The Boltzmann-Langevin approach: A simple quantum-mechanical derivation

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Abstract

We present a simple quantum-mechanical derivation of correlation function of Langevin sources in the semiclassical Boltzmann-Langevin equation. The specific case of electron-phonon scattering is considered. It is shown that the

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We present a simple quantum-mechanical derivation of correlation function of Langevin sor Boltzmann-Langevin equation. The specific case of electron-phonon scattering is considered assumption of weak scattering leads to the Poisson nature of the scattering fluxes.

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1. Introduction

Nonequilibrium electrical noise in mesoscopic systems was always the subject of primary interest for Markus shot noise in quantum-coherent conductors became a cornerstone of modern theory of fluctuations of shot noise in different mesoscopic systems with noninteractinose in colored to the calculations of shot affect the electrons ranging from double-barrier resonant tunnel didodes [2] to quantum-coherent metallic diffusive wires [3]. However this method has difficulty in describing interacting electrons or systems with dephasing. To circumvent it, one has to introduce dephasing probes [4, 5], i.e. ficting electrons or systems with dephasing. To circumvent it, one has to introduce dephasing probes [4, 5], i.e. ficting electrons or systems with dephasing. To circumvent of quantum-coherent electrons in the conductor by electrons of the dephasing probes have to be somehow related to the rate of actual microscopic scattering processes.

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even the noise in hybrid superconductor – normal-metal systems at voltages much higher than the Thouless energy [11]. More recently, it was extended to the case of spin-flip scattering in ferromagnetic spin valves [12] and applied to Coulomb drag in clean double-layer systems [13]

The key point in the Boltzmann – Langevin approach

is the derivation of the correlation function of Langevin sources. Kogan and Shul'man derived it assuming that the noise arises due to the randomness of electron scattering by impurities and phonons. It was also assumed that all scattering events are independent, hence the scattering of electrons between a pair of states at a given space point presents a Poisson process, whose spectral density is

Surprisingly, there were few attempts to derive the correlation function of Langevin sources directly from quantummechanical principles. In paper [14], this correlation function was calculated using a sophisticated extension of Keldysh diagrammatic technique, which involved time-ordering on a four-branch temporal contour. The current paper presents a much simpler quantum-mechanical derivation of this quantity, which does not require a diagrammatic technique.

The standard semiclassical distribution function of electrons $n_{\mathbf{p}}(\mathbf{r}t)$ presents the statistical average of the number of electrons in an element of phase space $\Delta p^3 \times \Delta x^3$ divided by the number of quantum states in this element $\Delta N = \Delta p^3 \Delta x^3/(2\pi\hbar)^3$, which is centred at point (**p**, **r**). This implies that the statistical averaging is performed on top of the coarse-grained averaging [15]. Once the distribution function is known, one may easily calculate different measurable quantities like charge or current density as linear functionals of it. It can be shown in many different ways [15, 16, 17] that this distribution function obeys the

$$\frac{\partial \bar{n}_{\mathbf{p}}}{\partial t} + \mathbf{v} \frac{\partial \bar{n}_{\mathbf{p}}}{\partial \mathbf{r}} + e\mathbf{E} \frac{\partial \bar{n}_{\mathbf{p}}}{\partial \mathbf{p}} = I_{col}\{\bar{n}_{\mathbf{p}}\},\tag{1}$$

where I_{col} is the collision integral that accounts for the electron scattering by impurities and phonons or electron electron scattering. This equation is valid provided that the semiclassical approximation holds. First, the characteristic length of spatial variation of $n_{\mathbf{p}}$ must be larger than the microscopic scale responsible for the scattering \hbar/p_c , where p_c is the characteristic momentum of an electron. Second, both the characteristic time of its variation and the inverse scattering rate must be larger than the characteristic time of the collision with impurity or phonon \hbar/ε_c , where ε_c is the characteristic energy of an electron.

Along with the average distribution function, one may also be interested in the correlation function of its fluctuations $\langle \delta n_{\mathbf{p}_1}(\mathbf{r}_1 t_1) \, \delta n_{\mathbf{p}_2}(\mathbf{r}_2 t_2) \rangle$, where the fluctuations $\delta n_{\mathbf{p}_i} =$ $n_{\mathbf{p}_i} - \bar{n}_{\mathbf{p}_i}$ are only coarse-grained-averaged, and the statistical averaging is applied to their product [18]. This quantity immediately gives the correlation functions of different observables. To calculate it, let us take a closer look at Eq. (1). Apart from the time derivative, the terms in the left-hand side describe the deterministic motion of electrons in the phase space due to smooth spatial variations of the distribution function and electrical potential. In contrast to this, the collision integral describes quantum-mechanical transitions of electrons between the states with different momentum, which are assumed to be local in space and time. These transitions are random and should be considered as the source of noise if the semiclassical description is used.

As the structure of the Boltzmann equation without the drift terms resembles the equation of motion of the Brownian particle, one may write the corresponding Langevin equation for the distribution function. To this end, $\bar{n}_{\mathbf{p}}$ should be replaced by $n_{\mathbf{p}}$ and a random Langevin source $\delta J_{\mathbf{p}}^{ext}(\mathbf{r}t)$ with zero average should be added [19] to the right-hand side of Eq. (1). As the duration of an electron collision with an impurity or phonon is much smaller than characteristic time of variation of $n_{\mathbf{p}}$, this source may be assumed to be delta correlated in time. Similarly, it should be delta correlated in space because of the local nature of the collisions. The momentum-dependent coefficient of these delta functions may be calculated as follows [20]. Choose an interval of time Δt much longer than the collision time \hbar/ε_c but so short that the distribution function cannot significantly change during this period. The direct integration of the Boltzmann - Langevin equation over time gives the increment of $n_{\mathbf{p}}$

$$\Delta n_{\mathbf{p}} \equiv n_{\mathbf{p}}(\mathbf{r},t+\Delta t) - n_{\mathbf{p}}(\mathbf{r}t) = \int_{0}^{\Delta t} d\tau \, \delta J_{\mathbf{p}}^{ext}(\mathbf{r},t+\tau). \eqno(2)$$

Hence the correlation function of two such increments is given by a double integral

$$\langle \Delta n_{\mathbf{p}_1}(\mathbf{r}_1) \, \Delta n_{\mathbf{p}_2}(\mathbf{r}_2) \rangle = \int_0^{\Delta t} d\tau_1 \int_0^{\Delta t} d\tau_2 \times \langle \delta J_{\mathbf{p}_1}^{ext}(\mathbf{r}_1, t + \tau_1) \, \delta J_{\mathbf{p}_2}^{ext}(\mathbf{r}_2, t + \tau_2) \rangle. \quad (3)$$

The delta function of $\tau_1 - \tau_2$ eliminates one of the integrations, and the other reduces to a multiplication by Δt . Hence it follows from Eq. (3) that

$$\langle \delta J_{\mathbf{p}_{1}}^{ext}(\mathbf{r}_{1}, t_{1}) \, \delta J_{\mathbf{p}_{2}}^{ext}(\mathbf{r}_{2}, t_{2}) \rangle = \delta(t_{1} - t_{2}) \, \delta(\mathbf{r}_{1} - \mathbf{r}_{2})$$

$$\times \lim_{\Delta t \to 0} \frac{\langle \Delta n_{\mathbf{p}_{1}}(\mathbf{r}_{1}) \, \Delta n_{\mathbf{p}_{2}}(\mathbf{r}_{1}) \rangle}{\Delta t}. \quad (4)$$

This is our basic formula for calculating the correlation function of Langevin sources.

3. Equations of motion for the operators

To carry out the calculations to the end, we have to calculate the ratio in Eq. (4) using quantum mechanics. Consider the particular case of electron–phonon scattering in an elementary volume of size Δx much smaller than the characteristic length at which the average distribution function or the electrical potential essentially change but much larger than \hbar/p_c . This allows us to describe the scattering with a locally uniform Hamiltonian. The Hamiltonian of the system $\hat{H} = \hat{H}_0 + \hat{V}$ is the sum of the noniteracting part

$$\hat{H}_{0} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} \, \hat{a}_{\mathbf{p}}^{+} \hat{a}_{\mathbf{p}} + \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} \, \hat{b}_{\mathbf{k}}^{+} \hat{b}_{\mathbf{k}}$$
 (5)

and the part describing the electron-phonon interaction

$$\hat{V} = \sum_{\mathbf{p},\mathbf{k}} (V_{\mathbf{k}} \hat{b}_{\mathbf{k}} + V_{-\mathbf{k}}^* \hat{b}_{-\mathbf{k}}^+) \, \hat{a}_{\mathbf{p}+\hbar\mathbf{k}}^+ \hat{a}_{\mathbf{p}}, \tag{6}$$

where $V_{\bf k}$ are the matrix elements of electron–phonon interaction, and $\hat{a}_{\bf p}$ and $\hat{b}_{\bf k}$ are the annihilation operators for electrons and phonons, respectively. The time-dependent occupation-number operator $\hat{n}_{\bf p}=\hat{a}^{\dagger}_{\bf p}\hat{a}_{\bf p}$ obeys the Heisenberg equation [21]

$$\frac{d\hat{n}_{\mathbf{p}}}{dt} = \frac{i}{\hbar} [\hat{H}, \hat{n}_{\mathbf{p}}(t)]. \tag{7}$$

As $[\hat{n}_{\mathbf{p}}, \hat{H}_0] = 0$, the time dependence of this operator is determined only by the weak electron–phonon coupling and may be considered as slow. Therefore Eq. (7) may be solved by iterations in \hat{V} . To this end, we perform a unitary transformation of all operators

$$\tilde{A}(t,\tau) = e^{-\frac{i}{\hbar}\hat{H}_0(t)\tau} \,\hat{A}(t) \, e^{\frac{i}{\hbar}\hat{H}_0(t)\tau}, \tag{8}$$

which brings Eq. (7) to the form

$$\frac{d\tilde{n}_{\mathbf{p}}}{d\tau} = \frac{i}{\hbar} \left[\tilde{V}(t, \tau), \tilde{n}_{\mathbf{p}} \right]. \tag{9}$$

If the time interval Δt is much shorter than the relaxation time of the distribution function due to the collisions, the increment of $\tilde{n}_{\mathbf{p}}$ may be calculated to the second order in \tilde{V} , and then the transformation inverse to Eq. (8) gives

$$\Delta \hat{n}_{\mathbf{p}} \equiv \hat{n}_{\mathbf{p}}(t + \Delta t) - \hat{n}_{\mathbf{p}}(t)$$

$$= \frac{i}{\hbar} \int_{0}^{\Delta t} d\tau \left[\tilde{V}(t, \tau - \Delta t), \, \hat{n}_{\mathbf{p}}(t) \right] - \frac{1}{\hbar^{2}} \int_{0}^{\Delta t} d\tau' \int_{0}^{\tau'} d\tau''$$

$$\times [\tilde{V}(t,\tau'-\Delta t), [\tilde{V}(t,\tau''-\Delta t), \hat{n}_{\mathbf{p}}(t)]], \quad (10)$$

where

$$\tilde{V}(t,\tau) = \sum_{\mathbf{p},\mathbf{k}} \left(V_{\mathbf{k}} e^{i\omega_{\mathbf{k}}\tau} \hat{b}_{\mathbf{k}} + V_{-\mathbf{k}}^* e^{-i\omega_{-\mathbf{k}}\tau} \hat{b}_{-\mathbf{k}}^+ \right) \\
\times e^{\frac{i}{\hbar} (\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p} + \hbar\mathbf{k}})\tau} \hat{a}_{\mathbf{p} + \hbar\mathbf{k}}^+ \hat{a}_{\mathbf{p}} \quad (11)$$

and all fermionic and bosonic operators are taken at time t. The density matrix of the system is assumed to be factorized into the electron and phonon parts, which are diagonal in the same representation as \hat{H}_0 . Therefore upon averaging Eq. (10), the first summand vanishes and in the second summand, only diagonal terms are left. The average products of four fermionic operators are decoupled into products of pair averages, e.g. $\langle \hat{a}^+_{\mathbf{p}+\hbar\mathbf{k}}\hat{a}_{\mathbf{p}}\,\hat{a}^+_{\mathbf{q}-\hbar\mathbf{k}}\hat{a}_{\mathbf{q}}\rangle = \delta_{\mathbf{p}+\hbar\mathbf{k},\mathbf{q}}\,\hat{n}_{\mathbf{q}}(1-n_{\mathbf{p}})$. As a result, all the arguments of exponents are proportional to the difference $\tau'-\tau''$, and

$$\langle \Delta \hat{n}_{\mathbf{p}} \rangle = -\frac{2}{\hbar^{2}} \int_{0}^{\Delta t} d\tau' \int_{0}^{\tau'} d\tau'' \sum_{\mathbf{k}} \left(\left\{ |V_{-\mathbf{k}}|^{2} \cos[(\hbar \omega_{-\mathbf{k}} + \varepsilon_{\mathbf{p} + \hbar \mathbf{k}} - \varepsilon_{\mathbf{p}})(\tau' - \tau'') / \hbar \right] (1 + N_{-\mathbf{k}}) + |V_{\mathbf{k}}|^{2} \cos[(\hbar \omega_{\mathbf{k}} + \varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p} + \hbar \mathbf{k}})(\tau' - \tau'') / \hbar \right] N_{\mathbf{k}} \right\} \times n_{\mathbf{p}} (1 - n_{\mathbf{p} + \hbar \mathbf{k}}) - \left\{ |V_{-\mathbf{k}}|^{2} \cos[(\hbar \omega_{-\mathbf{k}} + \varepsilon_{\mathbf{p} + \hbar \mathbf{k}} - \varepsilon_{p})(\tau' - \tau'') / \hbar \right] N_{-\mathbf{k}} + |V_{\mathbf{k}}|^{2} \cos[(\hbar \omega_{\mathbf{k}} + \varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p} + \hbar \mathbf{k}})(\tau' - \tau'') / \hbar \right] (1 + N_{\mathbf{k}}) \right\} \times n_{\mathbf{p} + \hbar \mathbf{k}} (1 - n_{\mathbf{p}}), \quad (12)$$

where $N_{\mathbf{k}} = \langle b_{\mathbf{k}}^{+} b_{\mathbf{k}} \rangle$ are the occupation numbers of phonon states. The double integration of Eq. (12) over τ' and τ'' can be replaced by a sequential integration over $\tau' - \tau''$ and τ' . If Δt is much larger than the inverse characteristic frequencies in the arguments of the cosine functions, the integration over $\tau' - \tau''$ gives delta functions of its coefficients, and the subsequent integration over τ' reduces to a multiplication by Δt . Therefore

$$\frac{\langle \Delta \hat{n}_{\mathbf{p}} \rangle}{\Delta t} = \sum_{\mathbf{k}} [J(\mathbf{p} + \hbar \mathbf{k} \to \mathbf{p}) - J(\mathbf{p} \to \mathbf{p} + \hbar \mathbf{k})], \quad (13)$$

where

$$J(\mathbf{p} \to \mathbf{p} + \hbar \mathbf{k}) = \frac{2\pi}{\hbar} \left[|V_{-\mathbf{k}}|^2 \, \delta(\hbar \omega_{-\mathbf{k}} + \varepsilon_{\mathbf{p} + \hbar \mathbf{k}} - \varepsilon_{\mathbf{p}}) \right.$$

$$\times (1 + N_{-\mathbf{k}}) + |V_{\mathbf{k}}|^2 \, \delta(\hbar \omega_{\mathbf{k}} + \varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p} + \hbar \mathbf{k}}) \, N_{\mathbf{k}} \right]$$

$$\times n_{\mathbf{p}} \left(1 - n_{\mathbf{p} + \hbar \mathbf{k}} \right) \quad (14)$$

is the scattering flux from state \mathbf{p} to state $\mathbf{p} + \hbar \mathbf{k}$. The right-hand side of Eq. (13) is exactly the electron–phonon collision integral I_{col} that appears in Eq. (1).

Now we calculate the average product of two increments $\Delta \hat{n}_{\mathbf{p}_1}$ and $\Delta \hat{n}_{\mathbf{p}_2}$ with the same t and Δt . To the second order in the interaction potential,

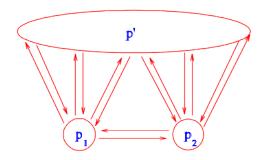


Figure 1: (color online). The scattering fluxes that appear in Eq. (16). States \mathbf{p}_1 and \mathbf{p}_2 exchange electrons with each other and with the rest of states \mathbf{p}' .

$$\langle \Delta n_{\mathbf{p}_1} \, \Delta n_{\mathbf{p}_2} \rangle = -\frac{1}{\hbar^2} \int_0^{\Delta t} d\tau_1 \int_0^{\Delta t} d\tau_2$$

$$\times \langle [\tilde{V}(t, \tau_1 - \Delta t), \, \hat{n}_{\mathbf{p}_1}(t)] \, [\tilde{V}(t, \tau_2 - \Delta t), \, \hat{n}_{\mathbf{p}_2}(t)] \rangle. \quad (15)$$

The averaging is performed according to the same rules as in Eq. (10), and the arguments of complex exponents appear to be proportional to the difference $\tau_1 - \tau_2$. Much like when calculating the collision integral, we replace the double integration over τ_1 and τ_2 by a sequential integration over $\tau_1 - \tau_2$ and over τ_1 . The former integration transforms the exponents in (15) into delta functions of coefficients of $\tau_1 - \tau_2$, and the latter reduces to multiplication by Δt . Therefore one obtains

$$\frac{\langle \Delta n_{\mathbf{p}_1} \, \Delta n_{\mathbf{p}_2} \rangle}{\Delta t} = \delta_{\mathbf{p}_1 \mathbf{p}_2} \sum_{\mathbf{k}} [J(\mathbf{p}_1 \to \mathbf{p}_1 + \hbar \mathbf{k})
+ J(\mathbf{p}_1 + \hbar \mathbf{k} \to \mathbf{p}_1] - J(\mathbf{p}_1 \to \mathbf{p}_2) - J(\mathbf{p}_2 \to \mathbf{p}_1), \quad (16)$$

where the scattering fluxes J are given by Eq. (14). The substitution of this ratio into Eq. (4) gives precisely the same expression for the correlation function of Langevin sources that was originally obtained in Ref. [6]. The full expression for it is given in Appendix A by Eq. (A.1). The physical interpretation of this expression was discussed in the above paper and is illustrated in Fig. 1. The arrows show the scattering fluxes between different pairs of local states, which are independent of each other. If states \mathbf{p}_1 and \mathbf{p}_2 coincide, all the outgoing and incoming scattering fluxes are correlated with themselves and give contributions proportional to their average rates. If these states are different, the contribution is given only by $J(\mathbf{p}_1 \to \mathbf{p}_2)$ and $J(\mathbf{p}_2 \to \mathbf{p}_1)$. Our calculations show that the independence of different scattering fluxes and their Poisson statistics are the direct consequences of the assumption of weak interaction of electrons with the scatterers.

The Boltzmann equation Eq. (1) is nonlinear because of the electron–phonon collision integral (13). However in actual problems, the fluctuations of the distribution function are effectively averaged over macroscopic lengths much larger than Δx and therefore are small. Hence the Boltzmann - Langevin equation may be obtained from Eq. (1) by linearizing it with respect to the fluctuation $\delta n_{\mathbf{p}}$ and

adding the Langevin source to the right-hand side

$$\frac{\partial \delta n_{\mathbf{p}}}{\partial t} + \mathbf{v} \frac{\partial \delta n_{\mathbf{p}}}{\partial \mathbf{r}} + e \mathbf{E} \frac{\partial \delta n_{\mathbf{p}}}{\partial \mathbf{p}} = \delta I_{e-ph} + \delta J_{\mathbf{p}}^{ext}.$$
 (17)

Together with the expression for the correlation function of Langevin sources given by Eqs. (4) and (16), it presents the full set of equations for calculating the noise.

4. Conclusions

We have derived the correlation function of Langevin sources in the Boltzmann – Langevin equation for the fluctuations of the semiclassical distribution function. It was postulated that the sources are delta correlated in space and time, but the momentum-dependent part was rigorously calculated using quantum-mechanical equation of motion for the operators of occupation numbers. If the electron–phonon interaction is assumed to be weak, the scattering fluxes between different pairs of electron states may be considered as independent Poisson processes, and the correlation function of Langevin sources is the sum of their correlation functions taken with the appropriate signs. This coincides with the conditions for the validity of the standard collision integral in the Boltzmann equation.

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Appendix A.

The full correlation function of Langevin sources for the case of electron–phonon scattering is given by the following expression:

$$\begin{split} \langle \delta J_{\mathbf{p}_{1}}^{ext}(\mathbf{r}_{1},t_{1}) \, \delta J_{\mathbf{p}_{2}}^{ext}(\mathbf{r}_{2},t_{2}) \rangle &= \delta(t_{1}-t_{2}) \, \delta(\mathbf{r}_{1}-\mathbf{r}_{2}) \\ \times \frac{2\pi}{\hbar} \sum_{\mathbf{k}} \Biggl\{ \delta_{\mathbf{p}_{1},\mathbf{p}_{2}} \Big[|V_{\mathbf{k}}|^{2} \, \delta(\varepsilon_{\mathbf{p}_{1}+\hbar\mathbf{k}}-\varepsilon_{\mathbf{p}_{1}}-\hbar\omega_{\mathbf{k}}) \, (1+N_{\mathbf{k}}) \\ &+ |V_{-\mathbf{k}}|^{2} \, \delta(\varepsilon_{\mathbf{p}_{1}+\hbar\mathbf{k}}-\varepsilon_{\mathbf{p}_{1}}+\hbar\omega_{-\mathbf{k}}) \, N_{-\mathbf{k}} \Big] \\ &\times n_{\mathbf{p}_{1}+\hbar\mathbf{k}} \, (1-n_{\mathbf{p}_{1}}) \\ &+ \delta_{\mathbf{p}_{1},\mathbf{p}_{2}} \Big[|V_{-\mathbf{k}}|^{2} \, \delta(\varepsilon_{\mathbf{p}_{1}}-\varepsilon_{\mathbf{p}_{1}+\hbar\mathbf{k}}-\hbar\omega_{-\mathbf{k}}) \, (1+N_{-\mathbf{k}}) \\ &+ |V_{\mathbf{k}}|^{2} \, \delta(\varepsilon_{\mathbf{p}_{1}}-\varepsilon_{\mathbf{p}_{1}+\hbar\mathbf{k}}+\hbar\omega_{\mathbf{k}}) \, N_{\mathbf{k}} \Big] \\ &\times n_{\mathbf{p}_{1}} \, (1-n_{\mathbf{p}_{1}+\hbar\mathbf{k}_{1}}) \\ &- \delta_{\hbar\mathbf{k},\mathbf{p}_{2}-\mathbf{p}_{1}} \Big[|V_{\mathbf{k}}|^{2} \, \delta(\varepsilon_{\mathbf{p}_{2}}-\varepsilon_{\mathbf{p}_{1}}-\hbar\omega_{\mathbf{k}}) \, (1+N_{\mathbf{k}}) \\ &+ |V_{-\mathbf{k}}|^{2} \, \delta(\hbar\omega_{-\mathbf{k}}+\varepsilon_{\mathbf{p}_{2}}-\varepsilon_{\mathbf{p}_{1}}) \, N_{-\mathbf{k}} \Big] \\ &\times (1-n_{\mathbf{p}_{1}}) \, n_{\mathbf{p}_{2}} \\ &- \delta_{\mathbf{k},\mathbf{p}_{1}-\mathbf{p}_{2}} \Big[|V_{\mathbf{k}}|^{2} \, \delta(\varepsilon_{\mathbf{p}_{1}}-\varepsilon_{\mathbf{p}_{2}}-\hbar\omega_{\mathbf{k}}) \, (1+N_{\mathbf{k}}) \end{split}$$

$$+ |V_{-\mathbf{k}}|^2 \delta(\hbar\omega_{-\mathbf{k}} + \varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}_2}) N_{-\mathbf{k}} \Big] \times n_{\mathbf{p}_1} (1 - n_{\mathbf{p}_2}) \bigg\}. \quad (A.1)$$

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