ISSN 1064-2269, Journal of Communications Technology and Electronics, 2015, Vol. 60, No. 6, pp. 572–576. © Pleiades Publishing, Inc., 2015.
Original Russian Text © A.P. Anyutin, I.P. Korshunov, A.D. Shatrov, 2015, published in Radiotekhnika i Elektronika, 2015, Vol. 60, No. 6, pp. 610–614.

On the Influence of the Medium Loss on Resonances of Surface Plasmons in a Cylinder

A. P. Anyutin, I. P. Korshunov, and A. D. Shatrov

Kotel'nikov Institute of Radio Engineering and Electronics (Fryazino Branch), Russian Academy of Sciences, Fryazino, Moscow oblast, 141190 Russia

e-mail: anioutine@mail.ru; korip@ms.ire.rssi.ru

Received December 3, 2014

Abstract—Two-dimensional problem of excitation of a circular cylinder made of a material with negative permittivity by a plane or a cylindrical wave is considered for the case of TM polarization. Conditions under which resonances of surface plasmons arise in cylinders thin compared to the wavelength are found with the use of strict numerical calculations.

DOI: 10.1134/S1064226915050010

1. FORMULATION OF THE PROBLEM

A 2D problem of diffraction of a plane linearly polarized wave

$$E_y^0 = \exp(-ikx), \quad H_z^0 = \exp(-ikx), \quad k = 2\pi/\lambda \qquad (1)$$

by a circular dielectric cylinder is considered (Fig. 1).

The Gaussian system of physical units is used. The time dependence of the fields is chosen to be $\exp(i\omega t)$ and λ is the wavelength in free space.

It is assumed that radius a of the cylinder is small compared to the wavelength,

$$ka \ll 1, \qquad (2)$$

and the permittivity of the cylinder satisfies the condition

$$\varepsilon \approx -1. \qquad (3)$$

Selection of the problem parameters in the neighborhood of the point $ka = 0$ and $\varepsilon = -1$ is associated with the fact that this point is singular. The problem has no solution at $ka = 0$ and $\varepsilon = -1$.

The solution to the problem is known in the static limit ($ka = 0$). The electric field inside the cylinder is uniform and

$$E_y = \frac{2}{\varepsilon + 1}. \qquad (4)$$

Let us add the field of the electric dipole to external uniform field $E_y^0 = 1$ outside the cylinder (at $r > a$). This field is created by bound charges on the surface of the cylinder. The charge density is

$$\sigma_b = \frac{1}{2\pi}\frac{\varepsilon - 1}{\varepsilon + 1}\sin\varphi. \qquad (5)$$

Expressions (4) and (5) tend to infinity at $\varepsilon = -1$. Note that the permittivity of the medium cannot be negative in electrostatics. However, in electrodynamics ($ka \neq 0$), the value of the complex permittivity $\varepsilon = \varepsilon' - i\varepsilon''$ can be unrestrictedly close to minus one.

It is more convenient to perform numerical study of the diffraction problem using z-component of the magnetic field: $U(r,\varphi) = H_z(r,\varphi)$. The boundary-value problem for function $U(r,\varphi)$ is scalar.

Total field $U(r,\varphi)$ satisfies the Helmholtz equation,

$$\frac{\partial^2 U(r,\varphi)}{\partial r^2} + \frac{1}{r}\frac{\partial U(r,\varphi)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 U(r,\varphi)}{\partial \varphi^2} + k^2\varepsilon(r)U(r,\varphi) = 0, \qquad (6)$$

where

$$\varepsilon(r) = \begin{cases} \varepsilon, & r < a, \\ 1, & r > a. \end{cases} \qquad (7)$$

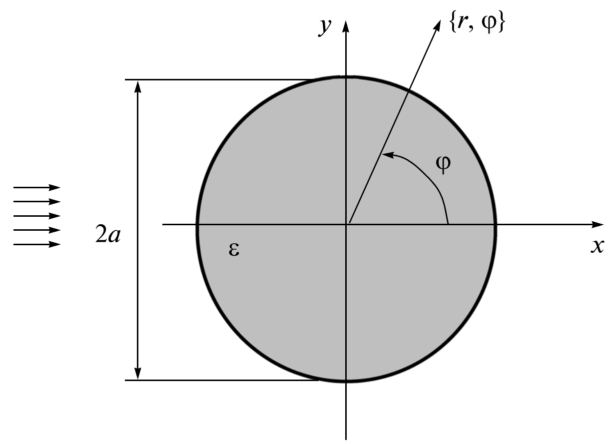

Fig. 1. Geometry of the problem.

The boundary conditions for function $U(r,\varphi)$ have the form

$$U(a-0,\varphi) = U(a+0,\varphi),$$
$$\frac{1}{\varepsilon}\frac{\partial U}{\partial r}(a-0,\varphi) = \frac{\partial U}{\partial r}(a+0,\varphi). \quad (8)$$

In the case of a plane wave, the incident field is given by the function

$$U^0 = \exp(-ikr\cos\varphi). \quad (9)$$

Outside the cylinder (at $r > a$), the total field consists of incident field U^0 and scattered field U^s fields:

$$U = U^0 + U^s, \quad r > a. \quad (10)$$

The scattered field in the far zone must satisfy the radiation condition,

$$U^s \sim \Phi(\varphi)\frac{1}{\sqrt{kr}}\exp(-ikr), \quad kr \to \infty, \quad (11)$$

where $\Phi(\varphi)$ is a scattering pattern.

Components of the electric field can be expressed using function $U(r,\varphi)$:

$$E_r = \frac{1}{ik\varepsilon(r)r}\frac{\partial U}{\partial \varphi}, \quad E_\varphi = -\frac{1}{ik\varepsilon(r)}\frac{\partial U}{\partial r}. \quad (12)$$

The analytical solution of problem (6)–(11) obtained via separation of variables is well known (the Rayleigh series [1]). In particular, when $ka \to 0$ the scattered field is given by the formula

$$U^s = -\frac{\pi}{2}\frac{\varepsilon-1}{\varepsilon+1}(ka)^2 H_1^{(2)}(kr)\cos\varphi, \quad r > a, \quad (13)$$

where $H_1^{(2)}(kr)$ is the Hankel function. Expressions for the case $kr \to 0$ are obtained from (12) and (13). These expressions coincide with the solution to the electrostatic problem:

$$E_r^s = \frac{\varepsilon-1}{\varepsilon+1}\frac{a^2}{r^2}\sin\varphi, \quad E_\varphi^s = -\frac{\varepsilon-1}{\varepsilon+1}\frac{a^2}{r^2}\cos\varphi, \quad r > a. \quad (14)$$

Equation (13) is obtained under the assumption of $ka \to 0$. However, if, at the same time, $\varepsilon \to -1$, expression (13) cannot be calculated. In [2], it is shown that, in this case, there are multipole resonances. These resonances occur at frequencies

$$(ka_m)^2 = -(m^2-1)(\varepsilon+1), \quad m \geq 2. \quad (15)$$

The azimuthal dependence of the near field at the resonance frequencies will contain only one harmonic $\cos(m\varphi)$. However, this effect can be realized only in the case of sufficiently small heat loss of the medium.

The aim of this work is to determine requirements on the loss of the medium under which multipole resonances become possible. Let us mention papers devoted to close subjects [3, 4] in which the dipole resonances of solid and hollow plasma cylinders were used for amplification of the radiation of a short linear antenna.

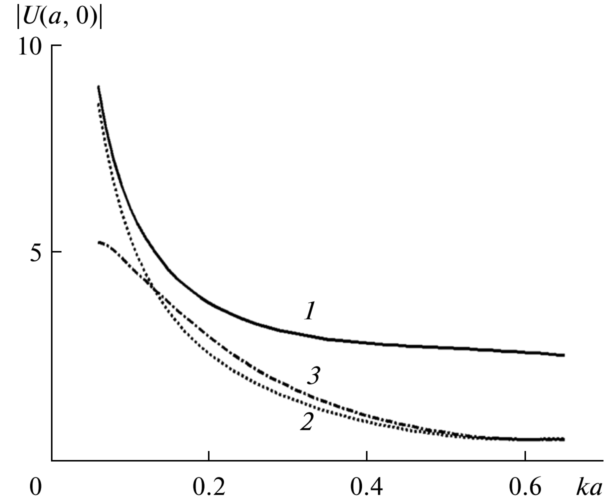

Fig. 2. Amplitude–frequency characteristics of the cylinder made of a material with ε' = −1 at different values of loss. Curves *1*, *2*, and *3* correspond to ε" = 0, 0.001, and 0.01, respectively.

2. NUMERICAL RESULTS

All numerical calculations of wave fields were performed using a modified method of discrete sources [5–7], which was previously applied in the study of allied problems [8–11].

Figure 2 shows a family of curves depicting the dependence of absolute value $|U|$ of the field on dimensionless parameter ka at coordinates $r = a$, $\varphi = 0$ located on the shady side of the cylinder for $\varepsilon' = -1$ and different values of ε''. It is seen that the field increases monotonically with decreasing radius of the cylinder.

Now let $\varepsilon' \neq -1$. Figure 3 shows a similar family of curves for the case $\varepsilon' = -1.04$. While the value of ε' changes only slightly, curves in Figs. 2 and 3 differ substantially. The curves in Fig. 3 contain resonance peaks at frequencies $ka = 0.11$, 0.32, and 0.55. The lowest frequency corresponds to the dipole resonance ($m = 1$), and the following frequencies correspond to multipole resonances ($m = 2$, 3). Let us note that approximate formula (15) gives the following values of resonance frequencies: $ka_2 \simeq 0.34$ and $ka_3 \simeq 0.56$. When $\varepsilon'' = 0$ and $\varepsilon'' = 0.001$, resonance values of $|U(a,0)|$ are by a factor of several tens greater than the amplitude of the incident plane wave $|U^0| = 1$ (curves *1*, *2*). Resonance $m = 3$ practically disappears at the loss $\varepsilon'' = 0.01$ (curve *3*).

The results shown in Figs. 2 and 3 characterize the behavior of the near field of the cylinder. Figure 4 shows the frequency characteristics of total scattering cross sections σ for the same values of ε as in Fig. 3. The value of σ was calculated using the formula

$$\sigma = \frac{1}{k}\int_0^{2\pi}|\Phi(\varphi)|^2\,d\varphi. \quad (16)$$

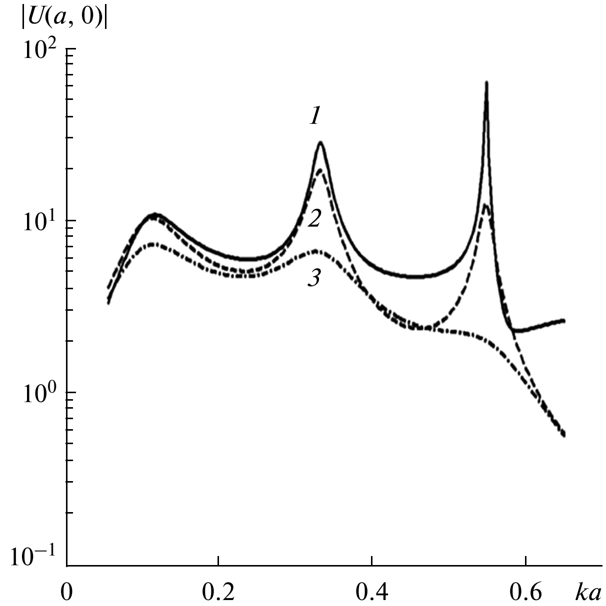

Fig. 3. Amplitude–frequency characteristics of the cylinder made of a material with $\varepsilon' = -1.04$ at different values of loss. Curves *1*, *2*, and *3* correspond to $\varepsilon'' = 0$, 0.001, and 0.01, respectively.

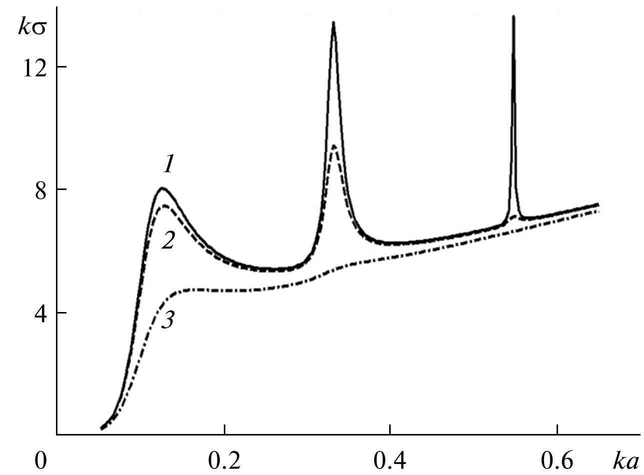

Fig. 4. Frequency dependence of the total scattering cross section for the cylinder made of a material with $\varepsilon' = -1.04$ for different values of loss. Curves *1*, *2*, and *3* correspond to $\varepsilon'' = 0$, 0.001, and 0.01, respectively.

Curve *1* corresponds to the case of zero heat loss, $\varepsilon'' = 0$, and curves *2* and *3* correspond to $\varepsilon'' = 0.001$ and $\varepsilon'' = 0.01$, respectively. It follows from the asymptotics of the Rayleigh series at $ka \to 0$ and $\varepsilon'' = 0$ that, at resonance frequencies satisfying $ka \ll 1$, the total scattering cross section is determined by a simple universal formula containing only the wavelength:

$$\sigma = \frac{4}{\pi}\lambda. \tag{17}$$

The magnitude of the resonance peak of the lowest oscillation ($ka_1 = 0.11$) agrees with this formula with a high accuracy ($k\sigma \approx 8$). The quality factor increases with increasing number m. Consideration for the loss (curves *2*, *3*) leads to weakening of resonance phenomena. This is primarily true for oscillations with greater quality factors. As can be seen from Fig. 4, all far-field resonances disappear at $\varepsilon'' \approx 0.01$.

Scattering patterns corresponding to the loss $\varepsilon'' = 0.001$ at resonant frequencies ka_m $(m = 1, 2, 3)$ are shown in Fig. 5. While the cylinder has small dimensions, the patterns have multilobe structure. The number of lobes ($2m$) depends on the number of the resonance.

Figure 6 illustrates the distribution of absolute value $|U|$ of the field along the surface of the cylinder at resonance frequencies ka = 0.11, 0.32, and 0.55 (curves *1*–*3*) at $\varepsilon'' = 0.001$. As in the case of far fields, the angular dependences of near fields at the resonant frequencies contain $2m$ clearly defined lobes like in the case of far fields.

The information on the spatial structure of the near field at resonance frequency ka_3 is presented in Fig. 7, which shows the lines of constant level of function $|U|$ for $\varepsilon'' = 0.001$. The resonance oscillation with azimuthal index m is a standing surface wave whose field is localized in the vicinity of the boundary $r = a$. In this case, the rate of decrease of the field with distance from the boundary increases with increasing index m [10]. Therefore, oscillations with large values of index m are weakly excited by a plane wave. They are efficiently excited by only the sources located close to the boundary $r = a$.

Figure 8 shows distributions of fields magnitudes $|U|$ at three resonance frequencies in the case of excitation of the plasma cylinder by a filament of magnetic current located near the boundary at the point ($r_0 = 1.2a$, $\varphi_0 = \pi$). In this case, the incident field has the form of a cylindrical wave:

$$U^0(r, \varphi) = H_0^{(2)}\left(k\sqrt{r^2 + r_0^2 + 2rr_0\cos\varphi}\right). \tag{18}$$

It is seen that, at frequency ka_3, field U can be approximated with a good accuracy by function $\cos(3\varphi)$ (curve *3*). Such a high degree of closeness of the resonance field to the field of an eigenmode oscillation are not observed in the case of excitation of the cylinder by a plane wave (see Fig. 6). Oscillations with smaller azimuthal indices are less sensitive to the cylinder excitation method (curves *1*, *2* in Figs. 6, 8).

It follows from the above results that multipole resonances of surface waves appear only at sufficiently small losses of the medium ($\varepsilon'' = 0.001$). Such losses are characteristic of the rarefied ionospheric plasma at an altitude of ~200 km [12]. The complex permittivity of the plasma is determined from the formula

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega(\omega - i\nu)}, \tag{19}$$

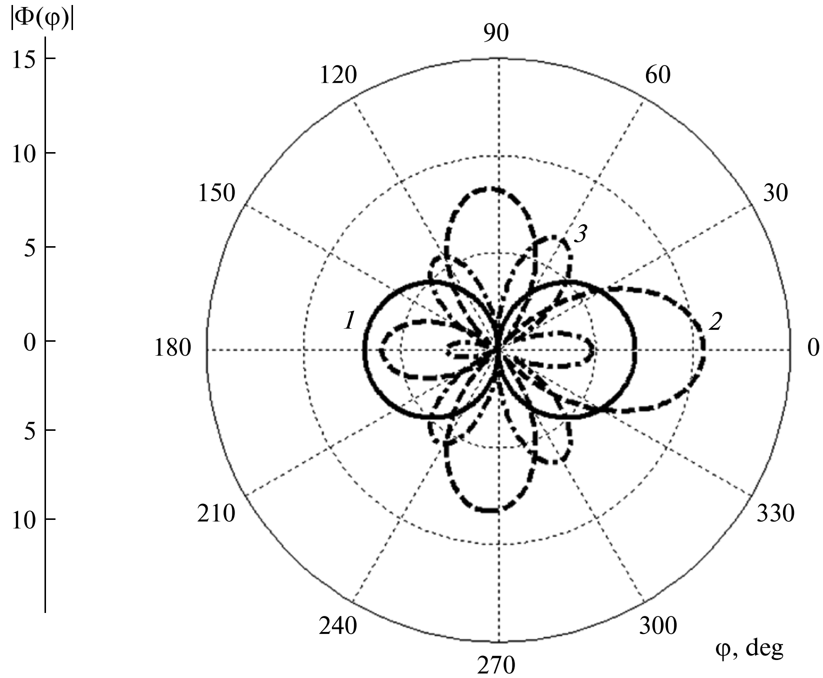

Fig. 5. Scattering patterns of the cylinder made of a material with parameters ε' = −1.04 and ε'' = 0.001 at different resonance frequencies. Curves *1*, *2*, and *3* correspond to ka = 0.11, 0.32, and 0.55, respectively.

where ω_p is the plasma angular frequency of electrons and ν is the frequency of collisions between electrons, ions, and neutral molecules. If we assume that $\nu/\omega_p \sim 10^{-3}$–$10^{-4}$, then, at frequency $\omega \approx \omega_p/\sqrt{2}$ corresponding to the HF band, we obtain values of ε that are required for realization of multipole resonances:

$$\varepsilon' \approx -1; \quad \varepsilon'' \approx 2\sqrt{2}\nu/\omega_p. \tag{20}$$

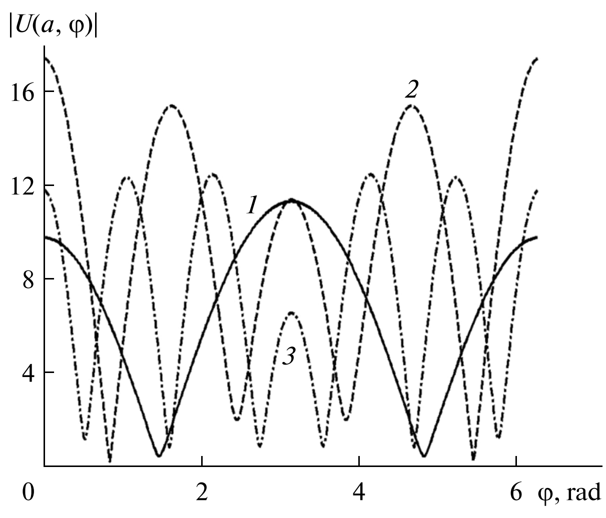

Fig. 6. Distribution of field magnitude $|U(a, \varphi)|$ along the surface of the cylinder made of a material with parameters ε' = −1.04 and ε'' = 0.001 at different resonance frequencies. Curves *1*, *2*, and *3* correspond to ka = 0.11, 0.32, and 0.55, respectively.

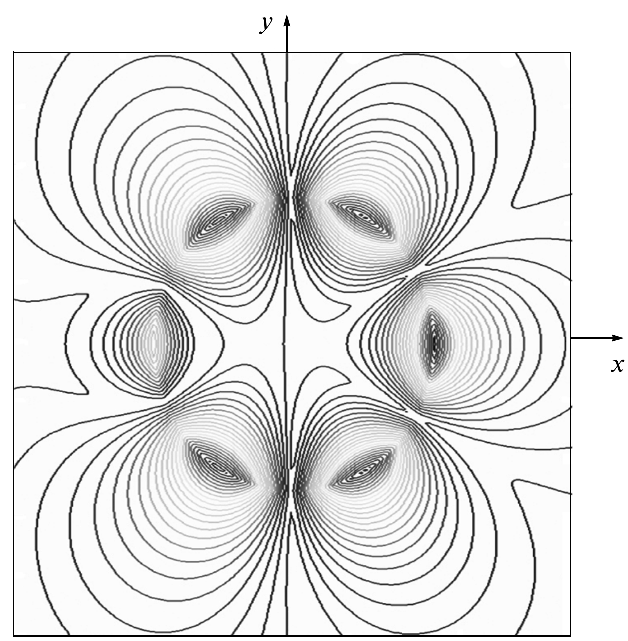

Fig. 7. Spatial distribution of the field for the cylinder made of a material with parameters ε' = −1.04 and ε'' = 0.001 at the resonance frequency ka_3 = 0.55.

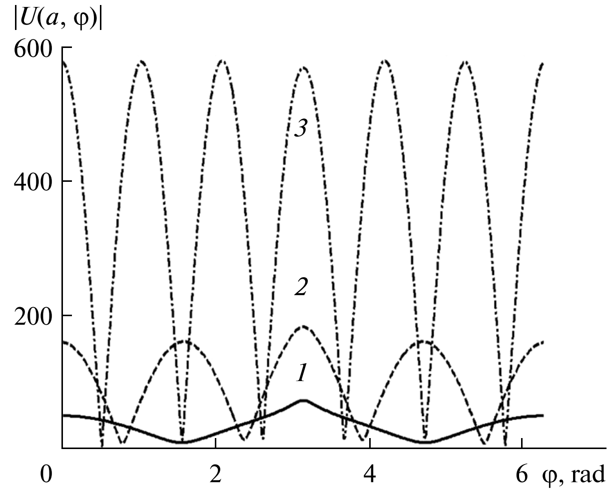

Fig. 8. Distribution of field magnitude $|U(a,\varphi)|$ along the surface of the cylinder made of a material with parameters $\varepsilon' = -1.04$ and $\varepsilon'' = 0.001$ at different resonance frequencies. Curves *1*, *2*, and *3* correspond to $ka = 0.11$, 0.32, and 0.55, respectively. The cylinder is excited by a linear source located at the point ($r_0 = 1.2a$, $\varphi_0 = \pi$).

Since the permittivity of plasma depends on frequency ω, the plots in Figs. 2–4, which describe the dependences of the field characteristics on dimensionless parameter ka, should be considered as functions of cylinder radius a.

Well-conducting metals have the properties of plasma in the optical wave band [13]. Thus, the condition $\varepsilon' \approx -1$ is fulfilled for silver at room temperature at the wavelength $\lambda \approx 340$ nm. In this case, $\varepsilon'' \approx 0.3$ [14]. As the temperature lowers to 90 K, the loss level becomes several times lower [15]. This level is insufficient for implementation of multipole resonances.

Thus, multipole resonances of surface plasmons in a cylinder are possible only at very low levels of the heat loss in the medium ($\varepsilon'' = 0.001$).

ACKNOWLEDGMENTS

This study was partially supported by the Russian Foundation for Basic Research (project no. 15-02-00954-a).

REFERENCES

1. B. Z. Katsenelenbaum, *High-Frequency Electrodynamics* (Nauka, Moscow, 1966; Wiley-VCH, Weinheim, 2006).
2. A. P. Anyutin, I. P. Korshunov, and A. D. Shatrov, J. Commun. Technol. Electron. **59**, 1087 (2014).
3. T. I. Bichutskaya and G. I. Makarov, Vestn. S.-Peterb. Univ., Ser. 4: Fiz., Khim., No. 2(11), 68 (1999) [in Russian].
4. T. I. Bichutskaya and G. I. Makarov, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **45**, 558 (2002).
5. A. G. Kyurkchan, S. A. Minaev, and A. L. Soloveichik, J. Commun. Technol. Electron. **46**, 615 (2001).
6. A. P. Anyutin, A. G. Kyurkchan, and S. A. Minaev, J. Commun. Technol. Electron. **47**, 964 (2002).
7. A. G. Kyurkchan and N. I. Smirnova, *Mathematical Modeling in Diffraction Theory with the Use of a Priori Information on Analytic Properties of the Solution* (ID Media, Moscow, 2014) [in Russian].
8. A. P. Anyutin and V. I. Stasevich, J. Quant. Spectrosc. Radiat. Transf. **100** (1–3), 16 (2006).
9. A. P. Anyutin, J. Commun. Technol. Electron. **55**, 132 (2010).
10. A. P. Anyutin, I. P. Korshunov, and A. D. Shatrov, J. Commun. Technol. Electron. **58**, 926 (2013).
11. A. P. Anyutin, I. P. Korshunov, and A. D. Shatrov, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **56**, 330 (2013).
12. K. Davies, *Ionospheric Radio Waves* (Blaisdell, London, 1969; Mir, Moscow, 1973).
13. V. V. Klimov, *Nanoplasmonics* (Fizmatlit, Moscow, 2009) [in Russian].
14. P. B. Johnson and R. W. Christy, Phys. Rev. B **6**, 4370 (1972).
15. P. Winsemius, F. F. van Kampen, H. P. Lengkeek, and C. G. van Went, J. Phys. F: Metal Phys. **6**, 1583 (1976).

Translated by A. Vyazovtsev