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Study of an electrodynamic system consisting of a laser cavity and an external weakly reflecting element

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Abstract. **The behaviour of the electromagnetic field in a three-mirror laser cavity is described using the method of integral equations. The results of numerical calculations and experimental studies for particular examples implementing the considered configuration are presented. The conditions for optimal tuning of a laser interferometer-deformograph with a three-mirror cavity are determined. The contribution of the reflected and scattered light and arising additional seismic noises to the resulting error of laser gravitational wave detectors is studied.**

Keywords: *laser, interferometer, deformograph, backscattering, gravitational wave detector.*

1. Introduction

The methods for calculating resonance frequencies (eigenfrequencies, modes) of an isolated quasi-optical resonator with spherical (in the 2D case, cylindrical) mirrors are developed and studied sufficiently well (see, e.g., review [1]). However, this two-mirror electrodynamic system incorporated in any laser measurement device as a constituent part acquires special characteristics and properties in the case of introducing a third reflecting or scattering element. This situation always exists in practice, since any optical load cannot be completely matched. Even if special measures have been undertaken, part of the reflected and scattered light returns into the cavity of the laser and introduces distortions into its spectral characteristics. The application of this effect, in particular, for open-cavity mode selection, is considered in Ref. [1]. Due to recent progress in high-stability lasers and high-accuracy measurement methods and devices, including interferometric ones, the influence of the unmatched load on the optical oscillator operation, even at rather weak backscattering, has become an extremely urgent problem.

The simplest model of such an electrodynamic system is a three-mirror cavity, in which an external weakly reflecting mirror plays the role of the unmatched load. The studies of three-mirror laser cavities, that began practically simultaneously with the invention of the first lasers [2, 3] are still going on [4, 5], which reflects the complexity of the problem and its importance for practical applications. In using the laser as a source of highly coherent light, e.g., in precise interference measurements, the instabilities of both the intensity and the frequency of the laser become important. The effect of frequency is particularly tangible in long-baseline interferometry. The analysis shows that the frequency instability is caused not only by random fluctuations of the laser cavity parameters, but also by fluctuations of the length and refractive index of the external cavity medium, formed by a certain reflecting or scattering optical element, e.g., a photodetector, in the laser frequency stabilisation system.

In the present paper as a two-dimension physical model of the laser with an unmatched load, we study a three-mirror cavity, consisting of partially transmitting mirrors, the transverse dimensions of which in the quasi-optical approximation are large in comparison with the transverse dimensions of the laser beam. We study three cases: the intermediate mirror is planar; the lengths of the partial cavities are equal; and the cavities are not matched. To describe the electromagnetic field in the three-mirror cavity the method of integral equations is used. We present the results of numerical calculations and experimental studies for some examples implementing such systems.

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2. Method for calculating eigenfrequencies of a three-mirror laser cavity

Consider the solution of the formulated problem by the example of a 2D (x, z) model of a compound cavity, formed by three mirrors M_1, M_2, and M_3. These mirrors are characterised by the following real-valued parameters: the transverse dimensions a_1, a_2, a_3; the radii of curvature R_1, R_2, R_3; and the transparencies ρ_1, ρ_2, ρ_3, related to the complex reflection coefficients r_1, r_2, r_3 of these mirrors by the formula [6]

$$r_n = -\frac{1}{1+2\mathrm{i}\rho_n}, \quad n = 1, 2, 3. \qquad (1)$$

As the potential $U(x,z)$ we choose the y-component of the electric vector of the electromagnetic field $U(x,z) = E_y(x,z)$. The function $U(x,z)$ satisfies the wave equation and the continuity conditions at each semitransparent mirror M_n. The jump of the normal derivative of the function $U(x,z)$ at the mirrors M_n is proportional to the currents on them. The values of these currents f_n are inversely proportional to the transparencies ρ_n and expressed as

$$f_n = \frac{1}{\rho_n} U_n, \quad n = 1, 2, 3, \qquad (2)$$

where $U_n(x,z) = E_{yn}(x,z)$ is the electric field at each of the semitransparent mirrors M_n.

If the transparency $\rho = 0$, then we have an ideally reflecting mirror described by the boundary condition $U = 0$ at its surface. If $\rho = \infty$, then the mirror is absolutely transparent and does not interact with the incident electromagnetic wave. Physically, the mirrors having such properties can be implemented as a short periodic (in the wavelength λ scale) aray of strip conductors. The conductors are oriented along the y axis perpendicular to the xz plane; the space factor determines the mirror transparency.

Calculating the fields at each of the mirrors, with allowance for the currents running on them, and using the Green function of the free space in the standard quasi-optical approximation, we arrive at a system of three integral equations [5]. The solution of these equations in the approximation of unlimited mirrors ($a_n = \infty$) is sought in the form of Gaussian beams. The equations are solved by integrating the terms in one side and equating the exponents of the terms in both sides of the equations. The resulting relations are valid if the matching conditions are satisfied for the spot dimensions in all three partial two-mirror cavities M_1M_2, M_2M_3, and M_1M_3. If these conditions are not satisfied, then there is no simple solution of the system in the form of Gaussian functions. We analysed the cases when the obtained system of integral equations can be reduced to a simpler systems of two integral equations, as well as the particular case $a_n = \infty$, $R_n = \infty$ (all three mirrors are unlimited and planar), when for the currents on mirrors it is possible to obtain a homogeneous system of two algebraic equations. The equality to zero of the determinant of this system yields the corresponding dispersion equation for the calculation of eigenfrequencies of the three-mirror cavity:

$$r_1 r_{23} \exp(-2ikL_{12}) = 1, \tag{3}$$

where

$$r_{23} = r_2 + \frac{t_2^2 r_3 \exp(-2ikL_{23})}{1 - r_2 r_3 \exp(-2ikL_{23})}; \tag{4}$$

$t_2 = 2i\rho_2/(1 + 2i\rho_2)$ is the transmission coefficient of the mirror M2 [6]; L_{12} and L_{23} are the lengths of the partial cavities; and k is the wavenumber.

In contrast to the standard dispersion equation for a two-mirror laser cavity, Eqn (3) allows for the dependence of one of the involved transmission coefficients on the length L_{23} of the external cavity and the laser frequency. At certain relations between the parameters of the coupled partial cavities M_1M_2 and M_2M_3 these dependences appear strong enough, and the insignificant variations of the length L_{23} (e.g., due to the microseismic or acoustic vibrations) lead to significant fluctuations of the resonance frequencies of the studied three-mirror system.

3. Numerical modelling of the behaviour of eigenfrequencies

Using the derived dispersion equation (3) for the three-mirror laser cavity, the frequency shift in the two-mirror cavity M_1M_2 under the addition of the mirror M_3 is calculated. In the approximation of weakly-coupled cavities, $|r_3| \ll |r_2|$, Eqn (3) has the analytic solution. The frequency shift $\Delta\omega$ under the additional condition [5]

$$\omega \ll c/L_{23}, \tag{5}$$

where c, the velocity of light, is a harmonic function of the length of the external cavity M_2M_3. To calculate this shift one can use the simplest formulae [3, 4]. If the above conditions of weak cavity coupling are not valid, the transcendental dispersion equation (3) becomes rather complicated and its solution requires numerical methods.

The calculations were carried out using MATLAB, which allowed the determination of the region of parameters, in which the frequency shift was described by a single-valued quasi-harmonic function of length L_{23} (Fig. 1a). We have also found the critical values of these parameters, at which the solution of Eqn (3) becomes multivalued, and the frequency shift corresponding to the variation of the length L_{23} of the external cavity can take two or three values (Fig. 1b). If the variations of the length L_{23} are of random sign-alternating character (e.g., due to turbulence fluctuations of air or microseismic ground vibrations), one can observe discontinuities, jumps and other nonlinear effects in the dependence of the frequency on L_{23} [7]. The position of the frequency in these regions becomes unstable, and the precision of the long-baseline interferometric measurements becomes worse by two-three orders of magnitude.

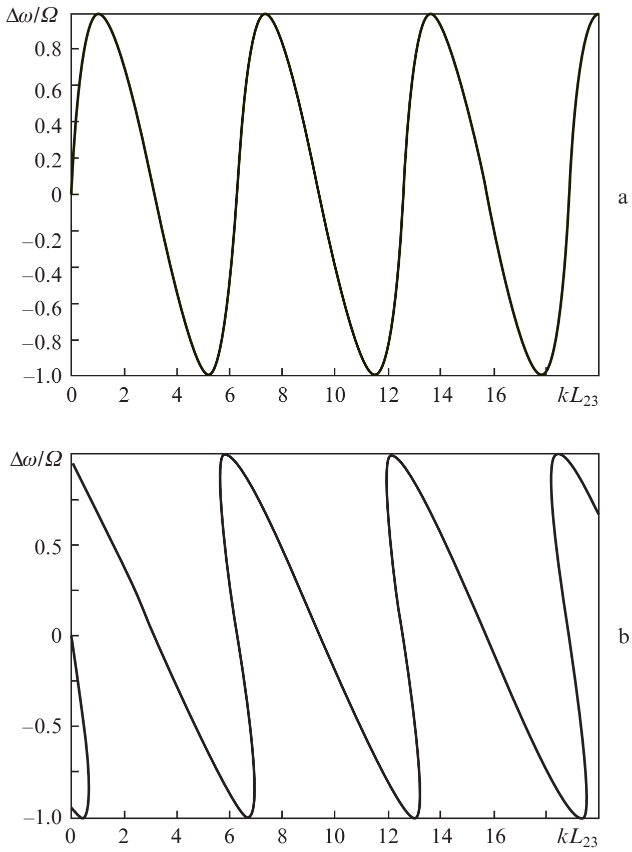

Figure 1. Frequency shift $\Delta\omega$ in a three-mirror laser interferometer as a function of length of the external cavity L_{23} for the coupling smaller than (a) and greater than (b) the critical one; $\Omega = c|r_3||t_2|^2/(2L_{12}|r_2|)$.

4. Experimental study of the three-mirror laser cavity operation

We studied the operating parameters of the three-mirror laser cavity, included into the scheme of a long-baseline interferometer with weak and critical feedback. The distinctive feature of the studied construction is the simultaneous He–Ne laser oscillation on two coupled transitions with $\lambda = 0.63$ and 3.39 μm [8]. We used the two-wave He–Ne laser with thermal regulation of the cavity length, providing the output power and frequency control. In the measurement scheme based on the three-mirror interferometer, the modulation of the laser light was implemented using the reflected and backscattered light due to the optical feedback. The experiments were carried out with the gas-discharge He–Ne tubes having internal mirrors and the cavity length 23 and 30 cm, providing the two- and three-frequency oscillation modes at $\lambda = 0.63$ μm and the single-frequency oscillation mode at $\lambda = 3.39$ μm. To control the temperature of the laser case, the high-precision multichannel temperature meter based on the MIT-8 device with a resolution 0.001 K within the range 273–378 K [9] was used. For the laser cavity fixed in the steel tube, the estimates of the temperature expansion coefficient α and the time constant τ, characterising the thermal inertia of the laser cavity, amount to $\sim$1.14 deg^{-1} and $\sim$1 h, respectively.

The two-wave laser considered above was used to construct three-mirror interferometric meters of displacements and deformations. The optical scheme of one of the versions of such a device is shown in Fig. 2 [10]. When the length of the laser cavity changed due to heating, we experimentally observed instability zones in the system of recording the interferogram shift, due to the impact of the reflected light on the spectral composition of laser radiation. Figure 3 illustrates typical examples of operation of two laser interferometers-deformographs having the baseline length 300 m (the laser beam protected by the underground beam guide, Fig. 3a) and 66 m (the laser beam propagates in the open atmosphere, Fig. 3b). Abrupt vertical discontinuities in both dependences (marked by an asterisk) correspond to the origin translation of the registration system by the quantity ΔL_N, a multiple of the half-wavelength: $\Delta L_N = N\lambda/2$ (N = 3, ..., 8). The amplitudes of microseismic ground vibrations and random fluctuations due to the air turbulence in both interferometers exceed the laser wavelength, i.e., $k\Delta L_{23} > 2\pi$.

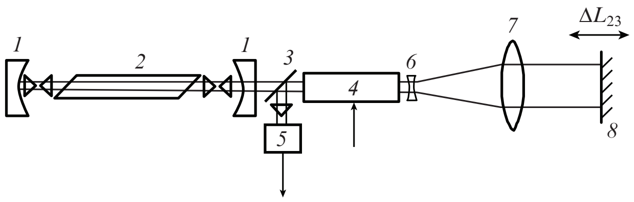

Figure 2. Optical scheme of the three-mirror laser interferometer-deformograph: (*1*) laser mirrors; (*2*) active medium of the laser; (*3*) beam splitter; (*4*) electro-optic modulator; (*5*) photodetector; (*6*, *7*) lenses of the matching telescope; (*8*) measuring mirror.

In the zone of unstable oscillation of the 66-metre three-mirror laser interferometer (the left-hand part of Fig. 3b), the operation of the registration system is disturbed due to the considerable fast fluctuations of the laser frequency. The recorded signal is subject to random chaotic distortions. With the cavity length in the free oscillation regime continuously scanned, such zones appear periodically. It is possible to avoid their appearance by tuning the laser oscillation frequency, i.e., varying the cavity length within ±157 nm, which corresponds to the width of its stable operation zone.

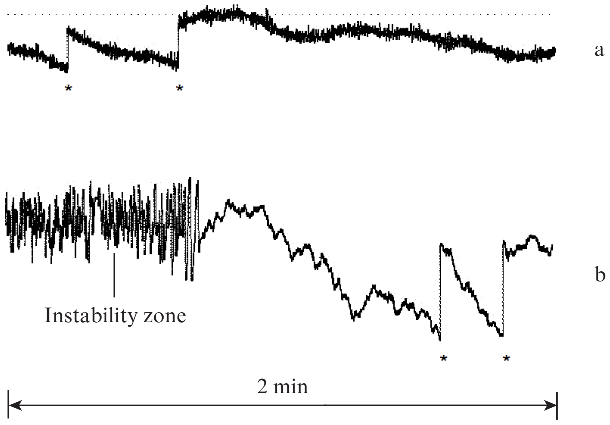

Figure 3. Optical length variations recorded by interferometers-deformographs with the baseline length 300 m and the underground beam guide (a) and the baseline length 66 m in open atmosphere (b). The asterisk marks the automatic translations of the origin of the registration system by $N = 3,\ldots,8$ periods of the interferogram.

5. Backscattering and seismic noises of the laser gravitational wave detector

More complicated electrodynamic systems are presented by laser interferometers based on the Michelson double-beam scheme. In particular, this relates to the laser interferometric gravitational wave detectors, which besides the basic (master and slave) laser cavities comprise a variety of additional optical elements and devices, forming external partial cavities.

The contribution of the reflected and scattered radiation to the resulting error of laser gravitational wave detectors was studied in Ref. [11]. It was shown that in the long-baseline (0.3–4 km) interferometers, constructed abroad according to the symmetric Michelson or Fabry–Perot schemes, the effects of backscattering can introduce additional uncontrollable errors into the measurement results. The fluctuations of intensity and frequency arising under these conditions in the used solid-state stabilised lasers [12] will be comparable with such major sources of interference as the technical and quantum noises in laser oscillators and photodetectors. The residual seismic noises and other natural and technogenic noises should be also taken into account [13, 14].

In the upgradable gravitational wave detectors, comprising complicated optical systems (recirculators, optical insulators, seismic compensators and other regulating and controlling devices) the reflected and scattered radiation will be inevitably present, which distort the resonance frequencies of both the operating mutimirror laser systems and the reference optical cavities, used to stabilise the frequency. The lengths of the partial cavities L_{2n}, not included in the automated control and vibrostability circuits, will experience random fluctuations under the impact of permanently present microseismic background and acoustic noises.

The arising additional noises in the systems of interferometric registration, providing the instrumental phase resolution 10^{-9}–10^{-10} rad in the frequency range 10 Hz–10 kHz, can appear comparable with the residual seismic and other technological noises or even exceed them. Indeed, in the up-to-date gravitational wave detectors the presence of backscattering at the level of 10^{-6} of the direct radiation power [15] (which corresponds to the parameter $|r_3| = 0.001$) will inevitably cause essential frequency errors. Without special measures undertaken, the distortions can arise, e.g., in the system stabilising the high-power lasers used in these detectors [12]. Thus, the centre frequency shift $\Delta\omega$ of the high-Q reference cavity (the parameter $|t_2|^2 \approx 10^{-4}$) [12] having the length L_{12} = 0.2 m will amount to ~70–80 s^{-1} (Figs 1 and 2), which is by one-two orders of magnitude greater than the limiting level of frequency noises for the constructions of these instruments [12–14].

The real presence of above noises, related to the reflected and scattered radiation, is confirmed by the results of a detailed analysis of error sources for the last and most advanced prototypes of gravitational wave detectors, Advanced LIGO Hl, Ll [15]. Thus, the effects considered in the previous Sections should be taken into account in the interpretation of the results for already existing gravitational wave detectors, as well as in the development of new improved devices.

6. Conclusions

Using the method of integral equations, we have studied the effect of the unmatched load on the operation of the optical oscillator at different levels of the backscattered and reflected radiation. The parameter regions were found, in which the frequency shift in the system is described by a single-valued quasi-harmonic function, as well as the critical parameter values, when the frequency shift can take two or three values. The frequency behaviour in these regions becomes unstable and the precision of the interferometric measurements is worsened.

In the three-mirror laser interferometer, we have found the zones of unstable laser operation in the two-wave oscillation regime. The conditions of optimal tuning, providing the stable operation of a laser interferometer-deformograph with a three-mirror cavity, are determined. The peculiarities of operation of the two-wave He–Ne laser in the oscillation regime on coupled transitions with λ = 3.39 and 0.63 μm are considered.

The possible contribution of the reflected and scattered light to the resulting error of laser gravitational wave detectors is analysed. The estimates of the arising additional seismic and acoustic noises in the interferometric registration systems are presented.

The results of the present work can be used in the design of two-wave sources of coherent laser light and the measurement facilities based on them for application in interferometry, metrology, geophysics, as well as for the analysis of the data from the existing gravitational wave detectors and development of novel advanced high-precision instruments of different purpose.

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