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The Resonance Properties of Multiturn Wire Helices Filled with a Metamaterial

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Abstract—The 2D problem of the excitation of a multiturn cylindrical wire helix filled with a metamaterial by electric- and magnetic-current filaments is considered. High-Q resonances in the low-frequency region are found and investigated. It is shown that, depending on the values of the constitutive and geometric parameters of the anisotropically conducting cylinder, the resonance fields can have both linear and circular polarizations. On the basis of a rigorous calculation, the amplitude-frequency characteristic of this structure and near- and far-field patterns are considered. Approximate analytic expressions are obtained for the resonance frequencies.

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INTRODUCTION

In study [1], an anisotropic cylinder with a surface perfectly conducting along helical lines is considered and the properties of 2D electromagnetic fields formed during diffraction of plane waves by such a cylinder are analyzed. The model of a cylindrical surface with helical anisotropic conductivity well describes single-turn and multiturn wire helices when the distance between the axes of neighboring conductors is much smaller than the wavelength and the value of gaps is within a certain interval [2, 3]. It is shown in [1] that, resonance phenomena are observed at low frequencies (when the cylinder's diameter is much smaller than the wavelength). These resonances occur only for the specific direction of rotation of the polarization plane of the incident circularly polarized wave [1, 3].

When the cylinder is filled with a magnetodielectric with the constitutive parameters $\varepsilon > 0$ and $\mu > 0$ and illuminated by a circularly polarized wave, the scattered field is elliptically polarized under the resonance conditions [4]. However, if $\varepsilon\mu = 1$, the polarization becomes circular and the resonance frequencies coincide with the resonance frequencies of the hollow cylinder. Below, an anisotropic cylinder whose interior medium is characterized by the parameters $\varepsilon = \mu = 1$ will be referred to as a hollow cylinder.

In [5], the 2D problem of excitation of a circular metamaterial cylinder ($\varepsilon < 0,\ \mu < 0$) by a filament source is investigated numerically. It is shown that high-Q resonances exist in cylinders of a small diameter. For the TM (TE) polarization, they occur for values of ε (μ) close to minus unity.

It is natural to suppose that new specific effects can manifest themselves in an object containing a chiral structure in the form of a metal helix and metamaterial filling. The purpose of this study is investigation of the resonance properties of an electrically small metamaterial cylinder under the assumption that the cylinder's surface exhibits perfect anisotropic conductivity along helical lines.

1. FORMULATION OF THE PROBLEM

Consider the problem of excitation of a circular metamaterial cylinder with parameters ε and μ by a filament source. Cylindrical coordinates (r, φ, z) (see Fig. 1) are used. Assume that, two-sided boundary conditions of the perfect anisotropic conductivity along helical lines are fulfilled on the cylinder's surface $r = a$ [3]:

$$\begin{gathered} E_z^+ = E_z^-, \quad E_\varphi^+ = E_\varphi^-, \\ E_z \cos\psi + E_\varphi \sin\psi = 0, \\ \left(H_z^+ - H_z^-\right)\cos\psi + \left(H_\varphi^+ - H_\varphi^-\right)\sin\psi = 0, \end{gathered} \tag{1}$$

where the plus and minus signs correspond to the outer ($r > a$) and inner ($r < a$) sides of the surface and ψ is the twist angle of the helix. For the sake of definiteness, helical lines are assumed to be right-handed ($0 < \psi < \pi/2$).

The cylinder is excited by electric- and magnetic-current filaments located beyond the cylinder at the point $r_0 > a$, $\varphi_0 = 0$ (Fig. 1). The exciting currents are assumed to be independent of the z coordinate. In this case, we deal with a 2D but double-potential problem. As potentials, we choose the functions

$$U_1(r,\varphi) = E_z(r,\varphi),\quad U_2(r,\varphi) = H_z(r,\varphi). \tag{2}$$

Below, we use a vector notation, for example,

$$\vec{U} = \{U_1, U_2\}. \tag{3}$$

Vector function $\vec{U}(r,\varphi)$ satisfies the inhomogeneous Helmholtz equation

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} + k^2\varepsilon(r)\mu(r)\right]\vec{U}(r,\varphi) = -\frac{4i}{r}\vec{A}\delta(r-r_0)\delta(\varphi), \tag{4}$$

where k is the wavenumber in free space, functions $\varepsilon(r)$ and $\mu(r)$ are determined by the formulas

$$\varepsilon(r) = \begin{cases}\varepsilon, & 0 < r < a,\\ 1, & r > a,\end{cases}\qquad \mu(r) = \begin{cases}\mu, & 0 < r < a,\\ 1, & r > a,\end{cases} \tag{5}$$

$\delta(\ldots)$ is the Dirac delta function, and the A_1 and A_2 components of vector $\vec{A}$ specify the amplitudes of the electric and magnetic exciting currents.

Quantities E_φ and H_φ, entering boundary conditions (1) are expressed through U_1 and U_2 as

$$E_\varphi = -\frac{1}{ik\varepsilon(r)}\frac{\partial U_2}{\partial r},\quad H_\varphi = \frac{1}{ik\mu(r)}\frac{\partial U_1}{\partial r}. \tag{6}$$

Field $\vec{U}(r,\varphi)$ should satisfy the radiation condition, i.e., as $kr \to \infty$, have the form

$$\vec{U}(r,\varphi) = \vec{\Phi}(\varphi)\left(\frac{2}{\pi kr}\right)^{1/2}\exp(-ikr + i\pi/4). \tag{7}$$

Exciting field $\vec{U}^0$ is a solution to the inhomogeneous Helmholtz equation in free space, and it is determined from the formula

$$\vec{U}^0(r,\varphi) = \vec{A}H_0^{(2)}\left(k\sqrt{r^2 + r_0^2 - 2rr_0\cos\varphi}\right), \tag{8}$$

where $H_0^{(2)}$ is the Hankel function. The pattern of field $\vec{U}^0(r,\varphi)$ has the form

$$\vec{\Phi}^0(\varphi) = \vec{A}\exp(ikr_0\cos\varphi). \tag{9}$$

Equation (4), boundary conditions (1), and radiation conditions (7) completely determine the boundary value problem for field $\vec{U}(r,\varphi)$.

2. THE METHOD OF SOLUTION

The problem formulated above can be solved analytically by means of the method of separation of variables [1, 4, 5].

Let us introduce vectors $\vec{L}^{(m)}$, $\vec{M}^{(m)}$, and $\vec{N}^{(m)}$ and scalar $W^{(m)}$:

$$\vec{L}^{(m)} = \left\{H_m^{(2)\prime}(ka)\sin\psi, iH_m^{(2)}(ka)\cos\psi\right\}, \tag{10}$$

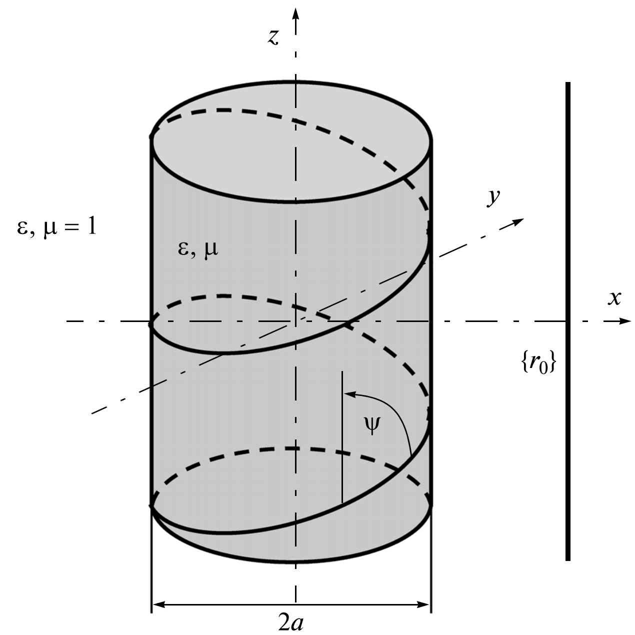

Fig. 1. Geometry of the problem.

$$\vec{M}^{(m)} = \left\{H_m^{(2)\prime}(ka)\sin\psi, -iH_m^{(2)}(ka)\cos\psi\right\}, \tag{11}$$

$$\vec{N}^{(m)} = \left\{\frac{n}{\varepsilon}J'_m(kna)\sin\psi, iJ_m(kna)\cos\psi\right\}, \tag{12}$$

$$\begin{aligned} W^{(m)} = {} & H_m^{(2)}(ka)J_m(kna)\left[\frac{n}{\varepsilon}H_m^{(2)}(ka)J'_m(kna)\right.\\ & \left. - H_m^{(2)\prime}(ka)J_m(kna)\right]\cos^2\psi + \frac{n}{\varepsilon}H_m^{(2)\prime}(ka)J'_m(kna)\\ & \times\left[\frac{n}{\mu}H_m^{(2)}(ka)J'_m(kna) - H_m^{(2)\prime}(ka)J_m(kna)\right]\sin^2\psi, \end{aligned} \tag{13}$$

where

$$n = \sqrt{\varepsilon\mu}, \tag{14}$$

$J_m(kna)$ are Bessel functions, and the prime denotes differentiation with respect to the corresponding argument.

The total field inside the cylinder ($r < a$) can be represented in the form

$$\vec{U}(r,\varphi) = \sum_{m=0}^{\infty}\delta_m H_m^{(2)}(kr_0)\vec{B}^{(m)}J_m(kna)\cos(m\varphi), \tag{15}$$

where

$$\delta_m = \begin{cases}1, & m = 0,\\ 2, & m \geq 1,\end{cases} \tag{16}$$

$$\vec{B}^{(m)} = \frac{2i\left(\vec{A}, \vec{M}^{(m)}\right)}{\pi ka W^{(m)}}\vec{N}^{(m)}. \tag{17}$$

In (17), $\left(\vec{A}, \vec{M}^{(m)}\right)$ denotes the scalar product

$$\left(\vec{A}, \vec{M}^{(m)}\right) = A_1 M_1^{(m)} + A_2 M_2^{(m)}. \quad (18)$$

The field outside the cylinder ($r > a$) consists of two terms that are the incident and scattered fields:

$$\vec{U}(r,\varphi) = \vec{U}^0(r,\varphi) + \vec{U}^s(r,\varphi). \quad (19)$$

The scattered field is determined from the formula

$$\vec{U}^s(r,\varphi) = \sum_{m=0}^{\infty} \delta_m H_m^{(2)}(kr_0)\left(\vec{C}^{(m)} + \vec{D}^{(m)}\right) \times H_m^{(2)}(kr)\cos(m\varphi), \quad (20)$$

where

$$\vec{C}^{(m)} = \left\{-A_1 \frac{J_m(ka)}{H_m^{(2)}(ka)}, -A_2 \frac{J'_m(ka)}{H_m^{(2)\prime}(ka)}\right\}, \quad (21)$$

$$\vec{D}^{(m)} = \frac{2i\left(\vec{A}, \vec{M}^{(m)}\right)\frac{n}{\varepsilon} J_m(kna) J'_m(kna)}{\pi ka H_m^{(2)}(ka) H_m^{(2)\prime}(ka) W^{(m)}} \vec{L}^{(m)}. \quad (22)$$

The terms from formula (20) that contain vectors $\vec{C}^{(m)}$ and $\vec{D}^{(m)}$ will conventionally be referred to as the nonresonance and resonance components of the scattered field. Note that the nonresonance field component coincides with the field formed during scattering by an isotropically conducting metal cylinder.

Scattering pattern $\vec{\Phi}^s(\varphi)$ can be represented in the form of a series:

$$\vec{\Phi}^s(\varphi) = \sum_{m=0}^{\infty} \delta_m (i)^m H_m^{(2)}(kr_0)\left(\vec{C}^{(m)} + \vec{D}^{(m)}\right)\cos(m\varphi). \quad (23)$$

The formulas obtained in this section can be applied to cylinders made from standard materials ($\varepsilon > 0$, $\mu > 0$) and to cylinders made from metamaterials ($\varepsilon < 0$, $\mu < 0$). Refractive index n (see (14)) is assumed to be positive for these cases.

Below, we consider only electrically small cylinders, i.e., such that

$$ka \ll 1, \quad kna \ll 1. \quad (24)$$

Using the asymptotic representations of function $H_m^{(2)}(ka)$ for small values of the argument, we obtain for $m \geq 1$ the form of vectors $\vec{L}^{(m)}$ and $\vec{M}^{(m)}$ accurate to factors

$$\vec{L}^{(m)} = \left\{\frac{m\tan\psi}{ka}, -i\right\}, \quad \vec{M}^{(m)} = \left\{\frac{m\tan\psi}{ka}, i\right\}. \quad (25)$$

Hence, it follows from formulas (20) and (22) that, at the frequency

$$ka = m\tan\psi, \quad m \geq 1 \quad (26)$$

the harmonic with number m of the resonance component of the scattered field is a right-handed circularly polarized wave and that this harmonic is not excited by a left-handed circularly polarized field when $A_2 = iA_1$.

3. LOW-FREQUENCY RESONANCES

Expressions (15) and (19) for near diffraction fields and (23) for far diffraction fields contain resonance denominators $W^{(m)}(ka)$ determined by formula (13). Let us examine the frequency dependence of these denominators. Expression (13) is a complex function of the parameter ka, and it does not vanish at the real values of ka. Under conditions (24), the real part of expression (13) substantially exceeds its imaginary part. The real parts of denominators $W^{(m)}(ka)$ vanish at the points that are resonance frequencies. At these frequencies, the only azimuthal harmonic, $\cos(m\varphi)$, dominates in the diffraction field.

To determine the resonance frequencies, we apply the known asymptotic decompositions of cylindrical functions for small values of the argument. We use two terms of the decomposition in positive powers for Bessel functions and two terms of the decomposition in negative powers of the argument for Hankel functions. As a result, we obtain the following biquadratic equation for resonance frequencies ka:

$$(ka)^2\left\{(1+\varepsilon) + \frac{(ka)^2}{2m}\left[\frac{1}{m-1} + \frac{\mu\varepsilon^2}{m+1}\right]\right\}\cos^2\psi - m^2\left\{\left(1+\frac{1}{\mu}\right) - \frac{(ka)^2}{2m}\left[\frac{1}{m-1} + \frac{\varepsilon}{m+1}\right]\right\}\sin^2\psi = 0, \quad (27)$$
$$m \geq 2.$$

Recall that the applicability of this equation is restricted by the condition $ka \ll 1$.

Consider the case when the expressions $(1+\varepsilon)$ and $(1+\mu)$ are not small quantities. These conditions are always fulfilled for materials with $\varepsilon > 0$ and $\mu > 0$. Then, the second terms in the curly brackets in expression (27) can be neglected. As a result, we obtain

$$ka = m\sqrt{\frac{1+1/\mu}{1+\varepsilon}}\tan\psi. \quad (28)$$

This expression matches the results from study [4]. When $\varepsilon\mu = 1$, formula (28) for the resonance frequencies coincides with condition (26), which provides for the circular polarization of the harmonic with number m. Thus, the scattered field in this case is right-handed circularly polarized at all of the resonance frequencies.

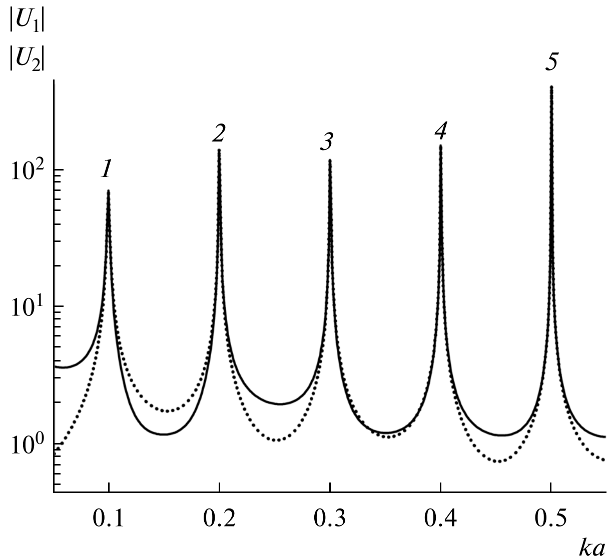

Fig. 2. Amplitude–frequency characteristic of a hollow cylinder for $\varepsilon = \mu = 1$; $A_1 = 1$, $A_2 = -i$; $\psi = 0.1$; $\varphi = \pi$; $r_0 = 1.2a$, and $r = 0.99a$: (solid curve) U_1, (dotted curve) U_2, and the numerals denote resonance numbers m.

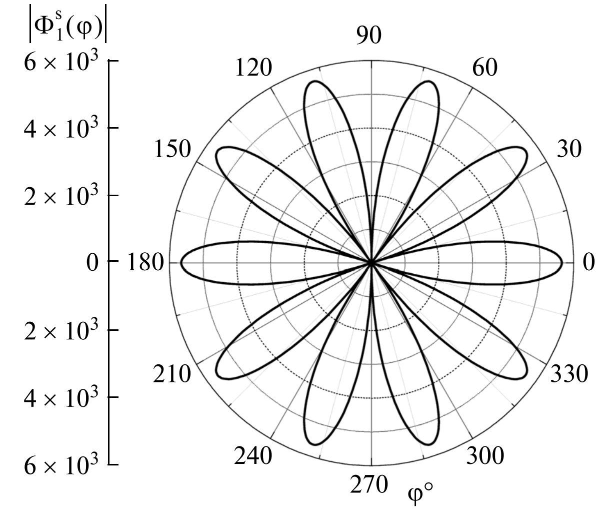

Fig. 3. Absolute value of the pattern of a hollow cylinder for $\varepsilon = \mu = 1$; $A_1 = 1$, $A_2 = -i$; $\psi = 0.1$; $\varphi = \pi$; $r_0 = 1.2a$; and $ka_5 = 0.4990602297$.

4. NUMERICAL RESULTS

The numerical results presented below are obtained by summing series (15), (20), and (23) and with the help of the modified method of discrete sources [6, 7]. The results of these calculations are in complete agreement. In all cases, the source coordinate is assumed to be $r_0 = 1.2a$.

The resonance phenomena observed during diffraction of plane waves by a hollow cylinder are investigated in [1]. First, let us discuss the spatial and frequency properties of the fields in the case when a hollow cylinder with anisotropic conductivity along helical lines is excited by a filament source situated in the exterior of the cylinder.

We investigate the cylinder's amplitude–frequency characteristic (AFC) that is considered to mean the dependence of the absolute value of the field at the point $r = 0.99a$, $\varphi = \pi$ on the dimensionless parameter ka proportional to the frequency. The AFC for the case $\psi = 0.1$ is depicted in Fig. 2. The excitation is assumed to be circularly polarized: $A_1 = 1$ and $A_2 = -i$. The solid and dotted lines in this figure characterize the behavior of the absolute values of the U_1 and U_2 components. The frequency characteristic is a sequence of resonance peaks. The resonances are enumerated so that, at resonance frequency ka_m, the azimuthal harmonic $\cos(m\varphi)$ dominates in the field.

It is seen that the amplitudes of components U_1 and U_2 are equal at the resonance frequencies, a result that indicates the circular polarization of the resonance field.

Figure 3 shows component $\Phi_1^s(\varphi)$ of the vector scattering pattern at the resonance frequency ka_5. It contains 10 identical lobes. Second component $\Phi_2^s(\varphi)$ of the vector scattering pattern graphically coincides with $\Phi_1^s(\varphi)$.

The dependence of field component U_1 of the resonance oscillation ka_5 on radial coordinate r is depicted in Fig. 4. It is seen that function $|U_1(r,\pi)|$ is localized within a narrow interval near the surface $r = a$. Function $|U_2(r,\pi)|$ practically coincides with function $|U_1(r,\pi)|$ and, therefore, is not presented. Thus, the spatial field structure in this case is the same as that of a surface wave propagating over a medium interface.

Figure 5 illustrates the radial dependence of the absolute value of field component U_1 for the oscillation with the azimuthal index $m = 1$. We have found that component $U_1(r,\pi)$ is a linear function of radius r in the interior of the cylinder. The same property is exhibited by function $U_2(r,\pi)$. Therefore (see (6)), $E_\varphi(r,\pi) = \mathrm{const}$ and $H_\varphi(r,\pi) = \mathrm{const}$ inside the cylinder. This result is in agreement with the statement that components E_y and H_y such that $E_y(r,\varphi) = \mathrm{const}$ and $H_y(r,\varphi) = \mathrm{const}$ dominate in the field of the resonance oscillation with the index $m = 1$ inside the cylinder [3].

Let us investigate low-frequency oscillations in wire helices filled with a metamaterial. The numerical computation has shown that, depending on the values of parameters ε, μ, and ψ, the fields of these oscilla-

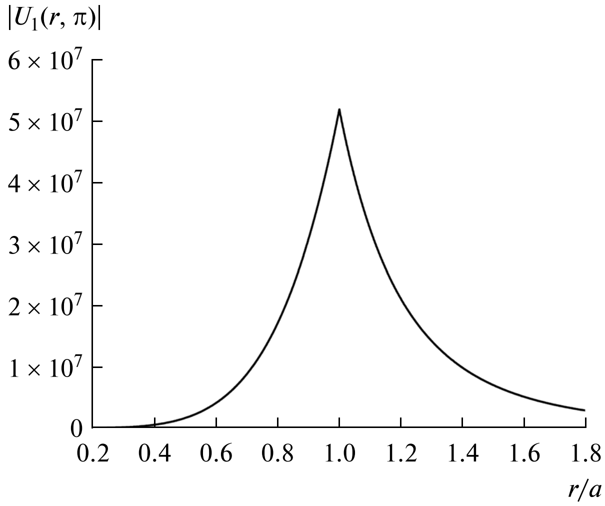

Fig. 4. Distribution of the absolute value of field component $|U_1(r,\pi)|$ along the radius of a hollow cylinder for $\varepsilon = \mu = 1$; $A_1 = 1$, $A_2 = -i$; $\psi = 0.1$; $\varphi = \pi$; and $r_0 = 1.2a$ at the resonance frequency $ka_5 = 0.4990602297$.

tions can exhibit various properties. Consider two specific examples.

(i) Let a cylinder be made from a metamaterial with parameters ε and μ close to minus unity and let $\varepsilon = \mu$. Then, Eq. (27) yields the following expression for the resonance frequencies:

$$(ka)^2 = -m(m^2 - 1)(1 + \varepsilon) = -m(m^2 - 1)(1 + \mu), \quad m \geq 2. \tag{29}$$

Note that the resonance frequencies prove to be independent of twist angle ψ. Expression (29) matches the results from [5], where metamaterial cylinders are investigated in the cases of the TM and TE polarizations. However, in contrast to [5], in the case under study, the eigenmode field contains both components E_z and H_z, and angle ψ determines the relationship between their amplitudes. In particular, the circular polarization can be provided for a specific eigenmode with the appropriately chosen angle ψ (see (26)).

The AFC of a cylinder filled with a metamaterial characterized by the parameters $\varepsilon = -1$ and $\mu = -1$ is depicted in Fig. 6 (dashed curve). Such a cylinder does not exhibit resonance properties, which follows from formula (29). However, when the values of the constitutive parameters slightly deviate form minus unity, high-Q oscillations appear. This effect is illustrated by the solid curve in Fig. 6.This curve is the ACF of a cylinder with the parameters $\varepsilon = \mu = -1.003$ and $\psi = 0.1$. Approximate formula (29) yields the following approximate values of resonance frequencies: $ka_2 = 0.134$, $ka_3 = 0.268$, and $ka_4 = 0.424$. These quantities are in good agreement with the positions of peaks on the solid curve. The numerical computation has shown that the positions of these peaks on the solid curve do not depend on angle ψ, a result that matches Eq. (29).

At the resonance frequency ka_4, the field is circularly polarized ($U_2 = -iU_1$) at the helix twist angle $\psi = 0.1$. In the case of the right-handed circularly polarized excitation ($A_1 = 1$ and $A_2 = -i$), the scattering pattern is described by the function $\Phi_4^s = \alpha\cos(4\varphi)$ with $\alpha \approx 10^3$.

Figure 7 illustrates the influence of helix twist angle ψ on the resonance frequencies of the cylinder for the

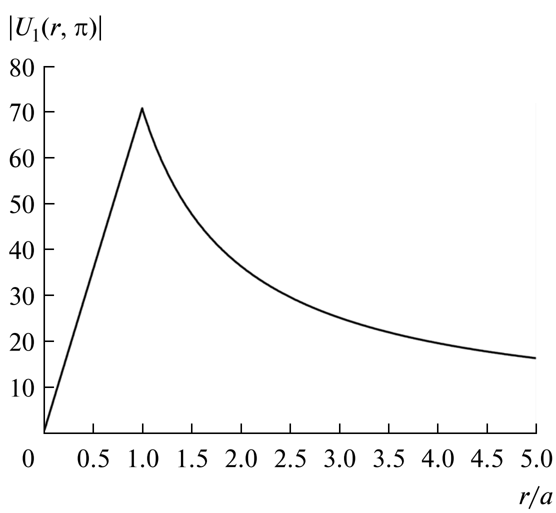

Fig. 5. Distribution of the absolute value of field component $|U_1(r,\pi)|$ along the radius of a hollow cylinder for $\varepsilon = \mu = 1$; $A_1 = 1$, $A_2 = -i$; $\psi = 0.1$; $\varphi = \pi$; and $r_0 = 1.2a$ at the resonance frequency $ka_1 = 0.0989872$.

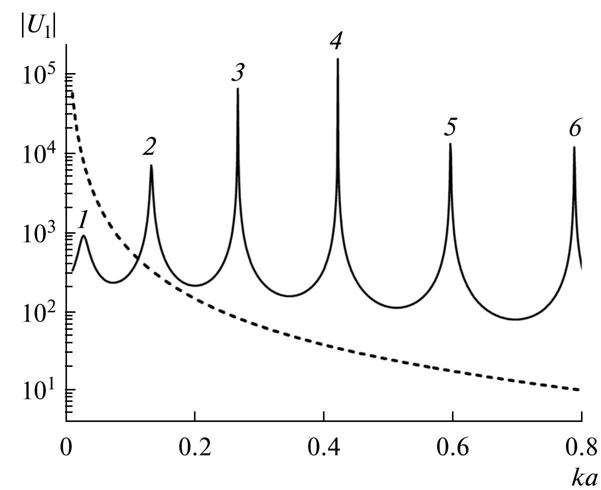

Fig. 6. Amplitude–frequency characteristic of a cylinder filled with a metamaterial for $A_1 = 1$, $A_2 = -i$; $\psi = 0.1$; $\varphi = \pi$; $r_0 = 1.2a$, and $r = 0.99a$: (dashed curve) $\varepsilon = -1$ and $\mu = -1$ and (solid curve) $\varepsilon = -1.003$ and $\mu = -1.003$.

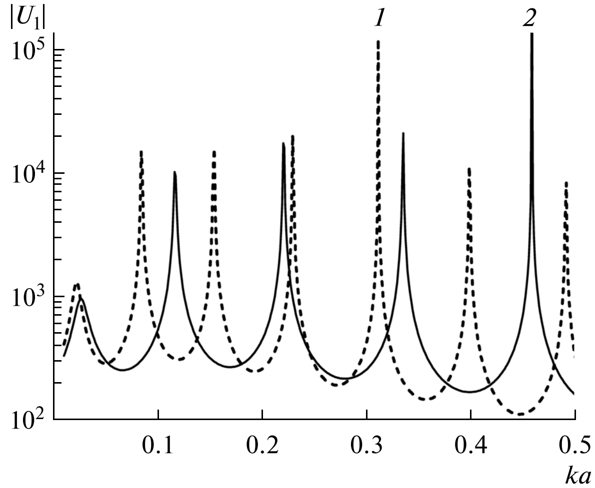

Fig. 7. Amplitude–frequency characteristic of a cylinder filled with a metamaterial for $A_1 = 1$, $A_2 = -i$, $\varphi = \pi$, $r_0 = 1.2a$, $r = 0.99a$, $\varepsilon = -1.0003$, and $\mu = -1.003$: (dashed curve) $\psi = 0.03$ and (solid curve) $\psi = 0.1$.

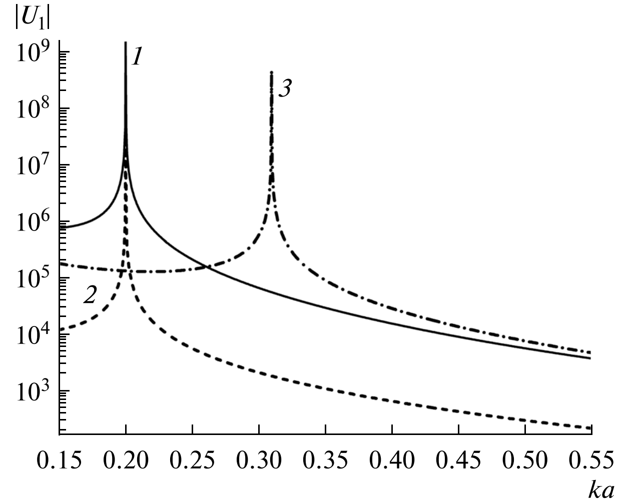

Fig. 8. Amplitude–frequency characteristic of the cylinder filled with a metamaterial for $\varphi = \pi$; $r_0 = 1.2a$, $r = 0.99a$; $\varepsilon = -1.3$, and $\mu = -1.0$: (curve *1*) $\psi_5 = 1.084957521$, $A_1 = 1$, and $A_2 = 0$; (curve *2*) $\psi_5 = 1.08495752$, $A_1 = 0$, and $A_2 = 1$; (curve *3*) $\psi_5 = 1.0832075$, $A_1 = 1$, and $A_2 = 0$.

case when $\varepsilon \neq \mu$. Curves *1* and *2* are the AFCs of cylinders with the parameters $\varepsilon = -1.0003$ and $\mu = -1.003$ for the angles $\psi = 0.1$ and $\psi = 0.03$, respectively. It is seen that the resonance frequencies in this case substantially depend on angle ψ; in particular, as ψ grows, the frequency spectrum rarefies.

(ii) Consider the case when the metamaterial is characterized by the parameters $\varepsilon = -1.3$ and $\mu = -1.0$. Curves *1* and *2* in Fig. 8 are the AFCs of a cylinder with the aforementioned constitutive parameters for the twist angle $\psi = 1.0849\ldots$ under two excitation conditions. Note that, in contrast to the situation illustrated in Fig. 7, in this case, the form of the AFC is radically changed: an only one resonance occurs. The figure depicts only the plots for the U_1 component, because the values of the U_2 component are smaller by two orders of magnitude. Curve *1* corresponds to the excitation by an electric-current filament ($A_1 = 1$ and $A_2 = 0$), and curve *2* corresponds to the excitation by a magnetic-current filament ($A_1 = 0$ and $A_2 = 1$). The comparison of curves *1* and *2* shows that the efficiency of the excitation by the magnetic current is lower than the excitation by the electric current by two orders of magnitude. Therefore, the considered 2D vector problem can be regarded approximately as the scalar problem corresponding to the case of the TE polarization. Below, we consider only component U_1 and set $A_1 = 1$ and $A_2 = 0$.

A substantial difference of Fig. 8 from Figs. 6 and 7 is the presence of only one resonance in a wide frequency interval. The index $m = 5$ is ascribed to twist angle ψ_5 corresponding to curves *1* and *2*, because, at this angle, the oscillation with the azimuthal index 5 is excited at the resonance frequency $ka = 0.2$. Curve *3* corresponds to the twist angle differing from ψ_5 by $0.1°$. When the twist angle changes so insignificantly, the resonance frequency increases by a factor exceeding 1.5. Therefore, when angle ψ is specified arbitrarily, the considered frequency interval may contain no resonance. A resonance occurs only when angle ψ belongs to narrow intervals and is near specific discrete values ψ_m. These discrete values are associated with resonances with different azimuthal indices *m*. To elucidate this situation, the absolute value of field $U_1(r,\varphi)$ observed at the point $r = 0.99a$, $\varphi = \pi$ at the frequency $ka = 0.2$ is shown in Fig. 9 as a function of twist angle ψ. It is seen that the curve exhibits a resonance character and that, the only azimuthal harmonic $\cos(m\varphi)$ prevails in the field at the resonance values of angles ψ_m.

Setting $\mu = -1$ in Eq. (27), we obtain an approximate formula for the determination of resonance twist angles ψ_m:

$$\tan^2\psi_m = -\frac{2m(m^2-1)(1+\varepsilon)+(ka)^2\left[m+1-\varepsilon^2(m-1)\right]}{m^2\left[m+1+\varepsilon(m-1)\right]}, \quad m \geq 2. \tag{30}$$

For $\varepsilon = -1.3$ and $ka = 0.2$, we obtain from (30) $\psi_3 = 0.8183$, $\psi_4 = 0.9606$, and $\psi_5 = 1.0858$. These values are in complete agreement with the positions of resonance peaks in Fig. 9.

At angle ψ_5, the scattering pattern is described by the function $\alpha\cos(5\varphi)$ with the amplitude $|\alpha| = 2.1\times10^3$. The absolute value of the field of this

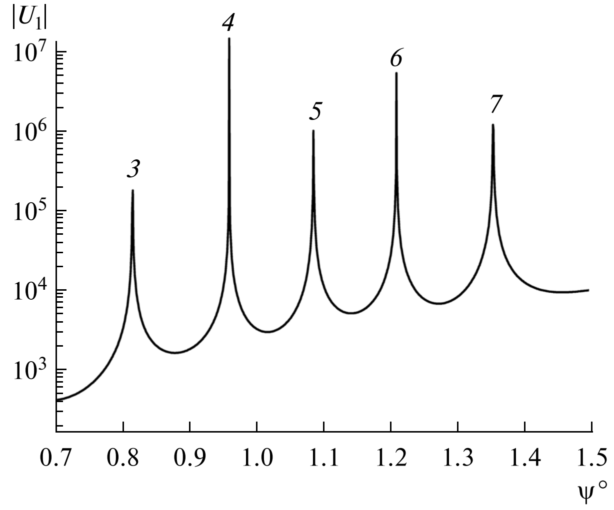

Fig. 9. Absolute value of the field at the point $\varphi = \pi$, $r = 0.99a$ vs. twist angle ψ of the helix for $\varepsilon = -1.3$, $\mu = -1.0$, $ka = 0.2$, $r_0 = 1.2a$, $A_1 = 1$, and $A_2 = 0$; the numerals denote resonance numbers m.

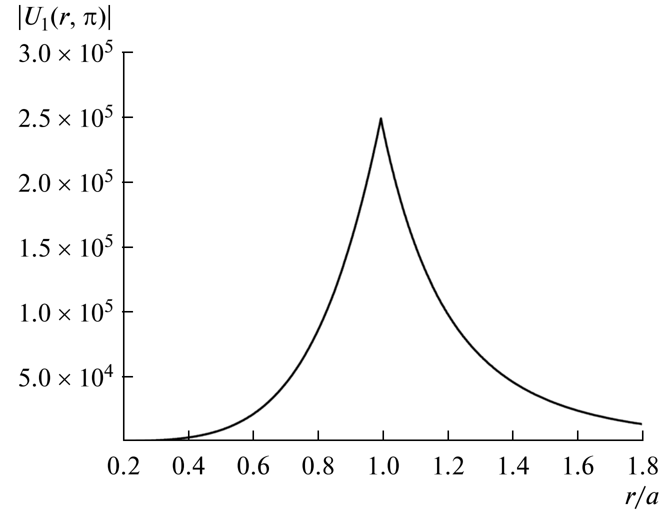

Fig. 10. Distribution of the absolute value of the field along the radius of the cylinder filled with a metamaterial for $r_0 = 1.2a$, $\varepsilon = -1.3$, $\mu = -1.0$, $ka = 0.2$; $A_1 = 1$, $A_2 = 0$; and $\psi_5 = 1.08495752$.

resonance oscillation is shown in Fig. 10 as a function of the radial coordinate. As in the case of a hollow cylinder (as well as a cylinder without a wire helix), the field has the form of a surface wave localized near the boundary $r = a$. Note that the value of the field amplitude at the point $r = a$, $\varphi = \pi$ is about 2.5×10^5 (5×10^7 for the hollow cylinder, see Fig. 4).

CONCLUSIONS

High-Q low-frequency resonances in a multiturn wire helix filled with a metamaterial have been found and investigated. It has been shown that the studied structure can be regarded as a ring resonator for surface waves formed by both the medium interface and the wire grating located on this interface. The polarization and spectral properties of the resonance fields have been analyzed. Note that these properties have not been observed for simpler structures previously investigated (e.g., structures without a metamaterial or a multiturn helix).

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