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Recovery of Images Distorted by an Instrument Function with Unknown Side Lobes

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Abstract—A method for compensating the influence of unknown side lobes of a distorting instrument function on the quality of image recovery is proposed. A processing algorithm, based on the knowledge of the main lobe of the instrument function and its spectrum together with a universal reference spectrum, is used to compensate the contribution of the unknown side lobes to the spectrum of the distorted image. The stability of the method to the noise inherent in the improved images and various forms of distorting instrument functions is analyzed. High efficiency of the method of compensation is demonstrated in the cases in which the total contribution from the unknown side lobes to the resulting brightness of the distorted image significantly (four-fold) exceeds the contribution from the main lobe.

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Spatially incoherent image formation in radio vision systems (both in radio and optical bands) often encounters situations in which the exact knowledge of the shape of the instrument function (IF) is impossible. This especially concerns the side lobes (SLs) of the IF and may result from the natural factors that cause the multibeam propagation or scattering of radio waves: atmospheric turbulence, haze, and clouds or from the design specifics of the antenna systems that do not allow for a sufficiently accurate 3D measurement of their directional pattern. In some cases, even if the SL level relative to the main lobe (ML) level is small (about one percent), the total contribution of the SLs to the resulting image brightness can be several times larger than the contribution of the ML. This is especially important for radio systems with a narrow ML, since, in this case, it is more likely that the total contribution from the unknown SLs to the resulting brightness of the distorted image will significantly (multiply) exceed the contribution from the ML. In this connection, it is important to formulate the problem of the recovery of images distorted by an instrument function with unknown side lobes.

In many cases, the image formation can be described by the convolution equation [1]

$$g(x,y) = \iint dx'dy' a(x-x', y-y') f(x', y') + n(x,y), \quad (1)$$

where $g(x,y)$ is the image of an object $f(x,y)$, formed by an instrumental function $a(x,y)$, and $n(x,y)$ is an additive noise.

The image quality is influenced by both the properties of the instrument function (IF) and the noise characteristics. To recover the image $f(x,y)$, it is necessary to know both the IF and the noise characteristics. The IF is usually known within the main lobe of an antenna system, and the influence of the side lobes on the image quality is assumed to be insignificant. This is far from being true, since the more “acute” the ML, the larger the angular range occupied by the BL. The SLs of an IF (IFSLs) will be understood as all that lies beyond the ML of an antenna system. The shape and characteristics of the IFSLs depend not only on the antenna system but also on the medium between this system and the object. Therefore, the SL characteristics are difficult to measure. As shown in [2, 3], SLs can have a strong negative effect on the quality of the recovered image $f(x,y)$. In [4], the possibility of image recovery in the case of unknown SLs was investigated. However, the method based on the analysis of the IF shape was described in a rather complicated manner without analyzing the effect of noise and the SL shape.

There are original works that take into account the contribution of the unknown propagation medium to the distorting IF. In [5], a modification of the method for identifying a linear observation model using the relationship between the energy spectra of the input and output images was described. In the absence of a priori data on the distorting system, it was proposed to use a technique for the recovery of an unknown IF from the observed image using a mask of boundaries taken from geoinformation systems describing the boundaries of the objects in the image. In [6], it was proposed to combine atmospheric distortion models and a linear spectral mixture into a unified model. This method for determining the parameters of a linear spectral mixture for hyperspectral images makes it possible to eliminate the preliminary procedure for correcting the atmospheric distortions of the detected image.

A technology for the recovery of images subjected to defocusing distortions was proposed in [7]; it is based on synthesizing a filter with a finite impulse response in which the determination of the impulse response samples is replaced with the identification of the parameters of a continuous function approximating the impulse response. In this case, a parametric family of approximating functions, depending on a small number of parameters, is specified taking into account the desired frequency characteristics of FIR filters used to correct distortions.

As a rule, the works known to the authors are devoted to the study of the destructive effect of the unknown side lobes on the recovered images. In this work, for the first time, we study the possibility of compensating the effect of unknown side lobes on the quality of recovery of radio images.

In this paper, we consider the method of image recovery in the case of unknown SLs and study the influence of a noise $n(x,y)$ and the IF shape on the quality of image recovery.

It is worth noting again that an image-distorting IF is identical to the antenna IF only in the absence of objects surrounding the antenna and in the case of a non-absorbing and nonscattering propagation medium (vacuum). If the propagation medium is scattering, then the contribution of the ML to the image decreases with increasing scattering and the contribution of the SLs increases. However, in our opinion, until the "disruption" of the directional pattern has not occurred, it can be assumed in some approximation that the shape of the antenna ML is identical to the shape of the IFML. Henceforward, we assume it to be known. Side lobes are understood as the part of the IF that lies beyond the ML. The shape of the IFSLs varies significantly [8, p. 106].

This work is based on two statements:

1. Since, in the spatial domain, the IFML is always narrower than the IFSLs (at least because the SLs is that that remains after "cutting" out the ML), the spectrum of the SLs is much narrower than the spectrum of the ML.

2. On average, the absolute value of the spectrum of undistorted images has a shape varying only slightly from image to image. This made it possible to introduce into the image processing a universal reference spectrum (URS) as a model sample of the amplitude spectrum of an undistorted image [9]:

$$\mathrm{URS}(R) = 128[0.55\exp(-2.5R^{0.75}) + 0.45\exp(-1.5R^{0.12})],$$

where R is the squared absolute value of the spatial frequency and 128 is half the maximum brightness.

Leaning upon statement 1, with the help of the URS, it is possible to estimate the contribution of the ML to the image spectrum.

In the spectral representation, relationship (1) looks like an algebraic equation [1]:

$$G(u,v) = A(u,v)F(u,v) + N(u,v), \tag{2}$$

where $G(u,v)$, $F(u,v)$, and $N(u,v)$ are, respectively, the spatial spectra of the image $g(x,y)$, object $f(x,y)$, and noise $n(x,y)$ and $A(u,v)$ is the normalized spectrum of an IF $a(x,y)$. The normalized spectrum is understood as a spectrum normalized so that its maximum value is equal to unity.

Assume that the IF consists of the sum

$$a(x,y) = k_g a_g(x,y) + k_b a_b(x,y), \tag{3}$$

where $a_g(x,y)$ and $a_b(x,y)$ are the main and side lobes, respectively, and k_g and k_b determine the maximum level of the ML and the SLs, respectively. Denote the normalized spectra of $a_g(x,y)$ and $a_b(x,y)$ as $A_g(u,v)$ and $A_b(u,v)$, respectively. Let us temporarily exclude the noise $n(x,y)$ from consideration and return it when applying the method being developed. Substituting the spectrum of expression (3) into relationship (2), we obtain:

$$G(u,v) = k_g A_g(u,v)F(u,v) + k_b A_b(u,v)F(u,v). \tag{4}$$

In expression (4), the second term on the right-hand side is the contribution from the SLs. If the quantity $|A_g(u,v)||F(u,v)|$ is known, then $F(u,v)$ can be calculated by the following algorithm:

Step 1. Make the estimate

$$A_g(u,v)F(u,v) \approx |A_g(u,v)||F(u,v)|\frac{G(u,v)}{|G(u,v)|}. \tag{5}$$

Step 2. Find $F(u,v)$ using the Wiener filter [1] (see the Appendix).

Step 3. Find the inverse Fourier transform and obtain the image of the object $f(x,y)$.

Estimate (5) is valid under the condition of a small influence of the SLs on the phase spectrum $F(u,v)$.

Let us consider the above algorithm for image recovery, the SLs being unknown, on the example of an IF consisting of a combination of two Butterworth filters.

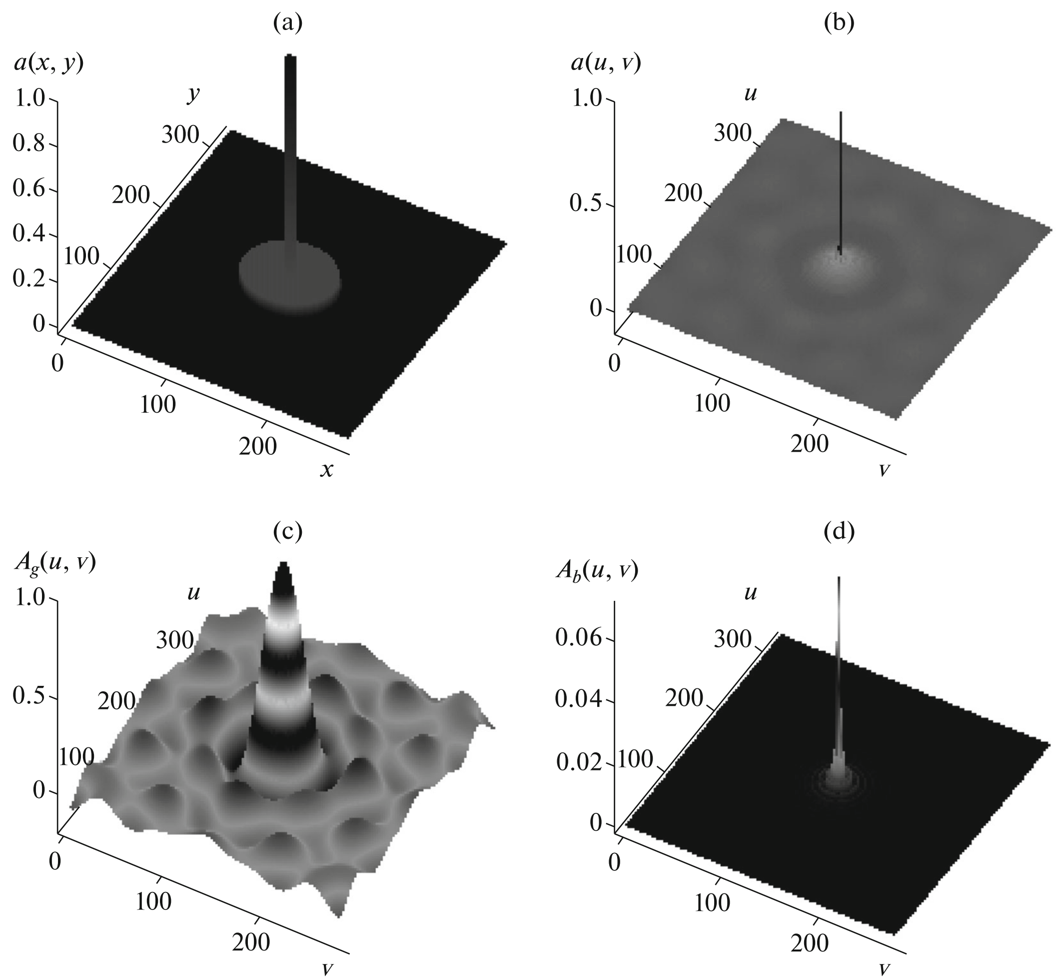

Fig. 1. (a) Instrumental function, (b) its spectrum, and the spectra of (c) the main lobe of radius $S_0 = 5$ and (d) side lobes of radius $S_b = 45$.

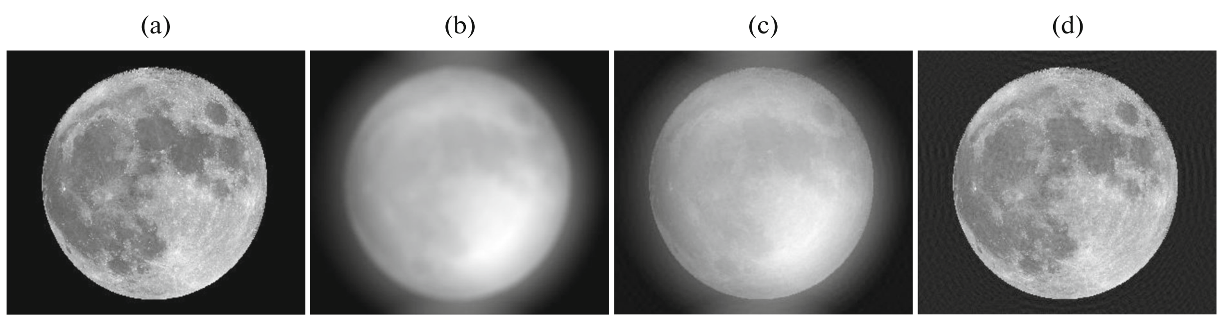

Fig. 2. Images of the Moon: (a) undistorted, (b) distorted when using the IF shown in Fig. 1, (c) recovered under the assumption of the absence of SLs, and (d) recovered on the basis of estimate (5).

Note that, when compensating the SLs, we never used the shape of the SL spectrum $A_b(u,v)$. We only used the normalized spectrum of the known ML, $A_g(u,v)$. The shape of the ML spectrum is not important; the only thing that matters is that it must correspond to the problem under consideration.

Figure 1 shows a test IF, its spectrum, and the spectra of the ML and SLs.

Figure 2 shows the following images: an undistorted image (the Moon) $f(x,y)$ (Fig. 2a), a distorted image obtained using the IF shown in Fig. 1 (Fig. 2b), an image recovered under the assumption that SLs are

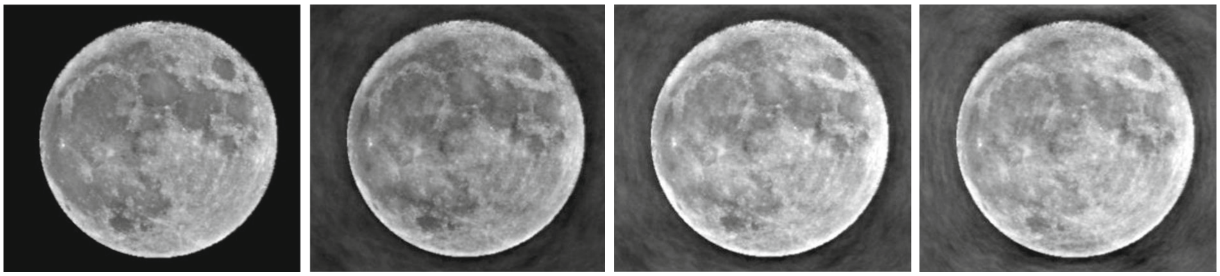

Fig. 3. Images of the Moon: (a) undistorted, (b–d) recovered on the basis of estimate (7) with the ratio between the sums of the SL and ML amplitudes of (b) 0, (c) 1, and (d) 4.

missing (Fig. 2c), and an image recovered on the basis of estimate (5) (Fig. 2d).

The analysis of Fig. 2 leads to the conclusion that the neglect of the SLs does not allow one to recover the image and the use of estimate (5) makes possible the image recovery with unknown SLs.

However, this approach cannot be used, since the quantity $|A_g(u,v)||F(u,v)|$ is usually unknown. Nevertheless, using the universal reference spectrum (URS) [9], one can estimate the quantity $|A_g(u,v)||F(u,v)|$ and obtain a recovered image comparable in quality with Fig. 2d. To this end, let us compare the graphs in Figs. 1c and 1d. As can be seen, the amplitude spectra of the ML and SLs are very different. This fact can be used for the correct estimation of the first term in (4). The corresponding algorithm is as follows:

Step 1. As an initial estimate of $|A_g(u,v)||F(u,v)|$, take the representation

$$|A_g(u,v)||F(u,v)| \approx |A_g(u,v)|\,\mathrm{URS}(u,v), \qquad (6)$$

where $\mathrm{URS}(u,v)$ is the URS.

Step 2. Approximate $|G(u,v)|$, the absolute value of the spectrum of the image distorted by the IF, by the method of least squares using estimate (6). In order to reduce the negative effect of noise and the SLs spectrum, in the approximation, exclude the region around the zero frequency with a radius of $2/\pi S_0$ and the region in which $|A_g(u,v)| < 0.1$. Thus, we estimate k_g.

Step 3. Finally estimate $|A_g(u,v)||F(u,v)|$ as

$$k_g A_g(u,v)F(u,v) = G(u,v), \qquad (7a)$$

if $|G(u,v)| < |A_g(u,v)|\,\mathrm{URS}(u,v)k_g$,

$$k_g A_g(u,v)F(u,v) = k_g\,|A_g(u,v)|\,\mathrm{URS}(u,v)\frac{G(u,v)}{|G(u,v)|}, \qquad (7b)$$

if $|G(u,v)| \ge |A_g(u,v)|\,\mathrm{URS}(u,v)k_g$.

Step 4. Find $F(u,v)$ using the Wiener filter [1] (see the Appendix).

Step 5. Using the inverse Fourier transform, find the image of the object $f(x,y)$.

It is advisable to normalize the obtained reconstructed image so that the mean brightness of the obtained image be the same as in the original distorted image.

Figure 3 shows the initial undistorted image of the Moon (Fig. 3a) and the images reconstructed on the basis of estimate (7a) and (7b), the ratio of the sum of the ML amplitudes to the sum of SL amplitudes, K_b, being equal to 0 (Fig. 3b), 1 (Fig. 3c), and 4 (Fig. 3d).

The comparison of the images in Fig. 3 leads to a conclusion that the recovered image depends on the SLs "power" relatively weakly in comparison to the ML "power".

Figure 4 shows the initial undistorted image of the Moon (Fig. 4a) and the images recovered on the basis of estimate (7a) and (7b) for $K_b = 4$ and the amplitude of the additive uniform noise equal to 0 (Fig. 4b), 0.5 (Fig. 4c), and 5 (Fig. 4d).

The comparison of the images in Fig. 4 leads us to a conclusion that the quality of recovered images is relatively stable against the negative influence of additive noise. An increase in the noise amplitude from 0 to 0.5, which corresponds to the usual quantization noise of digital photographs, gives rise to small artifacts. The further increase in the noise amplitude increases these artifacts tenfold, but the main details of the image remain almost unaffected.

Let us analyze the influence of the shape of the IF and the SL magnitude on the results of the recovery by the given method. Unfortunately (see, e.g., [10–13]), most of criteria for evaluating the image quality estimate the features of the image itself rather than its possible distortions. Therefore, we will evaluate the effect of the SLs on the magnitude of distortions and the quality of recovery by the CRI (Coefficient of Recoverability of an Image) criterion, which was proposed in [14] as a measure of reversible image distortions. If the CRI is close to zero, then the image spectrum is highly distorted but there is a chance to recover the image by spectral filtering. If the CRI is close to unity, then

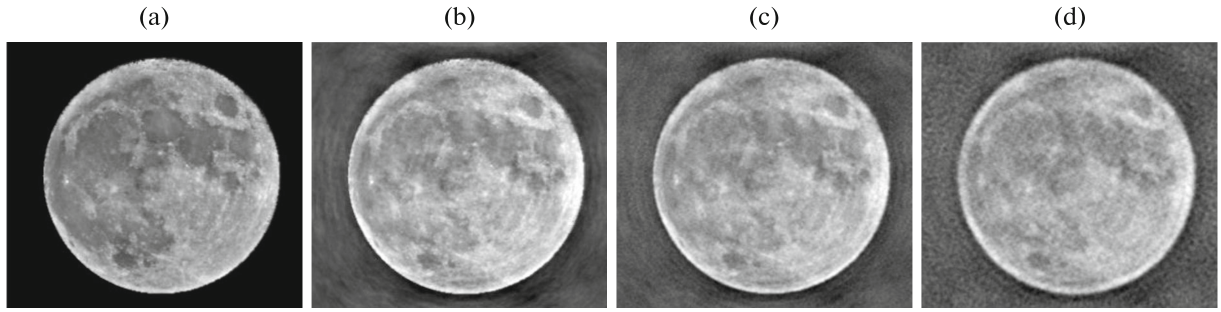

Fig. 4. Images of the Moon: (a) undistorted and (b–d) recovered on the basis of estimate (7) at $K_b = 4$ and $S_h =$ (b) 0, (c) 0.5, and (d) 5.

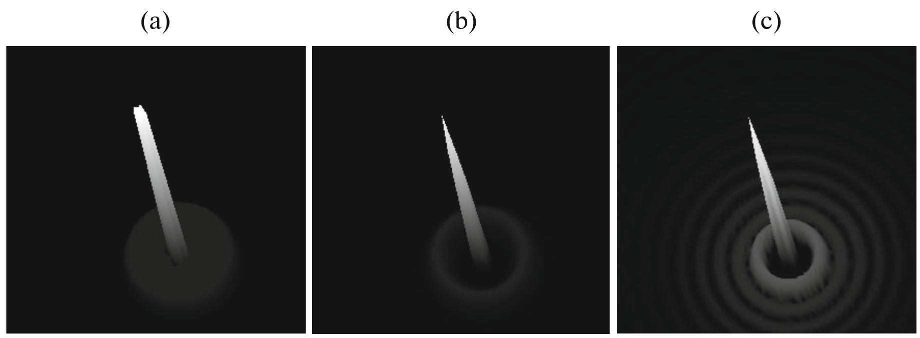

Fig. 5. Shapes the IF with SLs, used for testing the method at $S_0 = 2$: the (a) first, (b) second, and (c) variants. The SL level relative to the ML is 0.08; $K_b = 4$.

either the image is close to ideal or the image spectrum is subjected mostly to phase distortion. If there is not enough information about phase distortions, then the expectations of image improvement cannot be optimistic.

Let us consider the following variants of IFs:

(1) the above-described IF consisting of the sum of two Butterworth functions;

(2) the IF comprising a combination of cosines used in [2];

(3) the IF comprising a combination of the absolute values of the spectra of the Butterworth function, used in [15, 16].

Figure 5 illustrates the shapes of the IFs used for testing.

Figures 6a–6c show the CRI as a function of the ML radius S_0, the ratio K_b between the sums of the BL and GLs amplitudes, and the amplitude S_h of the additive noise for three different IFs corresponding to Figs. 5a–5c.

In Fig. 6a, the solid lines correspond to the absence of SLs and the dashed lines correspond to $K_b = 4$. It is seen that the dashed lines lie significantly lower than the solid lines, because the increase in ML may be interpreted as a broadening of the SLs.

This interpretation is also valid for explaining the decreasing dependence of the CRI on K_b (Fig. 6b).

As can be seen from Fig. 6c, the CRI increases with increasing noise amplitude. This illustrates the obvious fact that the presence of noise reduces the possibility of qualitative recovery of a distorted image.

Figure 7a shows the dependences of CRI for recovered images on the ML radius at $K_b = 4$ for three different IFs corresponding to Figs. 5a–5c. All graphs lie above the line CRI = 0.96. This indicates that the amplitude spectra have been reconstructed by the method considered with a sufficient quality.

Figure 7b shows the dependences of the CRI curves for the recovered images on the ratio between the sums of the ML and SL amplitudes at $S_0 = 5$ for the same IF. All the graphs lie above the line CRI = 0.98 and exhibit no explicit dependence on SL magnitude. This suggests that the method under consideration compensates the influence of SLs sufficiently well.

Figure 8 shows the rms deviation of brightness, Δ, for three methods of image recovery. The graphs illustrate a fairly accurate image recovery by the proposed method in the presence of unknown SLs.

Thus, we can make the following conclusions:

(1) Side lobes can significantly degrade the image quality.

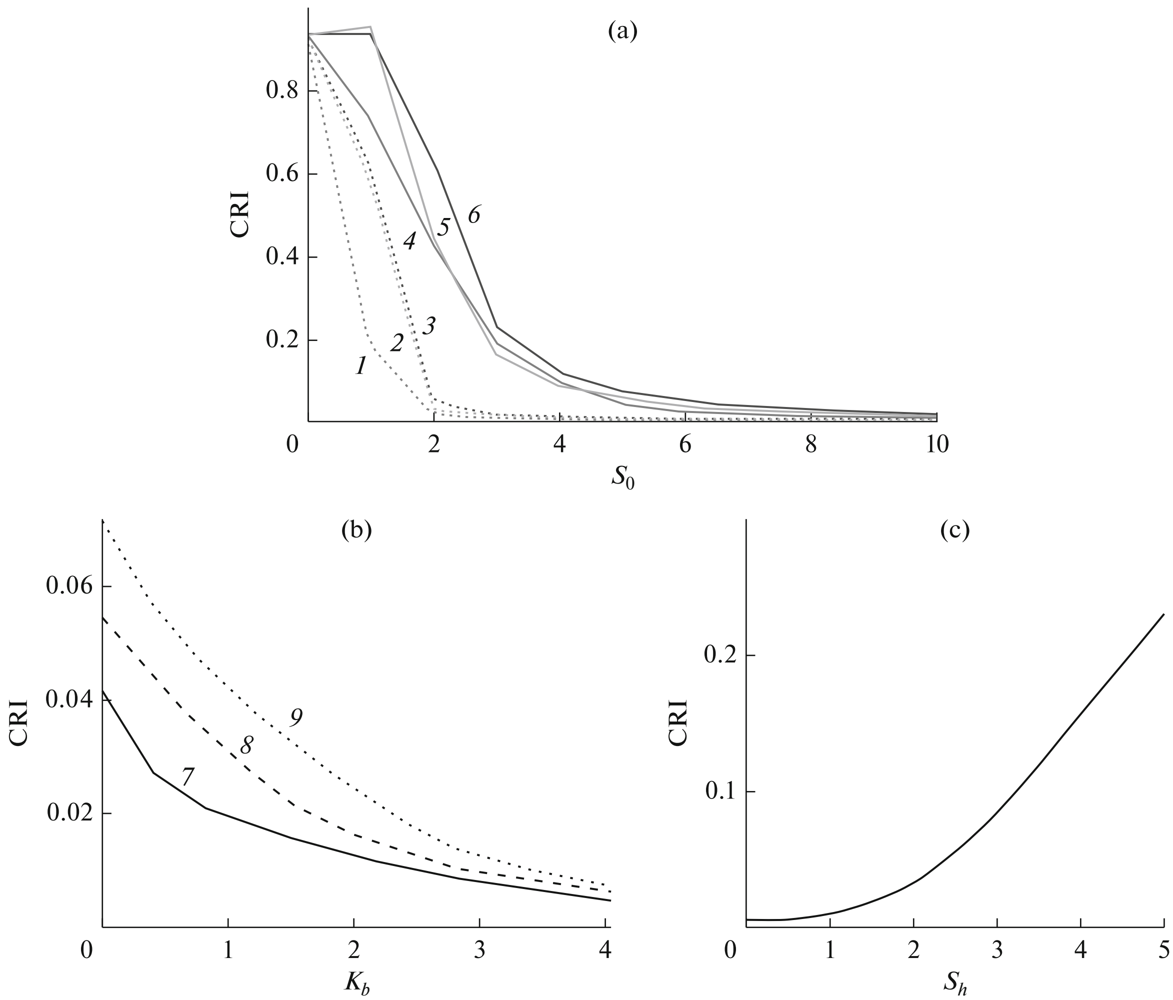

Fig. 6. CRI vs. (a) ML radius, (b) the ratio of the total contribution of the SLs amplitudes to the total contribution of the ML amplitudes, and (c) the effective noise amplitude; (a, b) (solid curves) the absence of SLs and (dashed curves) K_b=4; curves *1*, *4*, and *7* correspond to the IF in Fig. 5a; curves *2*, *5*, and *8*, to the IF in Fig. 5b; and curves *3*, *6*, and *9*, to the IF in Fig. 5c.

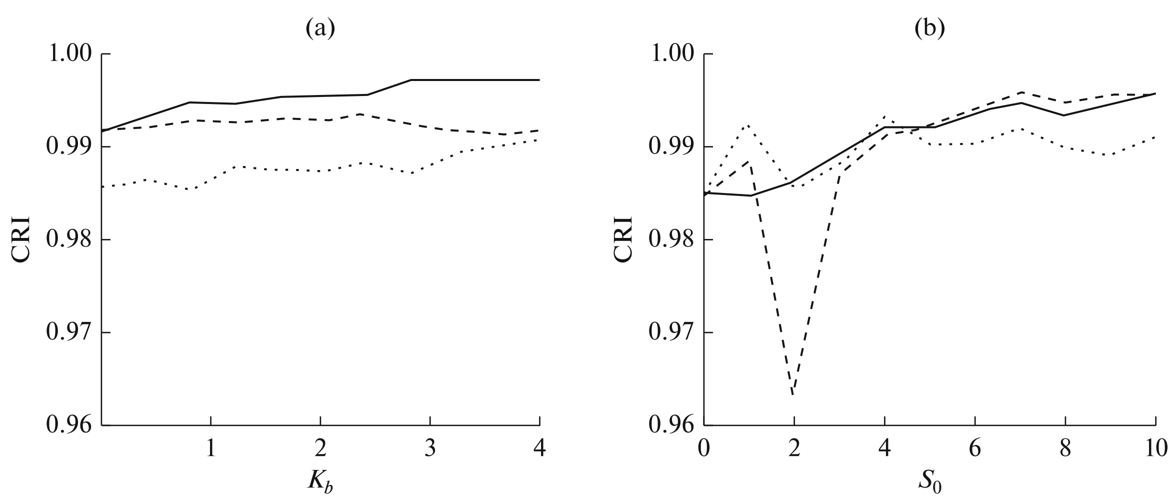

Fig. 7. CRI for recovered images vs. (a) the ratio of the total contributions of the SL and ML amplitudes and (b) the ML radius. The dotted curves correspond to the IF in Fig. 5a, dashed curves for IF in Fig. 5b, and solid curves, for the IF in Fig. 5c.

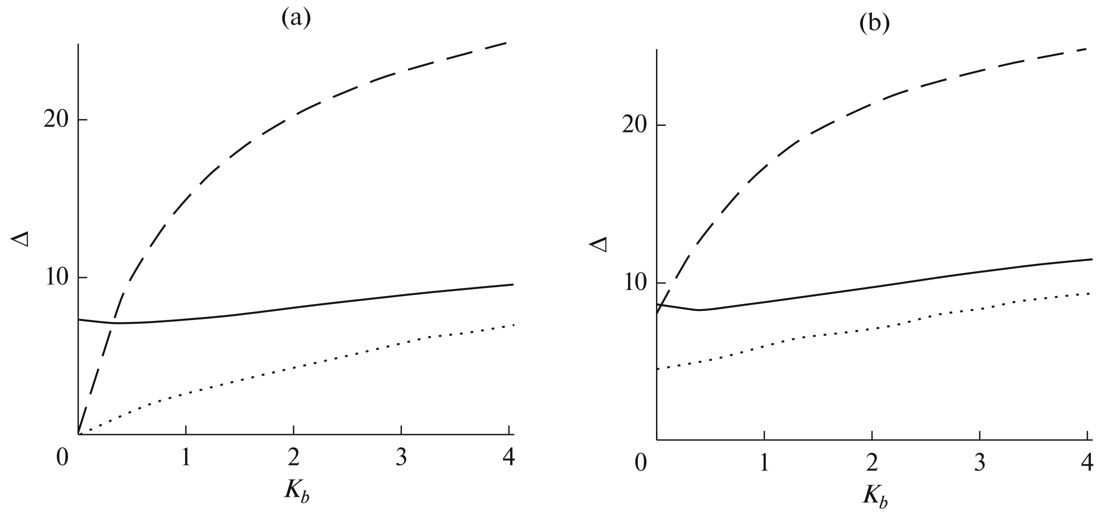

Fig. 8. Rms brightness deviation vs. the ratio between the total contribution of the SL and ML amplitudes for three methods for image recovery (a) in the absence of noise and (b) for a uniform noise with an amplitude of 0.5. The dashed curves correspond to the variant neglecting the influence of the SLs (recovery with an IF having only the ML); dotted curves correspond to the recovery with a completely known IF with SLs; solid curves correspond to the recovery using an IF with unknown SLs according to estimate (7).

(2) The influence of SLs can be very significant even if their magnitude is small compared to the ML.

(3) The recovery of images distorted by an IF with unknown SLs is very difficult, because the spatial spectrum of the SLs lies mainly in the low-frequency region.

(4) The method of SL compensation presented in this work is based on the universal reference spectrum and does not require additional information on the character of the SLs.

(5) In all the IF considered, the distorted images have been recovered with a quality comparable to the recovery with a known amplitude spectrum of the undistorted image.

APPENDIX

WIENER FILTERING

Formula (1) in the spectral representation looks like an algebraic equation [1]:

$$G(u,v) = A(u,v)F(u,v) + N(u,v), \qquad \text{(A.1)}$$

where $G(u,v)$, $F(u,v)$, and $N(u,v)$ are the spatial spectra of the image $g(x,y)$, object $f(x,y)$, and noise $n(x,y)$, respectively, and $A(u,v)$ is the normalized spectrum of IF $a(x, y)$. The normalized spectrum is understood as a spectrum normalized so that its maximum value is unity.

Under certain conditions [1], solution (A.1) can be written in the form

$$F(u,v) = \frac{1}{A(u,v)} \frac{|A(u,v)|^2}{|A(u,v)|^2 + K(u,v)},$$

where $K(u,v) = |N(u,v)|^2 / |F(u,v)|^2$, $|N(u,v)|^2$ is the energy spectrum of noise, and $|F(u,v)|^2$ is the energy spectrum of the undistorted image. If the energy spectra of noise and the undistorted image are unknown, $K(u,v)$ is a specially chosen constant [1].

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