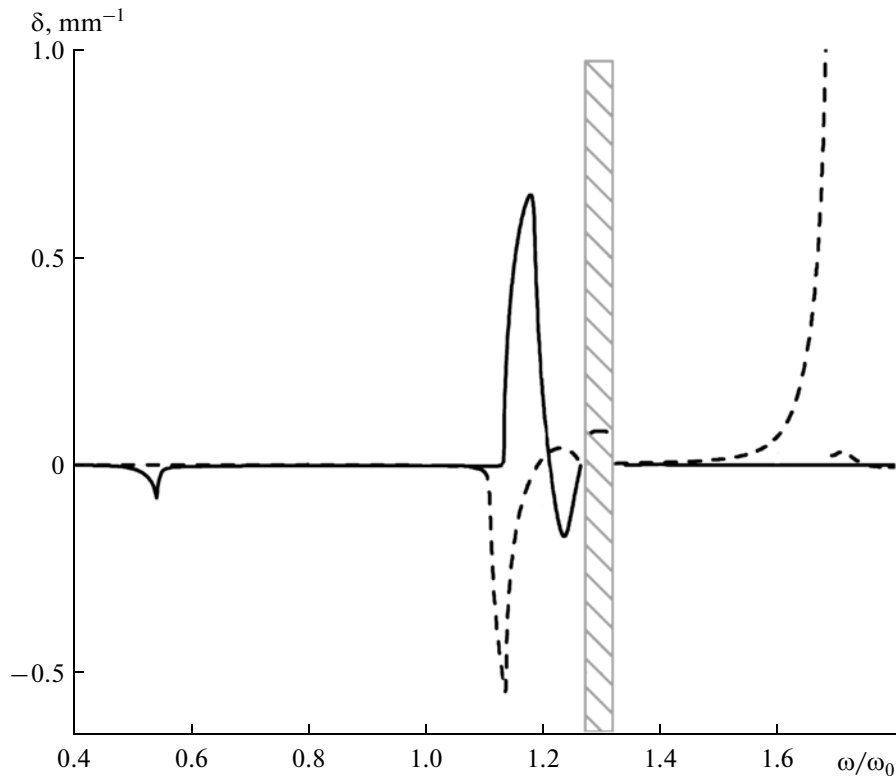
(ii) In Fig. 2, marked is the region where the signs of  $n'$  and  $n''$  are identical for the counterpropagating wave ( $n'' < 0$  and  $n'_- < 0$ ), i.e., the counterpropagating wave is backward. In the case of the inverted magnetostatic field ( $H_0 < 0$ ), the transmitted wave is backward ( $n''_+ > 0$  and  $n'_+ > 0$ ) in the same region.

(iii) In Fig. 3, there is a region with no propagating waves.

(iv) In Figs. 2b and 3b, the regions of slowed (with  $|n'_\pm| > 1$ ) and accelerated (with  $|n'_\pm| < 1$ ) waves are observed.



**Fig. 4.** Frequency dependences of specific nonreciprocity parameter  $\delta$  for  $H_0 > 0$ . Marked are the regions related with the FMR at (a)  $\omega_H/\omega_0 = 0.3$  and (c)  $\omega_H/\omega_0 = 1.4$  and (b) with RBME. The rectangle marks the region where the backward wave exists at  $\omega_H/\omega_0 = 0.3$ . Curves 1–3 and 4–6 are obtained for  $\omega_H/\omega_0 = 0.3$  and 1.4, respectively, at  $d = (1, 4) 0.15$ ,  $(2, 5) 1$ , and  $(3, 6) 4$  mm.



**Fig. 5.** Frequency dependences of the specific nonreciprocity parameter for  $H_0 < 0$  and  $d = 0.15$  mm. Resonance regions are not marked. The solid and dashed curves are obtained for  $\omega_H/\omega_0 = 0.3$  and 1.4, respectively. The rectangle marks the region where the backward wave exists at  $\omega_H/\omega_0 = 0.3$ .

(v) The aforementioned region of backward waves and the region without propagating waves do not coincide with such regions obtained in [6] for a bianisotropic plate with the same parameters of the bianisotropic medium. This circumstance indicates that the ferrite layer substantially affects the modes formed by the structure as a whole.

(vi) The frequency dependences of the slowing factors for transmitted ( $n_+$ ) and counterpropagating waves ( $n_-$ ) are nonsymmetric with respect to the zero point, which indicates the nonreciprocal wave propagation.

(vii) The inversion of the direction of the external magnetic field leads to the interchange of the transmitted and counterpropagating waves ( $n_+ \rightleftharpoons n_-$ ). This means that the change of the direction of the signal transmission is equivalent to the inversion of the direction of the magnetic field, a circumstance that is typical of all known experiments on the nonreciprocity of wave transmission.

The frequency dependences of the specific nonreciprocity parameter that are found with the use of the dependences from Figs. 2, 3 and similar dependences calculated for other values of air gap  $d$  between the ferrite and bianisotropic layer are depicted in Fig. 4.

Figure 5 shows the same dependences of the nonreciprocity parameter for inverted field  $H_0$  and  $d = 0.15$  mm.

In these figures, we can see frequency bands of nonreciprocal microwave transmission that are related with the FMR and RBME. We can also notice that, when FMR frequency  $\omega_H$  passes frequency  $\omega_0$  of the resonance of metamaterial elements in the case of the constant direction of the magnetostatic field, the sign of the nonreciprocity parameter is reversed in both the region related with the FMR (see, e.g., the passage from Fig. 4a to Fig. 4c) and the RBME region (curves 1–3 and 4–6 from Fig. 4b).

The reduction of the air gap leads, as a rule, to the increase of the absolute peak values of the nonreciprocity parameter of wave transmission and to small shifts of the maximum nonreciprocity frequency. The curves for  $d = 0.15$  mm and  $d = 0$  practically coincide, therefore, the latter are not presented in the figures. Curve 6 from Fig. 4c is an odd one among other dependences, apparently, because it refers to a special mode whose radiation concentrates, as in a waveguide, in the region between the bianisotropic and ferrite layers, which is possible at high frequencies when the gap is rather large. The inversion of the external magnetic field causes the reversal of the nonreciprocity sign (cf. the curves from Fig. 5 and Fig. 4 for  $d = 0.15$ ). Let us

compare some more conclusions from study [6] with the results illustrated in Figs. 4 and 5. We note the reversal of the sign of the nonreciprocity parameter at the boundary between the regions of forward and backward wave propagation. In addition, we see that the frequency of the maximal nonreciprocity in the RBME region for small  $d$  (see Fig. 4b) is close to frequency  $\omega_{\text{deg}} \approx 1.17\omega_0$  of local degeneration (corresponding to  $\zeta_{\text{deg}} \approx 1.37$  in Fig. 2 from [6]), at which the polarization of the magnetic field is the closest to circular.

## CONCLUSIONS

Thus, in the study, we have revealed the following:

(i) The transmission of microwaves along bianisotropic–ferrite metastructures is nonreciprocal;

(ii) There are two frequency bands of microwave transmission nonreciprocity: one is related with the resonance of structure elements, and the other is related with the FMR and tuned under the variation of the external magnetic field;

(iii) The sign of the microwave transmission nonreciprocity is reversed when the positional relationship of the aforementioned resonances changes;

(iv) The decrease of the thickness of the air gap between the ferrite and bianisotropic layers increases the nonreciprocity of wave transmission.

In addition, in study [15], the following conclusion has been drawn: the sign of the microwave transmission nonreciprocity is reversed when the ferrite and bianisotropic layers are interchanged in the structure without an air gap.

These features have been found in the experiments [3–6] with a ferrite plate and lattices of resonance elements. Hence, the developed theoretical approach makes it possible to explain (qualitatively at the current stage) the observed phenomena and experimental facts.

We should specially note that the nonreciprocal transmission of signals can be provided at frequencies substantially exceeding (by a factor greater than 3 in the above example with  $\omega_H = 0.3\omega_0$ ) the FMR frequency reachable with the available magnet (or the available power supply of an electromagnet). The use of hexaferrites and nanotechnologies can ensure the application of nonreciprocal structures of the considered types in the terahertz band.

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