In the paper we show that the boundaries of existence of limit cycles corresponding to in-phase and anti-phase activity are dangerous [2]. On these boundaries asymptotically stable limit cycles lose its stability due to subcritical bifurcations and principally different type of activity (chaotic bursting or chaotic spiking) appears. Strange attractors corresponding to both chaotic bursting and chaotic spiking are not structurally stable. In the parameter space of the system regions with chaotic activity alternates with the so-called stability windows in which asymptotically stable limit cycles appear.

We also show that stability windows inside regions of chaotic bursting activity correspond to the regular bursting activity, while in the stability windows inside chaotic spiking activity regular sequential regimes of various types appear.

Acknowledgements

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Hyperbolic Chaos in Coupled FitzHugh-Nagumo Model Neurons with Alternating Excitation of Relaxation Self-Oscillations

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The aim of the work is to show that in a simple model of two coupled FitzHugh-Nagumo neurons [1], under appropriate control by an external action providing al-ternating self-oscillations in the subsystems it is possible to accomplish the hyperbol-ic chaos corresponding to Smale-Williams solenoids [2].
Consider a model described by the following non-autonomous system of differ-ential equations:

$$\begin{aligned}
\dot{x} &= f\left(t/T+1/4\right)x-\tfrac{1}{3}x^3-u+\varepsilon(y-x),\\
\dot{u} &= ax-bu+I,\\
\dot{y} &= f\left(t/T-1/4\right)y-\tfrac{1}{3}y^3-v+\varepsilon(x-y),\\
\dot{v} &= ay-bv+I,
\end{aligned} \tag{1}$$

where the variables (x, u) relate to one and (y, v) to another FitzHugh-Nagumo sub-system; parameters a, b, I are assumed to be constant, ε is the coupling coefficient, and T is the modulation period. The modulation is described by a function f, which satisfies $f(\tau+1)=f(\tau)$, and on a single period it is defined by the relations

$$f(\tau)=\left\{A,\,0<\tau\le\tau_1;\ \frac{(A-C)\tau+C\tau_1-A\tau_2}{\tau_1-\tau_2},\,\tau_1<\tau\le\tau_2;\ \frac{(C-A)\tau+A\tau_2-C}{\tau_2-1},\,\tau_2<\tau\le1\right.$$

Bearing in mind that with growth of the value f, the frequency of self-oscillations of the FitzHugh-Nagumo oscillator decreases, parameter A can be chosen so that in the oscillation stage of large amplitude the basic frequency to be smaller than the fre-quency of linear oscillations by an integer .
Let us consider operation of the system in the regime with a hyperbolic attractor. We start with situation when one oscillator performs self-oscillation, and the second is inhibited. When the stage of excitation of the second subsystem comes, it begins to oscillate in a resonant manner due to the action of the M-th harmonic component of the oscillations of the first oscillator. Further, the process repeats again and again with a change in the roles of one and the other

subsystem. Each cycle of transmission of excitation is accompanied by transformation of the phase described by the ex-panding circle map . With compression along remaining directions in the state space, this corresponds to occur-rence of a Smale-Williams solenoid as at-tractor in the stroboscopic Poincaré map of the system.

In Fig. 1 panels (a), (b), (c) illustrate the transformation of the phases of oscilla-tions in a half-period of modulation, in three distinct modes corresponding to attrac-tors representing different topological types of the Smale-Williams sole-noids (respec-tively, with doubling, tripling and quadrupling of the number of turns at each step of the construction). In the first case, the largest Lyapunov exponent of the map for the period of modulation is close to ln2, and in the sec-ond – to ln3, and in the third – to ln4, i.e. to the values corresponding to the double, triple, and quadruple expanding circle map. Figure 2 shows a graph of the highest Lyapunov exponent versus the pa-rameter at larger modulation period. There one can observe plateaus corresponding to regions of existence of the attractors with multiplicity of the number of turns, re-spectively, 2, 3, 4, 5.

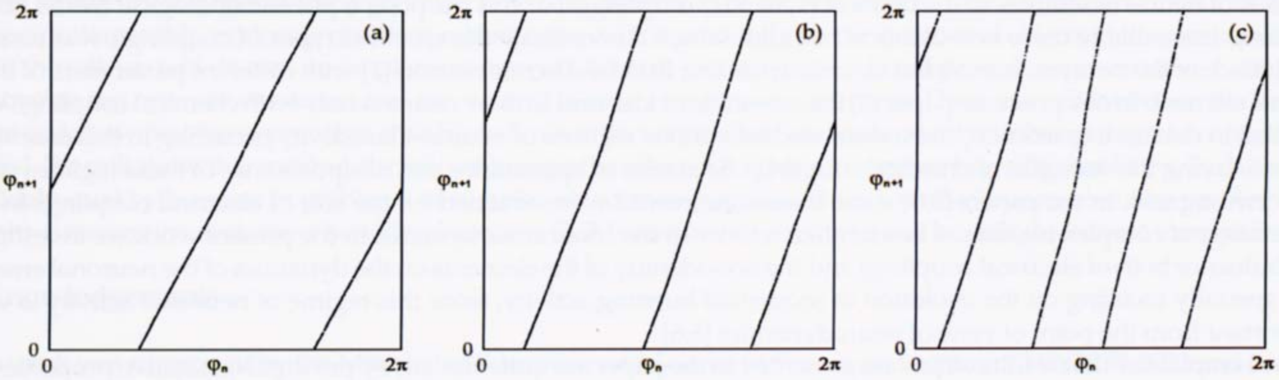

Fig.1. *The maps for the phase in half-period of modulation for A=5.49 (a), A=8 (b), and A=14 (c), where the Lyapunov exponent for the half-period is close, respectively, to ln2, ln3, and ln4, respectively. The remaining parameters are a=1, b=0, I=0.5, ε=0.01, T=400, τ1=0.4, τ2=0.5.*

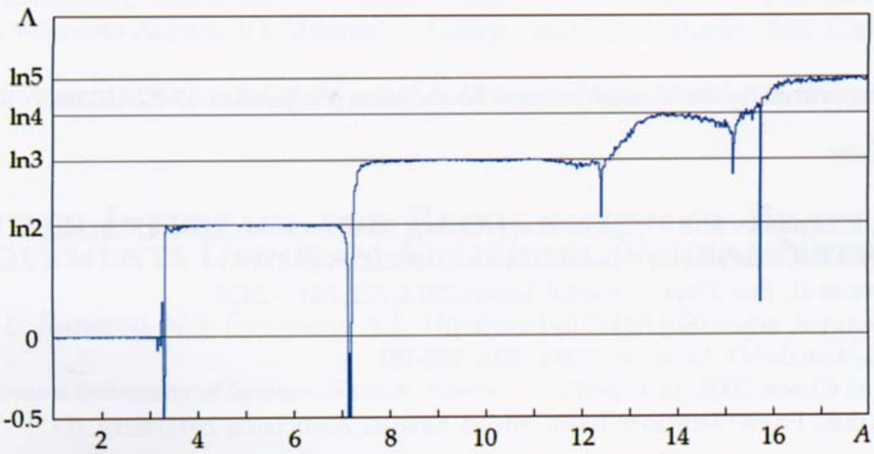

Fig.2. *Parameter dependence of the highest Lyapunov exponent of the mapping for half-period of the modulation at a=1, b=0, K=0.5, C=-2, ε=0.01, I=0.5, T=400, τ1=0.4, τ2=0.5.*

Our results testify to a possibility for systems of alternately excited coupled neurons to manifest hyperbolic chaos, char-acterized by the property of structural stability. This means a low sensitivity of chaos to variations of parameters, noises, interferences, which can be interesting both for understanding the functional capabilities of systems studied in neurody-namics, and for implementation of their technical counterparts.

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