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ACOUSTO- AND ELECTRO-OPTICAL MODULATION OF LIGHT https://doi.org/10.1070/QEL17023

Formation of light with controlled rotation of the polarisation plane by superposing linearly polarised waves

V.M. Kotov

Abstract. **A method is proposed for generating linearly polarised light, whose polarisation plane rotates at a given frequency. The method is based on the double passage of the light through an acousto-optic modulator made of an anisotropic crystal. An experimental prototype based on a paratellurite crystal is fabricated, with the help of which rotation of linearly polarised light of a He–Ne laser ($\lambda = 0.63$ μm) is obtained with the frequency controlled by the frequency of sound.**

Keywords: *acousto-optic diffraction, Bragg regime, rotation of the polarisation plane.*

1. Introduction

Light with controlled rotation of the polarisation plane is used in such fields of science and technology as ellipsometry [1], anemometry [2], as well as measuring the characteristics of anisotropic optical fibres [3], determining the orientation angle of optical insulators, thicknesses and refractive indices of thin films [4], etc. The most promising method for producing a light beam with a controlled rotation of the polarisation plane is the interference of two monochromatic waves with different frequencies. If the waves are circularly polarised in opposite directions, then their superposition yields a rotating linearly polarised wave, the rotation frequency of which is equal to half the frequency difference between the component waves [5, 6].

In Refs [7–9], acousto-optic (AO) Bragg diffraction in a gyrotropic crystal was used to rotate the plane of polarisation. The eigenwaves of such a crystal are circularly polarised, and the frequency difference between them appears due to AO coupling. Paratellurite (TeO_2), which exhibits an anomalously high AO quality factor M_2 of the material in the case of light diffraction by a 'slow' transverse acoustic wave [10] was used as the AO crystal. However, strictly speaking, the TeO_2 eigenwaves are circularly polarised only when light propagates along its optical axis. When the light deviates from the optical axis, the polarisations of the waves become elliptical, gradually turning into linear ones.

In this paper, we propose a method that allows controlled rotation of the polarisation plane by superposing linearly polarised waves. This significantly extends the potentialities of using a TeO_2 crystal, and widens the range of choice of crystals that can solve this problem. The device, developed basing on the proposed method, allows minimisation of optical losses, provides modulation of light intensity at a double acoustic frequency and is quite simple to implement.

V.M. Kotov Kotelnikov Institute of Radio Engineering and Electronics (Fryazino Branch), Russian Academy of Sciences, pl. Akad. Vvedenskogo 1, 141190 Fryazino, Moscow region, Russia;
e-mail: vmk277@ire216.msk.su

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2. Theory

Figure 1 shows the vector diagram of the AO interaction, which underlies the proposed method. The linearly polarised light with the wave vector $\boldsymbol{K}_0$ is incident on the xy face of a uniaxial crystal (assumed positive for definiteness) and split into two linearly polarised eigenwaves with wave vectors $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$. We assume that the polarisation of the incident light is oriented in such a way that the amplitudes of the waves $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ are equal. In Fig. 1, the z axis is chosen along the optical axis of the crystal and the xy face is orthogonal to it. According to Snell's law, the projections of the vectors $\boldsymbol{K}_0$, $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ onto the xy face are equal in absolute value: the projection of $\boldsymbol{K}_0$ is equal to $-X_1$, and the projections of $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ are equal to X_1. The wave surfaces of the crystal for ordinary and extraordinary waves are denoted by 1 and 2. In our case, the wave $\boldsymbol{K}_1$ is ordinary, its polarisation is orthogonal to the plane of the figure, and the wave $\boldsymbol{K}_2$ is extraordinary, its polarisation lying in the plane of the figure [11]. An acoustic wave propagates in the crystal with the wave vector $\boldsymbol{q}$ orthogonal to the yz plane. The conditions for AO interaction were chosen such that only the extraordinary wave $\boldsymbol{K}_2$ was involved

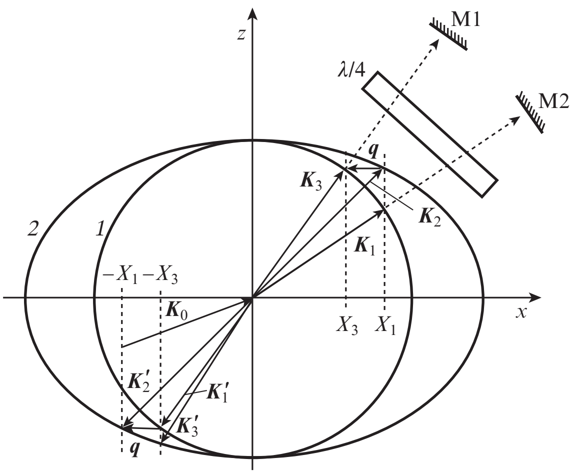

Figure 1. Vector diagram of AO interaction.

in the diffraction. The diffracted wave is ordinary and has the wave vector $\boldsymbol{K}_3$. Anisotropic diffraction of light by sound occurs. The projection of the vector $\boldsymbol{K}_3$ onto the xy face is denoted by X_3. Obviously, $X_1–X_3 = |q|$, where $|q|$ is the magnitude of the wave vector $\boldsymbol{q}$.

For theoretical consideration, we assume that a 100 % diffraction efficiency is realised, i.e., the wave $\boldsymbol{K}_2$ is completely diffracted into the wave $\boldsymbol{K}_3$. In this case, two ordinary waves $\boldsymbol{K}_1$ and $\boldsymbol{K}_3$ will propagate at the crystal output. After the crystal, a quarter-wave plate $\lambda/4$ and two mirrors M1 and M2 are installed, reflecting the rays in the strictly opposite direction. The $\lambda/4$ plate converts linear polarisations into circular ones, which after reflection from the mirrors remain circular, but with the opposite direction of rotation of the polarisation vector. Passing back through the $\lambda/4$ plate, the reflected rays will again acquire linear polarisations, but rotated by 90° relative to the polarisations of the initial rays $\boldsymbol{K}_1$ and $\boldsymbol{K}_3$ [12]. Thus, the polarisations of the rays propagating through the crystal in the opposite direction after reflection from the mirrors correspond to the polarisations of the extraordinary waves of the crystal. The reflected wave $\boldsymbol{K}_1$ propagates in the direction of the wave $\boldsymbol{K}_2'$, and the wave $\boldsymbol{K}_3$ propagates in the direction $\boldsymbol{K}_3'$. The projections of $\boldsymbol{K}_2'$ and $\boldsymbol{K}_3'$ on the xy face are $-X_1$ and $-X_3$, respectively. Only the extraordinary wave $\boldsymbol{K}_2'$ is involved in the re-diffraction, it diffracts into the ordinary wave $\boldsymbol{K}_1'$. The projection of $\boldsymbol{K}_1'$ on the xy face is equal to $-X_3$ and coincides with the projection of the wave vector $\boldsymbol{K}_3'$. The output waves are the waves $\boldsymbol{K}_1'$ and $\boldsymbol{K}_3'$, whose polarisations are mutually orthogonal, they are superposed at the output of the crystal to form a single radiation beam. The advantage of the method is that the direction of the input wave does not coincide with the direction of propagation of the output rays $\boldsymbol{K}_1'$, $\boldsymbol{K}_3'$ (their projections onto the xy face are different), i.e., there is an angular separation between the input and output rays. In addition, it is easy to see that in the case of 100 % diffraction efficiency the input wave is fully converted into the output light $\boldsymbol{K}_1'$, $\boldsymbol{K}_3'$.

Now we determine the frequencies of the waves $\boldsymbol{K}_1'$ and $\boldsymbol{K}_3'$. Let ω be the angular frequency of the incident wave . Then the frequencies of the split waves $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ are also equal to ω, and the frequency of the wave $\boldsymbol{K}_3$, as follows from Fig. 1, is equal to $\omega + \Omega$, where Ω is the angular frequency of sound. The diffraction occurs with increasing frequency. The frequency of the output wave $\boldsymbol{K}_3'$ is equal to the frequency of the wave $\boldsymbol{K}_3$, i.e., $\omega + \Omega$. The frequency of the wave $\boldsymbol{K}_1'$ formed as a result of diffraction of the wave $\boldsymbol{K}_2'$ by the same sound wave $\boldsymbol{q}$ is equal to $\omega - \Omega$; here the diffraction occurs with decreasing frequency. Thus, two waves with linear mutually orthogonal polarisations and frequencies $\omega + \Omega$, $\omega - \Omega$ are formed at the output of the device. If the $\lambda/4$ plate is placed in the path of these waves, then a linearly polarised wave is formed after the plate, whose plane of polarisation rotates with the angular frequency Ω.

Consider this issue in more detail. Figure 2 shows the $\lambda/4$ plate through which two linearly polarised waves propagate orthogonally to the plane of the figure. The polarisation corresponding to the propagation of the 'fast' wave through the plate material is directed along the x axis, and that of the 'slow' one along the y axis. Let the wave $\boldsymbol{K}_1'$ be incident onto the plate, the electric field of which is described by the expression $\boldsymbol{E}_1 = \boldsymbol{e}_1 E_1 \cos(\omega_1 t)$, where $\boldsymbol{e}_1$ is the unit vector directed along the field $\boldsymbol{E}_1$; E_1 is the field amplitude; ω_1 is its frequency; and t is time. Assume that the vector $\boldsymbol{e}_1$ is oriented at an angle of 45° to the x and y axes. Then the fields along these directions at the exit from the plate are described by the expressions:

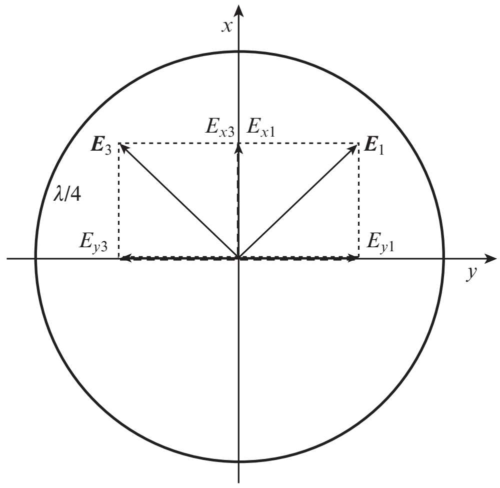

Figure 2. Passage of two optical waves through a $\lambda/4$ plate.

$$E_{x1} = \frac{E_1}{\sqrt{2}}\cos(\omega_1 t) \qquad E_{y1} = \frac{E_1}{\sqrt{2}}\cos(\omega_1 t), \tag{1}$$

i.e., a circularly polarised wave with right-handed rotation of the polarisation vector is formed at the output of the plate [11]. Let the wave $\boldsymbol{K}_3'$ be incident onto the same plate, whose electric field $\boldsymbol{E}_3 = \boldsymbol{e}_3 E_3 \cos(\omega_3 t)$ is oriented orthogonally to the field $\boldsymbol{E}_1$, and the field components of this wave along x and y at the output of the plate are expressed as

$$E_{x3} = \frac{E_3}{\sqrt{2}}\cos(\omega_3 t) \qquad E_{y3} = -\frac{E_3}{\sqrt{2}}\sin(\omega_3 t). \tag{2}$$

In this case, a circularly polarised wave is also formed, but with a left-handed rotation. Thus, two circularly polarised waves with right- and left-handed rotation and the frequencies ω_1 и ω_3, are superposed at the output of the plate. The resulting field will be a linearly polarised wave rotating with the frequency $(\omega_1 - \omega_3)/2$ [5]. Indeed, adding to each other the fields formed along the x and y directions (assuming that $E_1 = E_3 = E_0$), we obtain

$$\begin{aligned} E_x &= E_{x1} + E_{x3} = \frac{E_0}{\sqrt{2}}[\cos(\omega_1 t) + \cos(\omega_3 t)], \\ E_y &= E_{y1} + E_{y3} = \frac{E_0}{\sqrt{2}}[\sin(\omega_1 t) - \sin(\omega_3 t)]. \end{aligned} \tag{3}$$

After simple transformations, we have

$$\begin{aligned} E_x &= \sqrt{2}\,E_0 \sin\left[\frac{(\omega_1 - \omega_3)}{2}t\right]\cos\left[\frac{(\omega_1 + \omega_3)}{2}t\right], \\ E_y &= \sqrt{2}\,E_0 \sin\left[\frac{(\omega_1 - \omega_3)}{2}t\right]\cos\left[\frac{(\omega_1 + \omega_3)}{2}t\right]. \end{aligned} \tag{4}$$

From Eqns (4), it is seen that the fields oscillate with a frequency of $(\omega_1 + \omega_3)/2$, while their amplitudes

$$\sqrt{2}\,E_0 \cos\left[\frac{(\omega_1 - \omega_3)}{2}t\right], \ \sqrt{2}\sin E_0 \sin\left[\frac{(\omega_1 - \omega_3)}{2}t\right]$$

vary in the same way as the field components of a circularly polarised wave [see Eqn (1)], but the frequency of the field variation in this case is $(\omega_1 - \omega_3)/2$. Since $\omega_1 = \omega - \Omega$ and $\omega_3 = \omega + \Omega$, then $(\omega_1 - \omega_3)/2 = \Omega$. In other words, at the output of the device, linearly polarised light with the polarisation plane rotating with a frequency of Ω is obtained.

3. Experiment and discussion of results

Figure 3 shows an optical scheme for the implementation of the proposed method. The light generated by the laser is reflected from the mirror M1, passes through the polariser P1 and the AO modulator AOM, to the input of which an electric signal of frequency f from the generator is fed. The TeO_2 crystal with linearly polarised eigenwaves was chosen as the AO material of the modulator. The crystal size is 10 × 8 × 10 mm along the directions [110], [110] and [001], respectively. The direction [001] coincides with the direction of the optical axis z in Fig. 1, the direction [110] coincides with that of the x axis. A transverse travelling acoustic wave with the frequency $f \approx$ 40.04 MHz propagates along the [110] direction. The velocity of sound is 0.617×10^5 cm s^{-1}. Linearly polarised laser light with $\lambda = 0.63$ μm is incident on the optical face (001) of the crystal at an angle of $\sim 40°$ (the far-off-axis diffraction mode is used [10]). It is known [13] that the ellipticity ρ of the eigenwaves in a uniaxial gyrotropic crystal is determined by the expression

$$\rho = \frac{1}{2G_{33}} \times \sqrt{\tan^4\theta(n_e^{-2} - n_o^{-2})^2 + 4G_{33}^2} - \tan^2\theta(n_e^{-2} - n_o^{-2}), \quad (5)$$

where G_{33} is a component of the gyration pseudotensor; n_o and n_e are the principal refractive indices of the crystal; and θ is the angle between the wave vector of light and the optical axis z. For the light wavelength $\lambda = 0.63$ μm, the parameters of the TeO_2 crystal that enter Eqn (5) are as follows: $G_{33} = 2.62 \times 10^{-5}$; $n_o = 2.26$; $n_e = 2.41$; and the angle θ inside the crystal is $\sim 17°$. With these parameters, $\rho = 0.02$, i.e. the crystal eigenwaves are linearly polarised. The orientation of the polariser P1 ensured equal intensities of the eigenwaves.

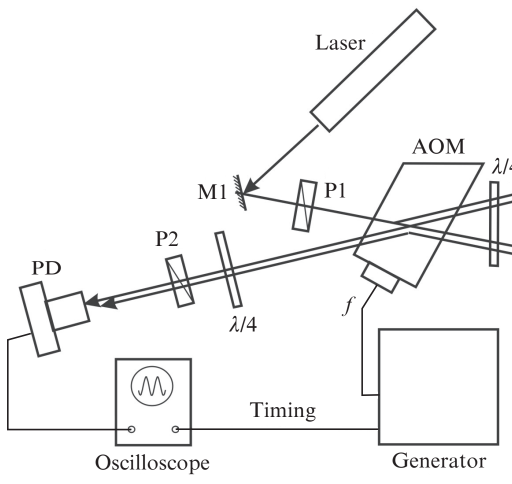

Figure 3. Optical scheme of the device.

The angle of light incidence and the frequency of sound are chosen to satisfy the Bragg phase-matching condition only for the extraordinary wave. The beams passing through the AO modulator are passed through the $\lambda/4$ plate and reflected from the mirrors M2 and M3 in the strictly opposite direction. After repeated passage through the AO modulator, two output beams are formed, propagating collinearly to each other. The presence of two beams is easily checked by blocking the mirrors M2 and M3 alternately. Beams emerging from the device are passed through another $\lambda/4$ plate, a polariser P2, and sent to the photodetector PD. The rotation frequency of the polarisation plane is f. However, since the photodetector operates in quadratic mode, the frequency of the measured signal is $2f$. Signals from the photodetector arrive at the oscilloscope. The reference signal from the generator is supplied to the same oscilloscope. To verify the fact of rotation of the polarisation plane, a polariser P2 is used.

Figure 4 shows the waveforms of the measured signals. The upper one corresponds to the signal taken from the photodetector, the lower one is the reference signal supplied from the generator. The frequency of the reference signal is 40.04 MHz, the frequency of the signal from the photodetector is twice as high. When the polariser P2 rotates, the signal from the photodetector shifts in the horizontal direction relative to the reference signal, while the amplitude of the signal from the photodetector does not change.

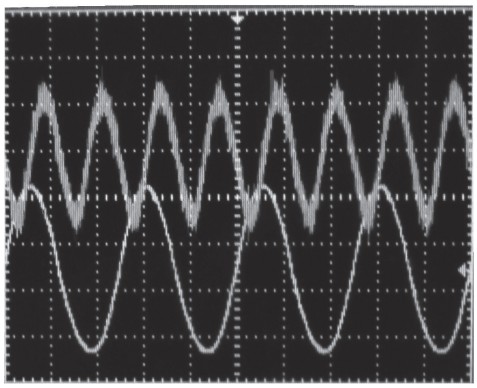

Figure 4. Oscillograms of electrical signals: the upper one is the signal from the photodetector, the division value is 1.0 mV; the lower one is the signal from the generator, the division value is 50 mV. The time scale of scanning is 10 ns div^{-1}.

Figure 5 shows the spectrogram of the signal taken from the photodetector and measured by the spectrum analyser. It can be seen that the centre frequency of the signal is 80.1 MHz, which is twice the frequency of the acoustic wave. The modulation depth of the signal from the photodetector shown in Fig. 4 is 20%. We associate a small depth of modulation, first, with the difference of the profiles of the interfering beams. The latter is due to the inhomogeneity of the diffracted field caused by two simultaneously acting factors, namely, the refraction of a Gaussian beam at the air–medium interface and the transfer of the angular spectrum of the beam during AO interaction [14, 15]. It was shown in Ref. [14] that the inhomogeneity of the diffracted beam field is enhanced with increasing angle of incidence and can reach 20% or more. To reduce this effect, crystal cuts can be used for which the angles of incidence of the rays on the optical face would be minimal.

Factors such as incomplete spatial overlap of superposed optical beams at the output of the device, uneven sensitivity of the photodetector area, etc., lead to a decrease in the modulation depth. In this work, the goal was not to obtain the optimal characteristics of the output light; the aim of the work was to confirm reliably the existing effect, which offers new possibilities for using AO diffraction.

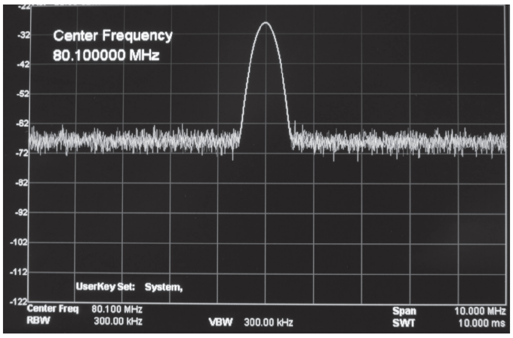

Figure 5. The frequency spectrum of the signal from the photodetector. The centre frequency of the signal is 80.1 MHz, the division value of the spectrogram is 1 MHz.

The main results of the work can be formulated as follows:

1. A method is proposed for generating linearly polarised optical light with a rotating plane of polarisation, based on the double passage of the light through an AO modulator made of a crystal whose eigenwaves are linearly polarised.

2. A prototype of a device based on TeO_2 AO crystal in the geometry with the linearly polarised eigenwaves was manufactured and tested. Using the device, the linearly polarised radiation of a He–Ne laser ($\lambda = 0.63$ μm) was converted into the light with a rotating plane of polarisation, the rotation frequency being controlled by the frequency of the sound wave. The results can be applied in various systems, where it is necessary to use linearly polarised radiation with the plane of polarisation rotating with a controlled frequency.

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