Influence of Electron Interference Effects on Reflection of Electron Waves from Potential Barrier in 2D Semiconductor Nanostructures

V. A. Petrov*, A. V. Nikitin

***Abstract*— The influence of the interference of electron waves in the case of their reflection from potential barrier on the inhomogeneous spatial distribution of the probability current density j_x (x, z) (or a quantum-mechanical current density ej_x (x, z), e is the electron charge) in 2D semiconductor nanostructure which is represented by rectangular narrow (x < 0, QW_1) and wide (x > 0, QW_2) quantum wells (QWs) sequentially oriented along the direction of the propagation of electron wave has been studied theoretically. We investigated behaviour of the ej_x (x, z) at falling of the electron wave on rectangular semi-infinite potential barrier (a potential wall) in height V_0 in semiconductor 2D nanostructure providing existence of the interference effects. We have considered a situation when in the 2D nanostructure at the left, from QW_1 the electronic wave of unit amplitude with energy $E_x < V_0$ on such barrier in QW_2 falls.**

I. Introduction

At present, advances in nanotechnology allow to create semiconductor nanostructures in which linear dimensions 1D or 2D of the conductive channel in the direction of propagation of the electron wave are smaller than the mean free path of the electron. In such a channel particles move in a ballistic regime that allows to study experimentally the effects of ballistic transport in such structures, in particular, various electron interference effects [1].

In the present work we theoretically investigated behaviour of the probability current density j_x (x, z) at falling of the electron wave on rectangular semi-infinite potential barrier (a potential wall) in height V_0 in semiconductor 2D nanostructure. We have considered a situation when in 2D nanostructure, consisting in the direction of propagation of the electron wave two rectangular quantum wells (QWs) of different width, at the left, from reg.1 (x <0, QW_1) on the first quantum-confined electron subband the electronic wave of unit amplitude with energy $E_x < V_0$ on such barrier in reg. 2 (x > 0, QW_2) falls. Different width of the QWs in the reg. 1 and 2 provide nonorthogonality of the wave functions in these regions. It results to confuse of the electronic subbands in different regions and to appearance of the electronic interference effects.

As is known [2], at falling of the electron wave with energy E_x (the x-axis is the direction of propagation of the electron wave) on a rectangular potential wall in height V_0 (x> 0) under condition of $E_x < V_0$ the quantum-mechanical current density ej_x (x, z) in the reg. 2 is equal to zero, because of the real exponent of the wave function in this region. Certainly, in this case exists exponentially dumped penetration of wave function of the particle into this region at x> 0.

It has been analytically demonstrated that in case of an electron wave falling along the first (lower) quantum-dimensional subband in QW_1 and its kinetic energy E_x being less than the energy positions of all the other subbands in QW_1 (i.e., the undamped propagation of the wave reflected from the barrier with real quasi-momentum is possible only along this lower subband) $ej_x^{(1)}$ (x,z) and $ej_x^{(2)}$ (x,z) are equal to zero. However, if a particle has such an energy that the refection of the wave with real quasi-momenta is possible along more than one (lower) subband, then the situation completely changes due to the interference of the reflected waves. In this case the interference leads to an existence of a complicatedly oscillating spatially inhomogeneous distribution $ej_x^{(1)}$ (x,z), and under the barrier in QW_2 it provides the appearance of exponentially damped at $x\to\infty$ and possessing a coordinate dependence of leakage $ej_x^{(2)}$ (x,z) under the barrier. Thus under a barrier there are three areas of distribution of the $ej_x^{(2)}$ (x, z): the central area (2), in which $ej_x^{(2)}$ it is directed in a positive direction of the x-axis and two lateral (1 and 3), in which $ej_x^{(2)}$ has a return direction. On two lateral zones 1 and 3 there is an outflow of a charge from under a barrier. We have obtained the dependences of the penetration length of the $ej_x^{(2)}$ from the E_x and V_0.

1. Results

In this section we give results of a numerical calculation of the refection effects for $ej_x^{(1)}$ (x,z) and $ej_x^{(2)}$ (x,z) in QW_1 and QW_2 for the symmetric on the z-axis semiconductor 2D nanostructure with concrete parameters on the basis of GaAs ($m^* = 0.067\ m_0$; m_0 is the free electron mass). Calculation is made in an infinitely hard-wall approximation for QWs. We have considered a problem of the

V. A. Petrov, A. V. Nikitin are with the Institute of Radio Engineering and Electronics, Russian Academy of Sciences, Moscow, 125009, Russia. (*corresponding author, fax: +74956293678; e-mail: vpetrov@mail.cplire.ru

.

scattering of the harmonic electron wave of unit amplitude spreading in the lower quantum-confined subband (m = 1) from narrow rectangular QW_1 (x < 0, the width a = 15 nm) to the wide rectangular QW_2 (x > 0, the width A = 50 nm). In addition was supposed, that on the boundary at x = 0 in reg. 2 exists semi-infinite potential barrier in height V_0 so energy of a falling particle $E_x < V_0$ (Fig.1). The particle motion along all the coordinates is assumed to be separated, and a particle wave vector is directed along the x-axis. We also assume that the potential energy in each of the QWs does not depend on x varying jump-wise at the point of the joint of the QWs (x = 0). Assuming that the QWs are defined by a infinite-height-wall potential along the z-axis, the energetic spectrums in this direction are completely discrete.

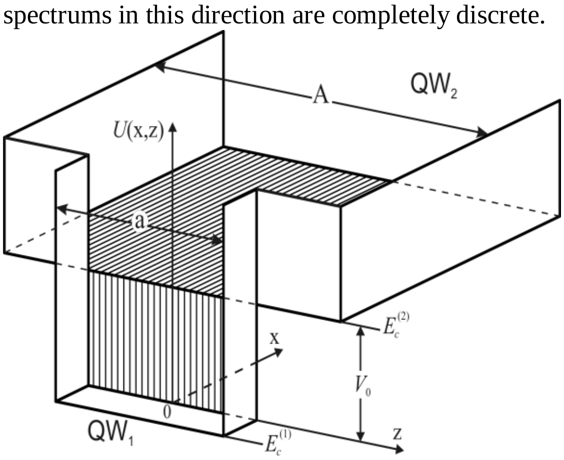

Fig.1. Schematic sketch of a symmetric 2D nanostructure on the base of two rectangular QWs with the different widths: a (QW_1) < A (QW_2). In the QW_2 it is created additional semi-infinite rectangular potential barrier V_0 .

We have obtained analytical expression for the longitudinal component of the probability current density $j_x^{(1)}$ (x,z) in the QW_1 and $ej_x^{(2)}$ (x,z) in the QW_2 :

$$j_x^{(1)}(x,z) = \frac{\hbar}{2m^*}\Big[2k_1\chi_1^2 - \sum_{i,j} B_j B_i^* \chi_i \chi_j (k_j + k_i^*) e^{i(k_i^* - k_j)x} + \\ + \chi_1 e^{-ik_1 x} \sum_j \chi_j B_j (k_1 - k_j) e^{-ik_j x} + \\ \chi_1 e^{ik_1 x} \sum_i B_i^* \chi_i (k_1 - k_i^*) e^{ik_i^* x}\Big] \quad (1)$$

$$ej_x^{(2)}(x,z) = \frac{\hbar}{2m^*}\left[\sum_{n,t} C_n C_t^* \varphi_n(z) \varphi_t^*(z) (k'_n + k'^*_t) \exp\left[i(k'_n - k'^*_t)\, x\right]\right] \quad (2)$$

Here, $\{\chi_{i,j\ (z)}\}$ and $\{\varphi_{n,t}\ (z)\}$ are eigenfunctions of the Schrödinger equations in the QW_1 and QW_2 accordingly , $B_{i,j}$ and $C_{n,t}$ are constant coefficients defining the amplitudes of the waves reflected in QW_1 on the subbands E_i ($k_i = [2m^*(E - E_i - E_y)]^{1/2}/\hbar$) and passed to the QW_2 through the subbands E_n ($k'_n = [2m^*(E - E_n - E_y)]^{1/2}/\hbar$). Let as note that if $E - E_y > E_i$ and E_n then k_i and k'_n are real, and the waves corresponding to them are spreading; at inverse inequality k_i and k_n are imaginary, and the waves are damped, with typical lengths of $l_i = |k_i|^{-1}$ and $l_n = |k'_n|^{-1}$. For the considered structures coefficients $B_{i,j}$ and $C_{n,t}$ are defined from the system of equations following from boundary conditions for wave functions and their derivatives in point x = 0.

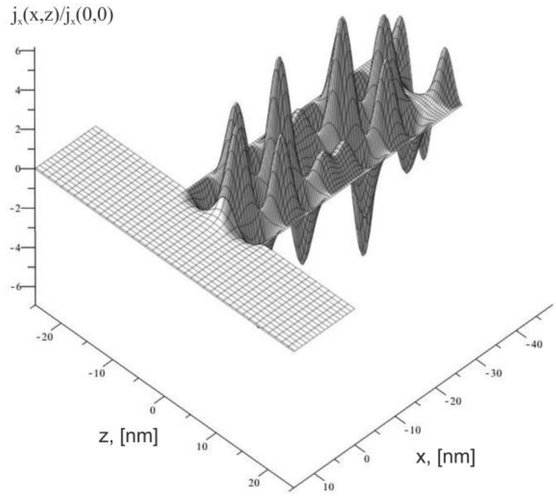

Fig.2. General view of inhomogeneous spatial distribution of the probability current density $j_x^{(1)}$(x,z) in QW_1 and $j_x^{(2)}$(x,z) in QW_2 in considered symmetric on the z-axis 2D nanostructure in an interval from x =15 nm up to x = - 49 nm.

At calculation three subbands with real k_{xi} and 28 subbands with imaginary in QW_1, and also 31 subbands with imaginary k_{xn} in QW_2 were taken into account. The particles kinetic energy ${}^1E_x$ in the QW_1 is 245 meV and the distance from the top of the potential barrier is 10,25 meV (V_0 =243 meV).

REFERENCES

[1] D. K. Ferry and S. M. Goodnick, *Transport in Nanostructures*. Cambridge Univ. Press, Cambridge 1997.

[2] L. D. Landau and E. M. Lifschitz, *Quantum Mechanics (Non – Relativistic Theory)*. Pergamon Press, Oxford, 1977.