ISSN 1063-7710, Acoustical Physics, 2018, Vol. 64, No. 6, pp. 665–669. © Pleiades Publishing, Ltd., 2018.
Original Russian Text © A.F. Kabychenkov, F.V. Lisovskii, E.G. Mansvetova, G.V. Arzamastseva, 2018, published in Akusticheskii Zhurnal, 2018, Vol. 64, No. 6, pp. 656–660.

PHYSICAL ACOUSTICS

Transformation of Hybrid Transverse Elastic Waves in Heterogeneous Micropolar Media

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Received March 7, 2018

Abstract—The features of propagation of coupled hybrid transverse elastic waves in a heterogeneously dense micropolar medium with spatial dispersion are studied. It is shown that in the region of the medium corresponding to the intersection point of unperturbed dispersion curves of elastic waves of different types, efficient transformation of a shear wave into a rotational wave or vice versa may occur.

Keywords: hybridization of spectra, coupled waves, rotational waves, shear waves, micropolar media, transformation of wave types, spatial dispersion

DOI: 10.1134/S1063771018060040

INTRODUCTION

Currently, there is a noticeable tendency toward expanding the range of theoretical and experimental studies on diverse processes and phenomena in condensed media and transmission lines due to the interaction, hybridization, and transformation of different types of waves. Particular attention is devoted to hybridization, which means the effects occurring in the vicinity of the intersection point of the unperturbed dispersion curves of two waves (crossover point) where their phase velocities coincide. If these waves have common components of the amplitudes of variables, splitting of these dispersion curves and their "gluing" with the formation of segments of perturbed dispersion curves, which correspond to the transformation of one wave into the other, occurs near the crossover point [1, 2].

The interest in such phenomena, which arose in the mid-20th century and was mainly due to advances in the development and implementation of methods for generating, transmitting, and transforming electromagnetic waves in the microwave region (see, e.g., [3, 4]), remains strong. The appearance of fiber and integrated optics gave significant additional impetus to the development of the theory of coupled waves. The first papers on the problem [5–8] were followed by numerous publications of other authors, including monographs (see, e.g. [9–11]). A modern view on the problem and an extensive bibliography are given in the monograph [12].

The results of experiments performed to date on various waveguide structures show that the hybridization of waves in such objects has interesting features. When studying the interaction of acoustic waves of zero and higher orders in lithium niobate plates, it was found that hybridization occurs only when one surface of the plate is electrically shorted (the other surface is free) or with a small deviation of the wave propagation direction from the axes of symmetry [13]. In langasite and bismuth germinate, the influence of an external electric field on the acoustic-wave hybridization effect was observed [14]. The number of such examples can easily be multiplied.

The presence of waveguide structures is not a necessary condition for wave hybridization. Moreover, such a phenomenon can exist in infinite condensed media due to the relationships between the electrical, magnetic, and elastic subsystems. The most well-known example of this effect is the existence of so-called magnetoelastic waves in magnetically ordered media, which arise due to hybridization of spin and elastic waves. This effect was first analyzed theoretically Turov and Irkhin for ferromagnets in 1956 [15] (see also [16, 17]); the first experiments that confirmed the conclusions of the theory were carried out in [18–20]. Around the same time, it was shown that electromagnetic and spin waves are also coupled in magnets [21]. Later, the authors of a series of papers [22–24] found that, using the hybridization of the spectra of elementary excitations, electromagnetic waves in an inhomogeneous magnetic field can efficiently excite long-wavelength magnons, which can then be converted into short-wavelength magnons, i.e., into spin waves. In turn, these waves can be transformed into elastic waves upon further propagation in an inhomogeneous magnetic field.

Relatively recently, the family of media in which coupled waves exist has been augmented by micropolar media, which are also called Cosserat continua [25], the elastic properties of which are described by the bending–torsion tensor and the asymmetric strain tensor. Historical information on the research of the discussed problem is given in [26], while a detailed description of the theory for the most widely used models and experimental data on some parameters of such media can be found in [27, 28] and [29, 30], respectively.

Nonclassical elasticity theories have found numerous applications in mechanics, condensed matter physics, and various fields of engineering. The physical objects it can be applied to include crystals with defects (including dislocations and disclinations), spin and dipole glasses, liquid crystals, biomaterials (e.g., bone tissue), nanomaterials, geomaterials, etc. [31]. The study of wave processes in micropolar media and the possibility of their practical application covers a variety of areas related to surface acoustic waves [32], elastic vibrations of plates and shells [33, 34], ferroelectric and transversely isotropic media [35, 36], identification of nanocrystalline media using acoustic spectroscopy [37], etc.

In Cosserat continua, due to the relationship between displacements and rotations, the transverse elastic waves corresponding to them are not independent, and the dispersion law for them in an infinite medium is described by a single equation:

$$\left(\omega^2 - v_{dt}^2 k^2\right)\left(\omega^2 - \omega_0^2 - v_{rt}^2 k^2\right) - ck^2 = 0, \quad (1)$$

where ω and k are the frequency and wavenumber; c is the coefficient of coupling between the shear and rotational waves with phase velocities v_{dt} and v_{rt}, respectively, at $c = 0$; and ω_0 is the activation frequency for rotational waves [27, 28].

The solution to Eq. (1), which was derived without consideration for spatial dispersion, gives two branches, the upper and lower, for the dependence $\omega(k)$, which do not intersect with each other. In this case, the hybridization is weakly pronounced and manifests itself only in the fact that the deformation of the medium during the propagation of rotational waves contains a small shear component, while the propagation of shear waves is accompanied by small turns.

INFLUENCE OF SPATIAL DISPERSION ON THE LAW OF DISPERSION OF HYBRID TRANSVERSE ELASTIC WAVES

Taking into account the influence of spatial dispersion on the propagation of elastic waves, Eq. (1) is modified as follows [15]:

$$\left[\omega^2 - \left(v_{dt}^2 + v_{dtk}^2\right)k^2\right] \times \left[\omega^2 - \omega_0^2 - \left(v_{rt}^2 + v_{rtk}^2\right)k^2\right] - c_0 k^2 = 0, \quad (2)$$

where v_{rtk}^2 and v_{dtk}^2 are the corrections for the squares of the transverse velocities of the displacement and rotational velocities, which are proportional to k^2, while $c_0 = C + C'k^2 + C''k^4$ is the modified coupling coefficient between the waves.

The explicit expressions for the variables introduced in (1) and (2) through the phenomenological constants in the expansion of the thermodynamic potential, which was used in [38], are not presented here, because below, we are only interested in the behavior of the dependences of such variables on the density ρ of the micropolar medium. Calculations have shown that in the considered case, the quantities v_{dt}^2, v_{rt}^2, v_{rtk}^2, v_{dtk}^2, and ω_0^2 are inversely proportional to ρ, while the quantities c_0, c_{00}, c_{01}, and c_{02} are inversely proportional to ρ^2 [38].

By introducing positive quantities $A = v_{dt}^2$, $A' = v_{dtk}^2/k^2$, $B = v_{rt}^2$, and $B' = v_{rtk}^2/k^2$ independent of k for further simplification and disregarding the dispersion of the coupling coefficient, i.e., assuming that $c_0 = C$, we obtain the following expressions for the roots of Eq. (2):

$$\omega_{1,2}^2 = Q_1 \pm \sqrt{Q_1^2 - Q_2},$$

where

$$Q_1 = \frac{1}{2}\left[\omega_0^2 + (A + B)k^2 + (A' + B')k^4\right],$$

$$Q_2 = \left[\left(A\omega_0^2 - C\right) + \left(AB + A'\omega_0^2\right)k^2 + (AB' + A'B)k^4 + A'B'k^6\right]k^2.$$

The position of the crossover points at $C = 0$, i.e., the intersection points of undisturbed dispersion curves when

$$\omega_1^2 = \omega_0^2 + \left(v_{rt}^2 + v_{rtk}^2\right)k^2 = \omega_0^2 + \left(B + B'k^2\right)k^2,$$

$$\omega_2^2 = \left(v_{dt}^2 + v_{dtk}^2\right)k^2 = \left(A + A'k^2\right)k^2,$$

corresponds to the following values of the squares of the wavenumbers (which are independent of ρ):

$$k_{1,2}^2 = \frac{1}{2(B' - A')} \times \left[(A - B) \pm \sqrt{(A - B)^2 - 4(B' - A')\omega_0^2}\right],$$

which must be real and positive. This is achieved for $A > B$ and $A' < B'$ (i.e., for $v_{dt} > v_{rt}$ and $v_{dtk} < v_{rtk}$), as well as for $\omega_0^2 < \omega_c^2$, where

$$\omega_c^2 = \frac{(A - B)^2}{4(B' - A')} = \frac{(v_{dt} - v_{rt})^2}{4(v_{rtk} - v_{dtk})}.$$

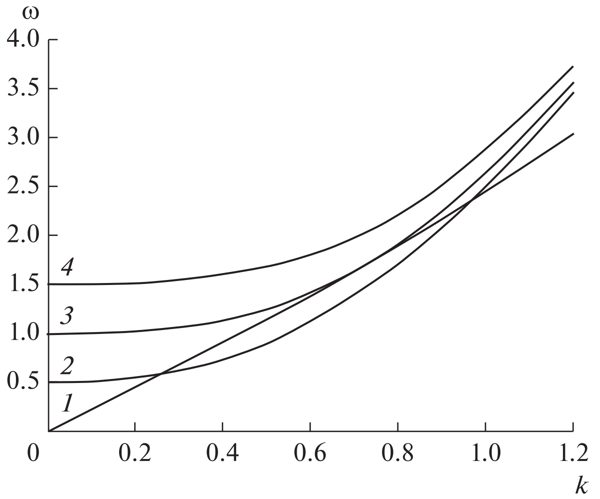

Fig. 1. Dispersion curves (for $A = 5$, $A' = 1$, $B = 1$, $B' = 5$, $\omega_c = 1$) for uncoupled transverse shear waves (curve *1*) and rotational waves (*2*, *3*, and *4* for $\omega_0 = 0.5$, 1.0, and 1.5, respectively).

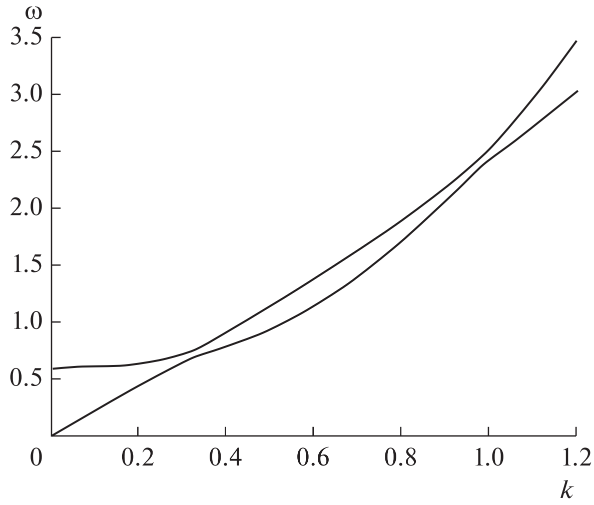

Fig. 2. Dispersion curves for $A = 5$, $A' = 1$, $B = 1$, $B' = 5$, $\omega_c = 1$, $\omega_0 = 0.5$, and $c_0 = 0.05$ for coupled hybrid transverse waves.

For $\omega < \omega_c$, two intersection points exist, which merge into one point at $\omega = \omega_c$; for $\omega > \omega_c$, the branches do not intersect, as shown in Fig. 1. If coupling exists, wave hybridization occurs (see Fig. 2).

SHEAR WAVE TRANSFORMATION INTO A ROTATION WAVE IN A MEDIUM WITH MONOTONIC DENSITY VARIATION

Figure 3a shows a general view of the graph of the "local" dispersion curves for transverse waves in regions of the medium with densities that differ twofold; Fig. 3b is an enlarged part of this graph that illustrates shear wave transformation into a rotational wave. The dotted line in Fig. 3b indicates the selected operating frequency; dots are the k value of the original shear wave and the k value of the rotational wave after this transformation. Obviously, the transformation of one wave to the other occurs in the region between two crossover points of the unperturbed dispersion curves. In all figures, arbitrary units are used for ω and k, since the values of the necessary elastic

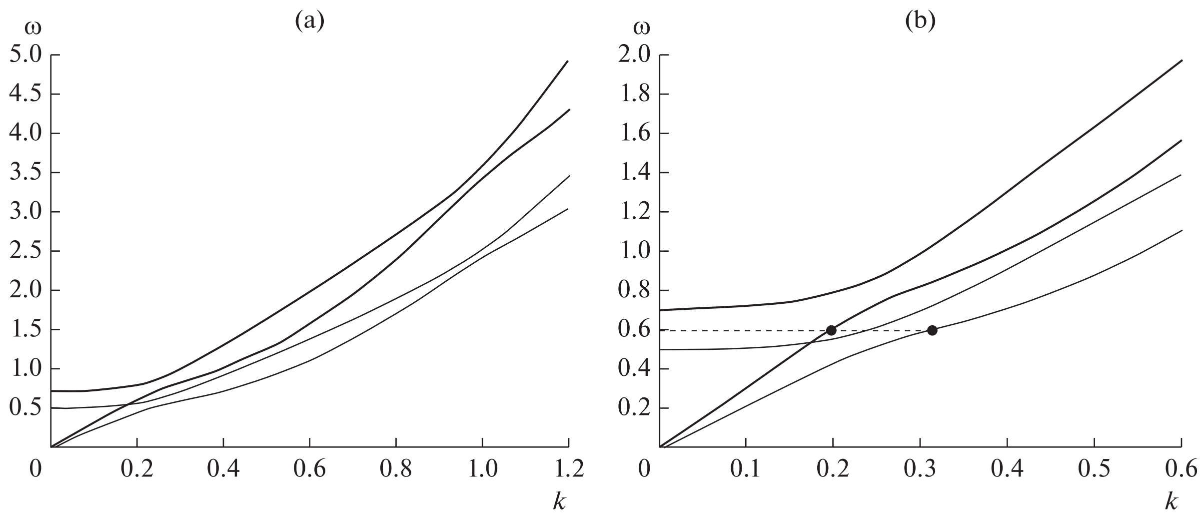

Fig. 3. Local dispersion curves for coupled hybrid transverse waves in region of ~~heterogeneous~~ micropolar medium with parameters $A = 5$, $A' = 1$, $B = 1$, $B' = 5$, $\omega_c = 1$, $\omega_0 = 0.5$, and $c_0 = 0.05$ (bold lines) and in region with parameters $A = 10$, $A' = 2$, $B = 2$, $B' = 10$, $\omega_c = \sqrt{2}$, $\omega_0 = 0.5\sqrt{2}$, $c_0 = 0.2$ (thin lines). (a) A general view and (b) enlarged part of plot (a).

constants for all of the previously studied real media are unknown [29, 30].

If the change in the wavenumbers of the coupled waves $k_{1,2}(\rho)$ in a medium with a density gradient $\rho(x)$ occurs rather slowly and satisfies the condition $dk_{1,2}/dx \ll k_{1,2}^2$, the Wentzel–Kramers–Brillouin method, i.e., the adiabatic approximation, can then be used to calculate the transformation coefficient, (see, e.g., [39]). This approach was used in [22–24] to analyze the transformation of different types of waves in magnets in ~~an inhomogeneous~~ internal field. Since the processes considered in these studies are described by wave equations similar in form to the equations that pertain to our case, the basic laws of transformation of different types of waves in both cases do not differ.

By analogy with the conclusions drawn in [24], the following can be stated. The transformation of a shear wave into a rotational wave occurs in the crossover region where their wavenumbers coincide at $C = 0$, while the transformation factor η_{dr} is mainly determined by the value of $\rho' = d\rho/dx$ in this region. Some critical value ρ'_{cr}, exists that depends on the parameters of the medium and the coupling coefficient between the waves. For $\rho' \gg \rho'_{cr}$, the transformation factor is small and determined by the expression $\eta_{dr} \approx \rho'_{cr}/\rho'$; for $\rho' \ll \rho'_{cr}$ and the transformation factor asymptotically tends to unity according to the law $\eta_{dr} \approx 1 - q\exp\left(-\rho'_{cr}/\rho'\right)$, where q is a numerical factor that depends on the particular form of the dependence $\rho(x)$. The transformation is accompanied by the origin of a reflected wave whose amplitude increases with ρ'.

CONCLUSIONS

The above analysis shows that effective mutual transformation of transverse elastic shear and rotational waves, which have spatial dispersion, is possible in a micropolar medium with monotonic density variation. Artificial micropolar media, e.g., epoxy composites with aluminum shot [29, 30], which make it easy to attain the necessary profile of changes in the mass density ρ, can serve as ideal media for observing this phenomenon. Note that strong interaction of transverse elastic shear and rotational waves may occur not only due to the gradient of ρ but also due to the density gradient of the moment of inertia J.

ACKNOWLEDGMENTS

The authors are grateful to O.A. Byshevskii-Konopko for help and useful advice.

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Translated by A. Seferov

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