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Generation of Coupled Modes in an Unmatched Three-Mirror Laser Cavity

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An extraordinary phenomenon of generation of coupled types of oscillations with a transverse field distribution corresponding to the TEM_{pq} eigenmodes with different pairs of indices p and q in the main laser cavity and in an external cavity has been experimentally detected. The excitation of coupled modes depends on the configuration and tuning of partial cavities. To solve the system of integral equations for a three-mirror cavity with mismatched spherical mirrors in the quasioptical approximation, it has been proposed to use modified boundary conditions including the coupling coefficients for eigenmodes in partial cavities.

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1. INTRODUCTION

An optical laser system including a three-mirror (generally multimirror) cavity is a fundamental electrodynamic model of any device employing coherent laser radiation. Any optical element placed in the path of a laser beam inside or outside of the main cavity of a laser is a source of reflected or scattered radiation and forms, together with the mirrors of the laser, a three-mirror cavity.

Theoretical and experimental studies of three-mirror optical cavities began almost immediately after the creation of the first cw gas lasers. However, since the strict analytical solution of this physical problem is complicated, electromagnetic fields in such optical system demonstrate an unusual behavior, and this optical system has a great applied significance, studies in this field continue to date. These works were stimulated by the necessity of the development of efficient methods for selection of types of oscillations present in laser radiation [1], by the possibility of development of active three-mirror laser interferometers [2, 3], and by the necessity of the determination of the energy and frequency characteristics of these devices [4–6]. Further investigation of the three-mirror optical system made it possible to create original devices for increasing the stability of the frequency of lasers [7, 8] and to develop new principles and methods of laser interferometric measurements, in particular, heterodyne frequency-modulated [9] and homodyne (self-mixing) interferometers [10, 11].

A linear cavity including three plane mirrors, one of which is partially transmitting, was used as the main physical model in cited works [2–11]. Thus, theoretical consideration was restricted to the solution of the one-dimensional problem in the approximation of plane waves with different frequencies. However, real experiments involve optical laser devices including spherical (cylindrical) mirrors and lenses and their description requires the solution of three-dimensional (two-dimensional) equations of electrodynamics. It was shown in [12] that the degree of matching of the optical elements affecting the frequency and spatial distributions of wave fields is an important characteristic of such systems. A necessary condition for the generation of stable oscillations of the electromagnetic field in a three-mirror system is the matching of transverse dimensions of spots of wave beams on each of three mirrors [1, 12]. If the beams are mismatched, losses to radiation appear in a three-mirror cavity including perfect mirrors and the wave equation does not have solutions in the form of undamped oscillations in such a system. However, the generation of undamped oscillations becomes possible if the system is supplemented by an active element, e.g., a gas-discharge lasing cell ensuring the compensation of these losses.

The aim of this work is to experimentally and theoretically study resonance conditions and features of the formation of electromagnetic wave beams with various spatial field distribution in a mismatched three-mirror laser cavity.

2. THREE-MIRROR CAVITY WITH A MISMATCHED EXTERNAL MIRROR

We consider a three-mirror laser cavity including three partially transmitting spherical (cylindrical in

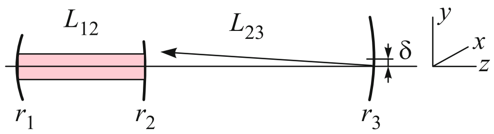

Fig. 1. (Color online) Three-mirror laser cavity $r_1r_2r_3$ including the mirrors r_1 and r_2 of the main laser cavity and mismatched external mirror r_3. An active lasing medium is placed between mirrors r_1 and r_2 and mirror r_3 is inclined or shifted in the transverse direction by δ.

the two-dimensional case) mirrors r_1, r_2, andr_3. Mirrors r_1 and r_2 spaced at a distance of L_{12} together with an active lasing medium between them form the main cavity of the laser. The third mirror r_3 placed at a distance of L_{23} from the main cavity r_1r_2 is mismatched with it. The mismatching of the partial cavities r_1r_2 and r_2r_3 means that the phase front of the beam that is formed by the cavity r_1r_2 and is emitted by the laser does not coincide with the surface of the mirror r_3. This mismatching can be due to two reasons (Fig. 1).

First, the radius of the phase front of the beam incident on the mirror r_3 can differ from the radius of curvature of the mirror r_3. In this case, noncoaxial beams outgoing to infinity in the transverse direction exist in partial cavities r_2r_3 and r_1r_3 and the emission of these beams is responsible for additional losses in such a laser system [12].

Second, the optical axis of the spherical (parabolic) mirror r_3 can differ from the optical axis of the main cavity r_1r_2 (the mirror r_3 is inclined or shifted in the transverse direction by δ). This also leads to additional losses to radiation.

If the indicated losses can be compensated, e.g., by the active lasing medium in the main cavity r_1r_2, then undamped oscillations at frequencies of the longitudinal and transverse eigenmodes of individual partial cavities r_1r_2, r_2r_3, and r_1r_3 should be expected in the mismatched three-mirror system $r_1r_2r_3$.

Indeed, in the approximation of infinite spherical mirrors, the system of integral equations describing the behavior of the electromagnetic field U in the three-mirror cavity can be represented in the form [12]

$$\frac{1}{r_n} f = a_{nl} f_l, \quad n, l = 1, 2, 3, \tag{1}$$

where r_n are the complex coefficients of reflection of mirrors, f_n are the currents in mirrors proportional to the scalar potential of the field U (e.g., the y component of the electric field) satisfying the wave equation, and A_{nl} is the matrix of kernels of integral operators corresponding to the Green's function of free space in the quasioptical approximation [12, 13].

Without loss of generality, the solution of the system of Eqs. (1) on the surface of the mirrors can be sought in the form of current proportional to the field distribution in Gaussian beams of the zeroth order,

$$F_{n0}(x, y) = \beta_{n0} \exp\left(-\frac{\alpha_n k(x^2 + y^2)}{2}\right), \tag{2}$$

and higher orders,

$$F_n^{pq}(x, y) = \beta_n^{pq} H_p(\chi_n x) H_q(\chi_n y) \times \exp\left(-\frac{\alpha_n k(x^2 + y^2)}{2}\right), \tag{3}$$

where k is the wavenumber; H_p and H_q are the Hermite polynomials of the orders p and q, respectively (p and q are integers denoting the orders of the transverse modes); α_n and β_n^{pq} are quantities to be determined; and $\chi_n = \sqrt{\alpha_n k}$ are the normalization coefficients.

3. ANALYSIS OF THE SOLUTION OF THE SYSTEM OF INTEGRAL EQUATIONS

Within the scheme of solving the system of integral equations describe in [12, 13], it is possible to show that the solution in the form of Gaussian beams of the zeroth order (2) and higher orders (3) exists if the conditions of matching of the spot sizes and phase fronts on mirrors are satisfied in all three partial two-mirror cavities r_1r_2, r_2r_3, and r_1r_3.

If the partial cavities are mismatched but the gain of the active medium in the laser cavity r_1r_2 is high enough, "partial" solutions of the system of Eqs. (1) are allowed in each cavity, e.g., the excitation of the zero-order Gaussian beam (2) in the main cavity r_1r_2 and higher order beams (3) in the external cavity r_2r_3. The stability of generation of such coupled oscillations can be achieved through their additional selection [1], e.g., by placing a diaphragm in the main cavity r_1r_2. To satisfy the boundary conditions (i.e., for the "matching" of fields and their normal derivatives) on the surface of the mirror r_2, it was proposed to use coupling coefficients [14] describing the conversion of the energy of the fundamental mode formed in the cavity r_1r_2 to the higher eigenmodes formed in the mismatched partial cavity r_2r_3.

The coupling coefficient between the zeroth-order mode (2) in the main cavity r_1r_2 and the pq-order mode (3) in the external cavity r_2r_3, e.g., at the transverse displacement of its optical axis along the y axis (see Fig. 1) from the cavity r_1r_2 by δ has the form

$$C_0^{pq}(\delta) = \int_{-\infty}^{\infty} H_p(\chi_2 x)H_q(\chi_2 y) \times \exp\left(-\frac{\alpha_2^{pq}k(x^2+(y-\delta)^2)}{2} - \frac{\alpha_2^0 k(x^2+y^2)}{2}\right)dxdy. \quad (4)$$

It is assumed that the spot sizes, which are inversely proportional to the coefficients α_2^0 and α_2^{pq}, can be different for the indicated modes. Thus, the boundary condition of continuity of the field U on the surface of the mirror r_2 [12] can be modified to the form

$$U_2^+ = C_0^{pq}(\delta)U_2^-, \quad (5)$$

where U_2 is the potential (y component of the electric field) on the mirror r_2, which is proportional to the current on the mirror r_2 [12, 13], and $U_2^\pm$ are the fields on the left and right of the surface of this mirror. Taking into account the current f_2 on the mirror r_2, the boundary condition corresponding to the discontinuity of field derivatives on the surface of this mirror can be written in the form

$$\frac{\partial U_2^+}{\partial(kN)} - C_0^{pq}\frac{\partial U_2^-}{\partial(kN)} = f_2, \quad (6)$$

where $\frac{\partial U}{\partial(kN)}$ is the derivative with respect to the normal to the surface of the mirror r_2.

Using the boundary conditions given by Eqs. (5) and (6) and following the scheme [12] of the solution of the system of integral equations (1), one can obtain a dispersion equation for determination of frequencies of eigenmodes excited in the studied three-mirror cavity.

In particular, in the approximation of infinite plane mirrors, this equation has the standard form [1, 12, 13]

$$r_1r_{23}\exp(-2ikL_{12}) = 1, \quad (7)$$

where

$$r_{23} = r_2 - \frac{t_2(1+r_2)r_3C_0^{pq}\exp(-2ikL_{23})}{1-r_2r_3\exp(-2ikL_{23})} \quad (8)$$

is the complex reflection coefficient of the composite mirror r_2r_3; L_{12} and L_{23} are the lengths of the corresponding partial cavities; t_2 and r_2 are the transmission and reflection coefficients of the mirror, respectively, which are related by the continuity relation $t_2 = (1+r_2)C_0^{pq}$ [12] and satisfy the energy conservation law $|r_2|^2 + |t_2|^2 = 1$; and i is the imaginary unit. The factor C_0^{pq} appears in the continuity relation because this coupling coefficient given by Eq. (4) determines the fraction of the energy of the fundamental mode (with zeroth index) generated in the cavity r_1r_2 of the laser that is transferred to modes of higher orders pq excited in the external cavity r_2r_3. The same coefficient determines the fraction of the energy of the modes with indices pq that returns to the main cavity of the laser. In other words, Eqs. (7) and (8) are equivalent to the dispersion equation of the two-mirror cavity whose second mirror has the reflection coefficient that not only depends on the frequency k but also includes the coupling coefficient of cavities C_0^{pq}.

If the partial cavities r_1r_2 and r_2r_3 have high Q factors ($|r_1| \sim |r_2| \sim |r_3| \sim 1$) and the mirror r_3 is mismatched with the main cavity r_1r_2 in the above sense, the coupling between these cavities is weak, and the fields of eigenmodes in the cavities are formed almost independently. In particular, if the TEM_{00} fundamental mode of the zero-order Gaussian beam (2) is generated in the main laser cavity r_1r_2, TEM_{01}, TEM_{02}, etc., eigenmodes (3) can be generated at a certain tuning of the mirror r_3 in the external cavity r_2r_3. An obvious condition for the excitation of coupled types of oscillations is the coincidence of the eigenfrequencies of coupled partial cavities.

4. EXPERIMENTAL STUDY OF THE GENERATION OF COUPLED MODES

Such an unusual regime of generation of eigenmodes in the three-mirror laser cavity was detected in our experiment at a setup in two modifications schematically shown in Fig. 2.

The main active cavity is formed by the mirrors M_1 and M_2 of a gas-discharge helium–neon laser with a length of L_{12}. The third mirror M_3 is located at a distance of L_{23} from the laser. Focusing lens F can be

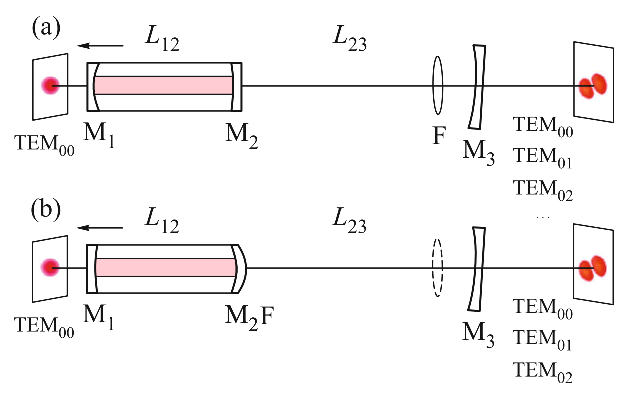

Fig. 2. (Color online) Schematics of two modifications of an experimental setup for the observation of generation of coupled modes: (M_1, M_2, M_3) mirrors forming the three-mirror cavity and (F) focusing lens.

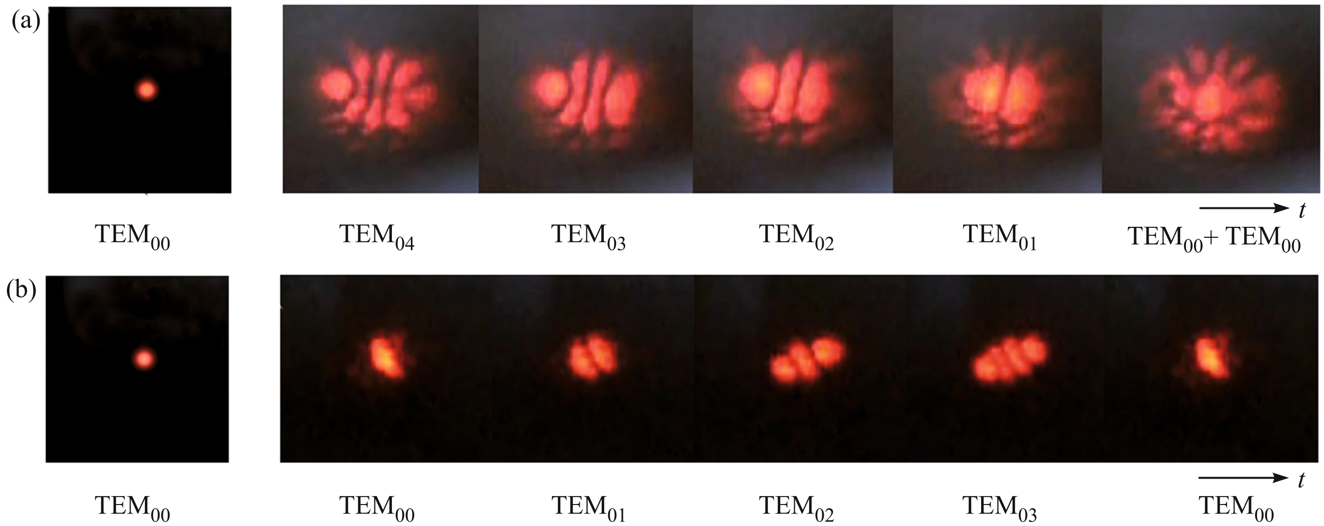

Fig. 3. (Color online) Photographs of field distributions at different times (left panels) at the output of the main laser cavity and (right panels) at the output of the external cavity (a) for the external cavity consisting of plane and spherical mirrors and (b) for the case where lens F is placed between mirrors M_2 and M_3.

placed between the mirrors M_2 and M_3. All three mirrors have a dielectric coating, are partially transmitting, and have the power reflection coefficients within the range $|r_{1,2,3}|^2 \sim 0.98–0.998$. Lasers with internal mirrors and the length of gas-discharge tubes of 110–230 mm are used. Brewster plates ensuring 633-nm plane polarized radiation at the output of the lasers are placed inside the tubes. The radii of curvature of spherical mirrors M_1 (Fig. 2a) and M_2 (Fig. 2b) are 0.3–0.5 m. Thus, the cavities formed by these mirrors together with plane mirrors M_2 and M_1, respectively, are nearly confocal. The third spherical mirror M_3 (radius of curvature of about 100 m) together with the lens F (focal length of about 1 m) makes it possible to significantly vary the parameter of nonconfocality of the external cavity by varying the length L_{23} in the range of 0.5–1 m. To modify the scheme of the experiment shown in Fig. 2b, the spherical mirror M_2 is combined with the lens F whose focal length is equal to the radius of curvature of the mirror M_2, which ensures the plane front of the wave leaving the laser. The optical element M_3F for the wave incident from the side of the mirror M_3 is equivalent to a plane mirror. The mirror M_3 is equipped with a two-coordinate angular tuning system, which allows varying the tilt angle of the mirror and the degree of mismatching of the main and external cavities. The output radiation on the left and right of the three-mirror cavity M_1–M_2–M_3 is observed on white screens and is recorded by a photo camera.

The results of observation of the generation of coupled TEM_{00} and TEM_{pq} modes (p, $q = 0, 1, 2, ...$) in the described laser system are shown in Figs. 3a and 3b. The left panels show the field distribution in the cross section at the output of the main laser cavity on the side of the mirror M_1. A series of right photographs show the time sequence of the field distribution at the output of the external cavity on the side of the mirror M_3 for the stable generation states. Such states were observed for the following pairs of coupled modes: TEM_{00}–TEM_{01}, TEM_{00}–TEM_{02}, ..., TEM_{00}–TEM_{24}. The successive variation of the generation states in time is due to the rearrangement of the frequency of the main laser cavity because of its heating after the switching-on of the pump source.

It is remarkable that the transition from one bound state to another state is characterized by change in the index q by unity in the external cavity M_2–M_3, whereas the field distribution in the main cavity M_1–M_2 does not change ($p = q = 0$). The sequence of transitions from states with a larger index q to that with a smaller index (Fig. 3a) is changed to the opposite sequence, i.e., from a smaller index q to a larger index (Fig. 3b) at the variation of the parameter of nonconfocality of the external cavity. This phenomenon can be explained by comparing the eigenfrequencies of the active laser cavity M_1–M_2 and passive external cavity M_2–M_3, which are specified by the solutions of the dispersion equations (7) and (8) for the corresponding two-mirror cavities.

We use the solution determining the eigenfrequency of the generated mode with the transverse indices p and q given by Eq. (3) in the cavities M_1–M_2 and M_2–M_3 (see Fig. 2), where one mirror is plane

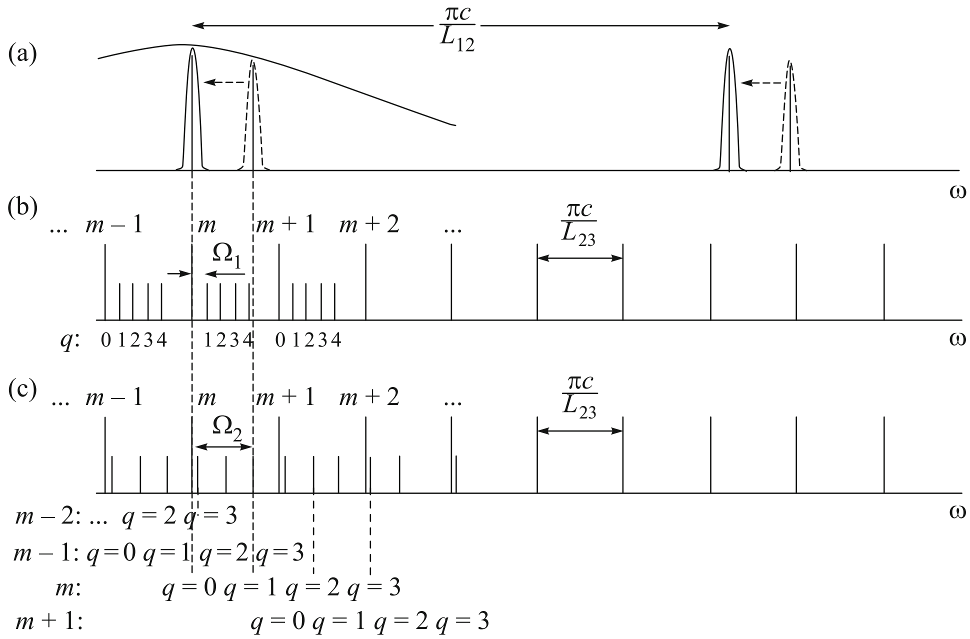

Fig. 4. Frequencies (a) in the main laser cavity and (b, c) in the external cavity at (b) $L_{23} < R_3/2$ and (c) $R_3/2 < L_{23} < R_3$ for various longitudinal m and transverse q indices.

and the second mirror is spherical [15, Chap. 7, Rus. p. 95]:

$$\omega_{mpq} = \frac{c}{L_{nl}}\left(\pi m + 2(p+q+1)\arctan\frac{L_{nl}}{((R_n - L_{nl})L_{nl})^{1/2}}\right), \quad (9)$$

where $\omega_{mpq} = k_{mpq}c$ is the resonance frequency, m is the longitudinal index (large integer), c is the speed of light, L_{nl} is the length of the partial cavity (n, $l = 1, 2, 3$), and R_n is the radius of curvature of the spherical mirror M_n, $n = 1, 2, 3$.

The relation between L_{nl} and R_n determines the degree of nonconfocality of the corresponding cavity. The cavity at $L_{nl} = R_n/2$ is called semi-confocal (one mirror is plane, $R_2 = \infty$). If $L_{nl} \ll R_n/2$, the cavity approaches a Fabry–Perrot cavity composed of two plane mirrors. If $R_n/2 \ll L_{nl} \le R_n$, the cavity is close to spherical (concentric). The case $L_{nl} > R_n$ corresponds to an unstable state of the field in the cavity, and the generation of undamped oscillations in it is impossible.

We consider the case where the TEM_{00} fundamental eigenmode ($p = q = 0$) is excited simultaneously in the laser, M_1–M_2, and external, M_2–M_3, cavities. The distance between neighboring resonance frequencies with the longitudinal indices m and $m+1$ for these cavities will be $\pi c/L_{12}$ and $\pi c/L_{23}$, respectively, as is shown in Fig. 4.

Thus, the resonances for the TEM_{00} mode are equidistant in frequency with the frequency distance between them inversely proportional to the length of the cavity. When the index p or q of TEM_{pq} modes (p, $q = 1, 2, 3$) changes by unity, the frequency positions of the modes remain equidistant, but the distance between resonance frequencies decreases at $L_{nl} < R_{nl}/2$ and increases at $R_{nl}/2 < L_{nl} < R_{nl}$, as follows from Eq. (9). These two situations are shown in Figs. 4b and 4c, respectively.

We consider the case where the configuration of the external cavity M_2–M_3 is determined by the condition $L_{23} < R_3/2$ (Fig. 4b). If the radius of curvature of the mirror M_3 becomes large enough (in our experiments, $R_3 \sim 100$ m), the distance between resonance frequencies Ω_1 given by Eq. (9) for TEM_{pq} and TEM_{pq+1} transverse modes will be much smaller than $\pi c/L_{23}$:

$$\Omega_1 = 2c(L_{23}R_3)^{-1/2} \ll \pi c/L_{23}, \quad (10)$$

as is shown in Fig. 4b. When the laser is heated, the length L_{12} of the main cavity M_1–M_2 increases. Consequently, all its resonance frequencies spaced by $\pi c/L_{12}$ from each other are shifted toward lower frequencies (see Fig. 4a). Thereby, the frequency of the TEM_{00} fundamental mode generated by the laser within the Doppler contour of the amplification line also decreases (indicated by an arrow in the figure). When this frequency coincides with any of the resonance frequencies of the external cavity, a higher mode with the corresponding transverse index is excited in it. In particular, for the case shown in Fig. 4b, the TEM_{04}, TEM_{03}, TEM_{02}, TEM_{01}, and TEM_{00} modes corresponding to the indices $q = 4$, 3, 2, 1, and 0, respectively, will be successively excited in the external cavity M_2–M_3. This situation is illustrated by a time sequence of photographs of field distributions at the output of the external cavity M_2–M_3 shown in the right panels of Fig. 3a.

The resonance frequencies of the TEM_{pq} transverse modes with the indices $q = 1$, 2, and 3 for the configuration of the external cavity M_2–M_3 satisfying the condition $R_3/2 < L_{23} < R_3$ are arranged as is shown in Fig. 4c. The distance Ω_2 between neighboring resonances with the indices q and $q + 1$ increases when the radius of curvature of the mirror R_3 approaches the length of the external cavity L_{23} and, according to Eq. (9), becomes maximal at $R_3 = L_{23}$:

$$\Omega_{2\,\max} = \pi c/L_{23}. \qquad (11)$$

The distance Ω_2 is about 0.7 of the maximum frequency difference $\pi c/L_{23}$ (see Fig. 4c) if the radius of curvature is $R_3 = 1.25L_{23}$, which is ensured by the introduction of the lens F with a focal length of about 1 m between the mirrors M_2 and M_3 (see Fig. 2). The distances between resonance frequencies with the indices $q = 0, 1, 2,...$ are larger, but resonances with other longitudinal and transverse indices $m - 1$, $q = 2$ and $m - 2$, $q = 3$ appear between resonances with the neighboring transverse indices, e.g., m, $q = 0$ and m, $q = 1$. Then, upon the heating of the cavity L_{12}, the frequency of generation of the TEM_{00} laser mode will decrease (as is shown by arrow in Fig. 4a) and will successively coincide with resonances determined by the following combinations of pairs of indices for the eigenmodes of the external cavity L_{23} (see Fig. 4c): m, $q = 1$; $m - 1$, $q = 2$; $m - 2$, $q = 3$; m, $q = 0$. Correspondingly, TEM_{01}, TEM_{02}, TEM_{03}, and TEM_{00} transverse modes will be successively excited in the external cavity L_{23}, as is demonstrated in the sequence of photographs of the field distributions shown in the right panels of Fig. 3b.

The revealed unusual phenomenon of generation of coupled oscillations with different pairs of transverse indices in partial laser cavities can significantly affect the formation of optical fields in high-precision laser systems involving back reflected or scattered radiation. Long-base laser interferometers applied in modern seismology [16, 17] and gravitational-wave astronomy [16, 18] belong to such systems. In particular, we showed in [13] that the energy of even relatively weak backscattered radiation at a level of 10^{-6} (corresponding to the parameter $|C_0^{pq}| \sim 0.001$) in the 4-km Advanced LIGO (laser interferometer gravitational-wave observatory) [18] can lead to an error in phase measurements that is one or two orders of magnitude larger than the allowed limit for this instrument.

The results obtained in this work can be applied to develop and create new optical measuring instruments, in cryptography, and in systems of protection, optical processing of information, and data transmission.

5. CONCLUSIONS

To solve the system of integral equations describing the spatial distribution of the electromagnetic field in a three-mirror laser cavity with mismatched mirrors in the quasioptical approximation, we have proposed to use modified boundary conditions including the coupling coefficients of eigenmodes with different pairs of transverse indices in partial cavities. In a laser with a three-mirror cavity, we have experimentally detected and studied an extraordinary generation of coupled types of oscillations with the transverse field distribution corresponding to the TEM_{00} eigenmode in the main laser cavity and TEM_{01}, TEM_{02}, TEM_{03}, ... eigenmodes in the external cavity. To our knowledge, such generation mode was not previously observed in lasers. Conditions for the excitation of coupled models depending on the configuration and tuning of partial cavities have been analyzed.

We dedicate this work to the memory of Aleksandr Dmitrievich Shatrov, our colleague and scientific advisor, who passed away untimely.

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