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Resonant Wave Scattering on a Finite Supercritical Plasma Layer

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Abstract—The two-dimensional problem of excitation of a finite supercritical plasma layer by a cylindrical wave has been numerically studied. It is established that the frequency dependence of the excited field intensity has a resonant character for plasma-layer parameters close to values corresponding to a triply degenerate surface wave in the case of an infinite plasma layer. The values of near and far fields at the resonance frequency are calculated.

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This Letter considers the two-dimensional (2D) problem of excitation of a finite supercritical plasma layer (or a plate of metamaterial with negative relative dielectric permittivity ε and positive relative magnetic permeability $\mu = 1$) by a cylindrical wave from a filamentary source. Let the plasma layer have a thickness of $2a$, a length of $2b$, and real negative permittivity $\varepsilon < 0$ (Fig. 1a). The incident wave is assumed to be TM polarized, whereby the electromagnetic field components are $H_z(x, y)$, $E_x(x, y)$, and $E_y(x, y)$. In this case, the problem of wave diffraction reduces to finding a scalar function $U(x, y) = H_z(x, y)$, which must satisfy the corresponding Helmholtz equations inside and outside the layer, as well as necessary boundary conditions on the plate surfaces and the condition of radiation at infinity. Incident wave field $U_0(r, \varphi)$ is set in the following form:

$$U_0(r, \varphi) = H_0^{(2)}(k\sqrt{r^2 + r_0^2 - 2rr_0\cos(\varphi - \varphi_0)}), \quad (1)$$

where $H_0^{(2)}$ is the Hankel function; k is the wavenumber of the free space; and (r, φ) and (r_0, φ_0) are the polar coordinates of the observation point and wave source, respectively. The temporal dependence of the fields is described by the factor $\exp(i\omega t)$.

The problem formulated above has been solved using a modified method of discrete sources [1, 2]. In application to 2D problems of diffraction on magnetodielectric bodies of rectangular shapes, this method has been described, e.g., in [3]. The properties of eigenwaves of the discrete spectrum of a plasma layer of infinite length ($a = \infty$) in the case of TM polarization have been studied in detail by Tamir and Oliner [4]. It was established in [4] that, in the case of $\varepsilon < 0$, the spectrum consists of an infinite number of complex waves and two (for $\varepsilon < -1.0363$ and $-1 < \varepsilon < 0$) or four (for $-1.0363 < \varepsilon < -1$) eigenwaves with real propagation constants. For some sets of layer parameters (ε, kb), these waves can be doubly or triply degenerate.

Recently, we have demonstrated [5] that high-Q near-field resonances appear in a layer with $a \gg b$ and the parameters (ε, kb) corresponding to doubly degenerate surface waves in the case of an infinite layer ($a = \infty$). This Letter presents the results of numerical calculations for the case of parameters

$$\varepsilon = -1.0363, \quad kb = 0.513, \quad (2)$$

which correspond to the triply degenerate surface waves [6]. Figures 1 and 2 present the results of calculations performed for $a = 10b$ in the case in which a source is situated under the wide bottom face of the plate ($r_0 = 2b$, $\varphi_0 = -\pi/2$).

Figure 1b shows the frequency dependence of the maximum absolute value of total field $|U|$ at the center of the upper wide face. As can be seen, the dependence has a resonant character with a Q value on the order of 10^2. Figure 2a shows a distribution of the absolute value of the total field along the upper wide face at a resonance frequency of $kb = 0.5122$. Here, the field structure is characterized by periodic oscillations (standing surface waves) with the maximum amplitude monotonically decreasing in the direction toward narrow edges of the plate. The $U(x, b)$ function can be approximated by a sum of three terms of the $U_m = A_m\cos(h_m, x)$ type, where h_m values are close to the propagation constant h_0 of a triply degenerate surface wave in a layer of infinite length. The terms are phased so that the resulting field almost vanishes at the plate ends ($x = \pm a$). However, it should be noted that the resonance phenomena under consideration cannot be explained entirely by re-reflection of the surface waves from the plate edges. Indeed, degeneracy of the forward and reverse surface waves is a more significant factor responsible for the appearance of resonance.

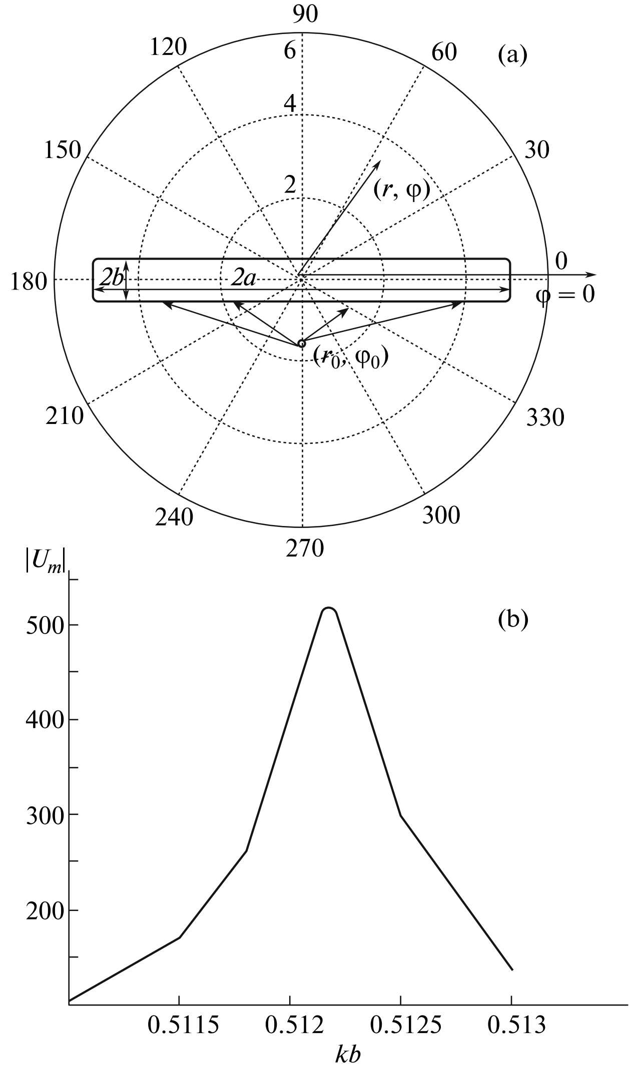

Fig. 1. Excitation of a plate by wave from a point source: (a) geometry of the problem; (b) frequency dependence of the maximum absolute value of the total field for a plate with $\varepsilon = -1.0363$.

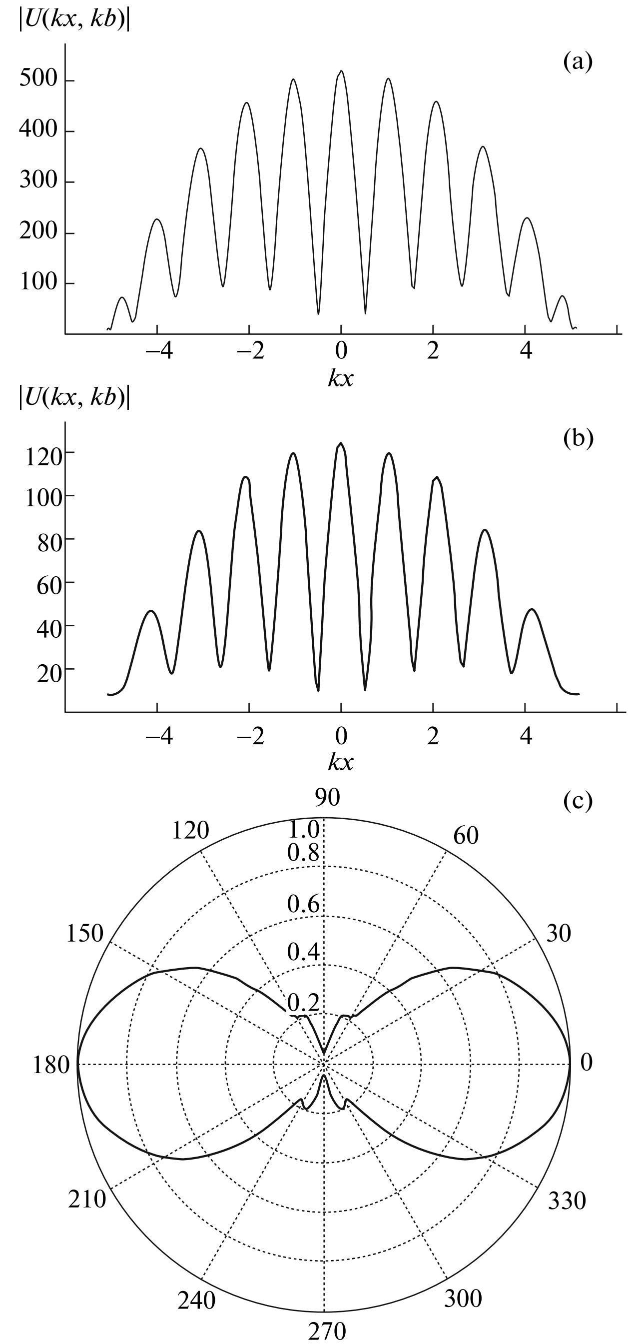

Fig. 2. Near and far fields at resonance frequency kb = 0.5122 for a source situated under the center of the lower wide face of the plate: (a) profile of the absolute value of the total field along the upper wide face, (b) profile of the absolute value of the total field along the $y = 2b$ plane, and (c) normalized scattering diagram.

The resonance also exists in a layer of infinite length [7].

The propagation constant of a degenerate wave is determined by the following formula [6]:

$$h_0 = k\sqrt{1 + (3\varepsilon/2kb)^2} \approx 3.2k. \tag{3}$$

Accordingly, number N of field oscillations over the plate length ($-a < x < a$) is estimated as

$$N \approx 2h_0 a/\pi = 10.5, \tag{4}$$

which is consistent with the profile in Fig. 2a.

Figure 2b shows the distribution of the absolute value of the total field in a plane that is parallel to the upper face of the plate and is spaced by distance b from this face. As can be seen, the $U(x, 2b)$ function is similar to $U(x, b)$, but has a smaller amplitude $U(x, 2b) \approx 0.24U(x, b)$. The field of a triply degenerate surface wave decays with increasing distance from the layer boundary as [8]

$$\psi(y) = \exp[-1.5|\varepsilon|(y - b)/b] \approx \exp[-1.55(y - b)/b], \quad y > b. \tag{5}$$

At $y = 2b$, the coefficient of field attenuation is $\exp(-1.55) \approx 0.21$. This value quite satisfactorily agrees with the results of numerical calculations, which confirms the role of degenerate surface waves in the resonance field formation.

Figure 2c presents a normalized scattering diagram of a plate at the resonance frequency. As can be seen, scattered fields in the far zone in the lower and upper half-spaces coincide to within a graphical accuracy. This scattering diagram has only two lobes ($\varphi = 0$ and $\varphi = \pi$) directed toward narrow faces of the plate. These features of the scattering diagram are related to the fact that the field of intrinsic oscillations of the plate predominates over initial field (1) of the point source and is related to resonances of the surface waves on the plate.

Almost complete absence of scattering in the directions perpendicular to wide faces of the plate ($\varphi = \pi/2$ and $\varphi = 3\pi/2$) is a nontrivial property of the scattering diagram. Accordingly, the plasma layer fully transmits radiation of the point source in the direction of $\varphi = \pi/2$ and does not reflect a signal in the opposite direction.

Analogous calculations have been performed for a source situated near the narrow edge face of the plate, at a point with the coordinates ($r_0 = a + 2b$, $\varphi = \pi$), while the other parameters remained unchanged. It was found that the character of the spatial distribution of the absolute value of the total field along the upper face of the plate ($y = b$) and in the plane $y = 2b$ remained qualitatively the same as that in the case considered above ($r_0 = 2b$, $\varphi_0 = -\pi/2$). However, the structure of the normalized scattering diagram changed to consist of three main lobes directed toward the narrow edge at $\varphi = 0$ (Fig. 3). At a plate length of $2a \approx 1.6\lambda$, where λ is the wavelength, these lobes possess rather small angular dimensions. This is a kind of "superdirectivity" effect exhibited by a plate excited in the given configuration.

Thus, it is established that, in a 2D problem of a thin finite plate with dimensions $2a \times 2b$ ($a \gg b$) and relative permittivity $\varepsilon = -1.0363$ excited by a point source, the system exhibits a high-Q resonance. This resonance appears at a frequency of $kb \approx 0.51$ and is related to the triple degeneracy of surface waves on the plate. At the resonance frequency, the field distribution on the wide face of the plate has the shape of a standing wave with the amplitude monotonically decreasing toward narrow edges of the plate. The structure of the normalized scattering diagram depends on the position of the source. For a source situated at the wide face of the plate ($r_0 = 2b$, $\varphi_0 = -\pi/2$), the diagram has two lobes directed toward the narrow faces of the plate, while scattering in the directions

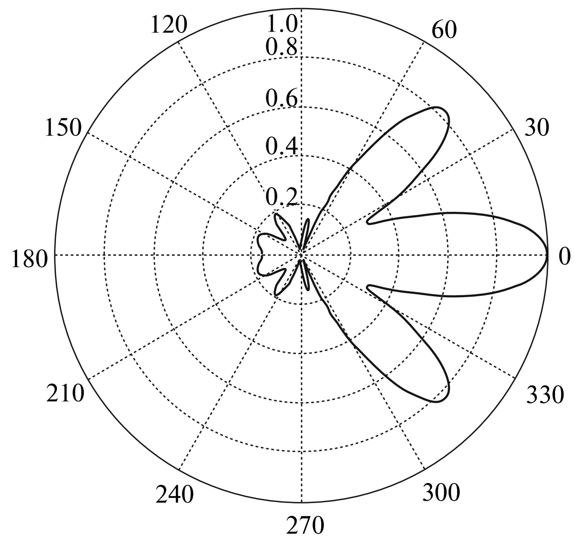

Fig. 3. Normalized scattering diagram at resonance frequency $kb = 0.5122$ for a source situated at the center of the left narrow side face of the plate.

perpendicular to the wide faces is absent. If the source is situated at a narrow face of the plate ($r_0 = a + 2b$, $\varphi = \pi$), the scattering diagram has three lobes directed toward the narrow edge ($\varphi = 0$) and exhibits the effect of "superdirectivity."

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REFERENCES

1. A. G. Kyurkchan, S. A. Minaev, and A. L. Soloveichik, J. Comm. Technol. Electron. **46**, 6154 (2001).
2. A. P. Anyutin, A. G. Kyurchkan, and S. A. Minaev, J. Comm. Technol. Electron. **47**, 858 (2002).
3. A. P. Anyutin, J. Comm. Technol. Electron. **53**, 387 (2008).
4. T. Tamir and A. A. Oliner, IEEE J, **51**, 317 (1963).
5. A. P. Anyutin and A. D. Shatrov, J. Comm. Technol. Electron. **57**, 1024 (2012).
6. A. P. Anyutin and A. D. Shatrov, J. Comm. Technol. Electron. **57**, 1232 (2012).
7. A. D. Shatrov and V. V. Shevchenko, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **17**, 1692 (1974).
8. V. P. Mal'tsev and A. D. Shatrov, J. Comm. Technol. Electron. **57**, 170 (2012).

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