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Degeneration of Quasi-Static Resonances in an Impedance Circular Cylinder Covered by a Metamaterial Layer

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Abstract—An impedance cylinder covered by a metamaterial layer is considered. The 2D problem of excitation of the cylinder by a magnetic-current filament is analyzed. High-Q surface-wave resonances formed in cylinders of small (as compared to the wavelength) dimensions are investigated. The effect of degeneration of oscillations with different indices m in the law $\cos(m\varphi)$, which describes the azimuthal dependence of resonance fields, is discovered. The spatial and frequency characteristics of degenerate oscillations are numerically investigated on the basis of rigorous methods.

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INTRODUCTION

Artificial media with negative relative permittivity ε and permeability μ are today conventionally called metamaterials [1]. Electromagnetic fields excited by sources located near metamaterial bodies exhibit a number of extraordinary properties [2–5]. In studies [6, 7], high-Q resonances are discovered in solid and hollow metamaterial cylinders that have electrically small dimensions and are characterized by the values of ε and μ approaching minus unity. Such cylinders can be considered as ring resonators supporting surface waves propagating over the metamaterial boundary. In this situation, the field is described by the only azimuthal harmonic $\cos(m\varphi)$. It is shown in [8] that the effect of degeneration of quasi-static resonances can be observed in a chiral anisotropically conducting cylinder filled by a metamaterial. In particular, this effect manifests itself in that the resonance field is described by the function $\cos(m\varphi)$ with different values of index m in the near and far zones. In this study, we demonstrate that the effect of degeneration of high-Q quasi-static resonances also occurs in impedance cylinders covered by a metamaterial layer. Here, we deal with a scalar diffraction problem that leads to results similar to those obtained from the solution of the vector problem studied in [8] but differs from the latter in the subject of investigation and by a simpler mathematical apparatus.

1. FORMULATION OF THE PROBLEM AND THE METHOD OF SOLUTION

An impedance circular cylinder covered by a metamaterial layer with parameters $\varepsilon < 0$ and $\mu < 0$ is considered. The 2D problem of excitation of the cylinder by a filament is analyzed. The case of the TM polarization is studied in cylindrical coordinates (r, φ, z). The time dependence of the fields is chosen to be $\exp(i\varphi t)$, and it is assumed that the source is situated outside the cylinder on the ray $\varphi = \pi$ at the point $r = r_0$ (Fig. 1).

The diffraction problem under study is reduced to determination of scalar function $U(r,\varphi) = H_z(r,\varphi)$ satisfying the inhomogeneous Helmholtz equation

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + k^2\varepsilon(r)\mu(r)\right] \times U(r,\varphi) = -\frac{4i}{r}\delta(r - r_0)\delta(\varphi - \pi), \quad (1)$$

where k is the wavenumber in free space and functions $\varepsilon(r)$ and $\mu(r)$ are specified as follows:

$$\varepsilon(r) = \begin{cases} \varepsilon, & a < r < b, \\ 1, & r > b, \end{cases} \qquad \mu(r) = \begin{cases} \mu, & a < r < b, \\ 1, & r > b. \end{cases} \quad (2)$$

We assume that the boundary conditions of the third kind are fulfilled on the cylinder's surface $r = a$:

$$a\frac{\partial U}{\partial r}(a,\varphi) = wU(a,\varphi), \quad (3)$$

where w is a dimensionless real parameter proportional to the impedance of the surface. On the boundary $r = b$, function $U(r, \varphi)$ satisfy the conditions

$$U(b - 0,\varphi) = U(b + 0,\varphi),$$
$$\frac{1}{\varepsilon}\frac{\partial U}{\partial r}(b - 0,\varphi) = \frac{\partial U}{\partial r}(b + 0,\varphi). \quad (4)$$

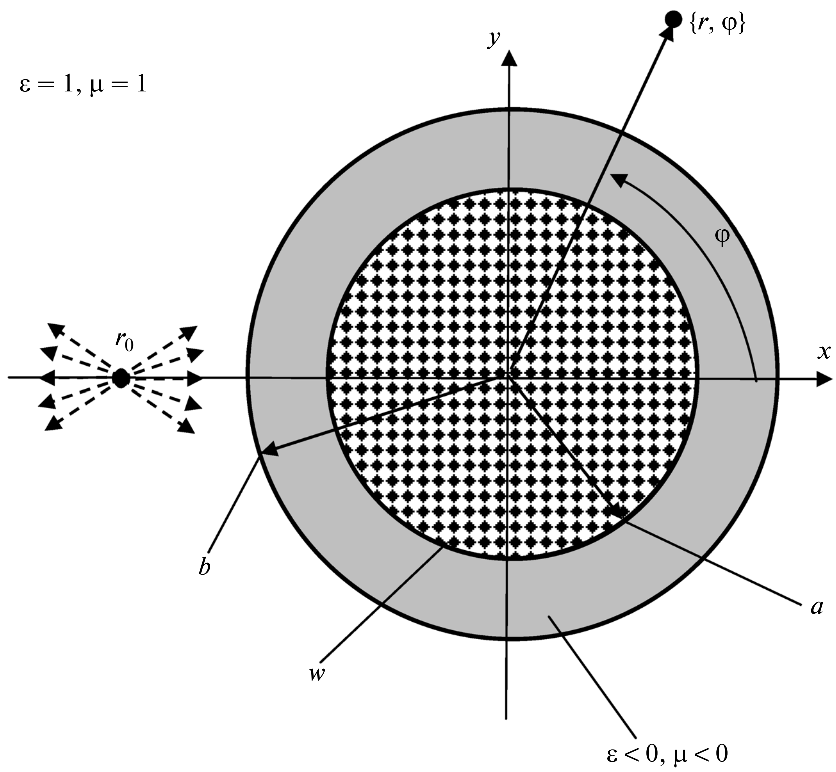

Fig. 1. Geometry of the problem.

Field $U(r, \varphi)$ should also satisfy the radiation condition at infinity.

The field beyond the cylinder ($r > b$) can be represented as the sum of the incident and scattered fields

$$U(r,\varphi) = U^0(r,\varphi) + U^S(r,\varphi). \tag{5}$$

It follows from Eq. (1) that the field of the incident cylindrical wave is specified in the form

$$U^0(r,\varphi) = H_0^{(2)}\left(k\sqrt{r^2 + r_0^2 + 2rr_0\cos\varphi}\right), \tag{6}$$

where $H_0^{(2)}$ is the Hankel function.

Scattered field U^S in the far zone $(kr \to \infty)$ can be represented as

$$U^S(r,\varphi) = \Phi^S(\varphi)\left(2/\pi kr\right)^{1/2}\exp\left(-ikr + i\pi/4\right), \tag{7}$$

where $\Phi^S(\varphi)$ is the scattering pattern. Then, the pattern of incident field $U^0(r, \varphi)$ has the unit amplitude:

$$\Phi^0(\varphi) = \exp\left(-ikr_0\cos(\varphi)\right). \tag{8}$$

The formulated diffraction problem can be solved by means of the method of separation of variables (involving the Rayleigh series [9]). Let us present the basic formulas of the Rayleigh method. We introduce the notation

$$\begin{gathered}
A_m(kb) = J_m(Nkb)J'_m(kb) - \frac{1}{\varepsilon}NJ'_m(Nkb)J_m(kb), \\
B_m(kb) = H_m^{(2)}(Nkb)J'_m(kb) - \frac{1}{\varepsilon}NH_m^{(2)'}(Nkb)J_m(kb), \\
C_m(kb) = J_m(Nkb)H_m^{(2)'}(kb) - \frac{1}{\varepsilon}NJ'_m(Nkb)H_m^{(2)}(kb), \\
D_m(kb) = H_m^{(2)}(Nkb)H_m^{(2)'}(kb) - \frac{1}{\varepsilon}NH_m^{(2)'}(Nkb)H_m^{(2)}(kb), \\
L_m(ka) = wJ_m(Nka) - NkaJ'_m(Nka), \\
M_m(ka) = wH_m^{(2)}(Nka) - NkaH_m^{(2)'}(Nka),
\end{gathered} \tag{9}$$

where

$$N = \sqrt{\varepsilon\mu}, \tag{10}$$

J_m are the Bessel functions, and the prime denotes the differentiation with respect to the argument.

The scattered field outside the cylinder ($r > b$) has the form

$$\begin{gathered}
U^S(r,\varphi) = \sum_{m=0}^{\infty}(-1)^m\delta_m H_m^{(2)}(kr_0) \\
\times \frac{M_m(ka)A_m(kb) - L_m(ka)B_m(kb)}{L_m(ka)D_m(kb) - M_m(ka)C_m(kb)}H_m^{(2)}(kr)\cos(m\varphi),
\end{gathered} \tag{11}$$

where

$$\delta_m = \begin{cases} 1, & m = 0, \\ 2, & m > 1. \end{cases} \quad (12)$$

The field in the layer $(a \le r \le b)$ is

$$U(r, \varphi) = \frac{2i}{\pi k b} \sum_{m=0}^{\infty} (-1)^m \delta_m H_m^{(2)}(kr_0) \times \frac{M_m(ka) J_m(Nkr) - L_m(ka) H_m^{(2)}(Nkr)}{L_m(ka) D_m(kb) - M_m(ka) C_m(kb)} \cos(m\varphi). \quad (13)$$

The formula for scattering pattern $\Phi^S(\varphi)$ can be obtained from expression (11) through replacing Hankel function $H_m^{(2)}(kr)$ by $(i)^m$.

2. LOW-FREQUENCY RESONANCES

The Raylegh series contain resonance denominators $L_m(ka)D_m(kb) - M_m(ka)C_m(kb)$. Let us analyze the frequency dependence of these denominators assuming that

$$kb \ll 1, \quad Nkb \ll 1, \quad \left(\frac{a}{b}\right)^{2m} \ll 1, \quad |\varepsilon + 1| \ll 1. \quad (14)$$

When conditions (14) are fulfilled, the real parts of the denominators substantially exceed their imaginary parts. The real part of a resonance denominator vanishes at the point kb_m, which is a resonance frequency. In order to determine resonance frequencies, we apply the known asymptotic expansions of cylindrical functions for small values of the argument. We use two terms of the expansion in positive powers of the argument for the Bessel functions and two terms in negative powers for the Hankel functions. Taking into account conditions (14), we obtain the following expression for resonance frequencies:

$$(kb_m)^2 = \frac{2m(m^2 - 1)}{1 - \mu + m(1 + \mu)} \times \left[\left(1 + \frac{1}{\varepsilon}\right) + \left(1 - \frac{1}{\varepsilon}\right)\frac{m - w}{m + w}\left(\frac{a}{b}\right)^{2m}\right], \quad m \ge 2. \quad (15)$$

In the case of the TM polarization, the permeability affects the resonance frequencies only slightly. Therefore, for definiteness, we assume below that $\mu = -1$. At the resonance frequency kb_m, the only azimuthal harmonic $\cos(m\varphi)$ dominates in field $U(r, \varphi)$.

Evidently, the above formulas can be applied for the solution of the diffraction problem when an impedance cylinder is replaced by a cylinder with the perfect electric ($w = 0$) or magnetic ($w = \infty$) conductivity of the surface.

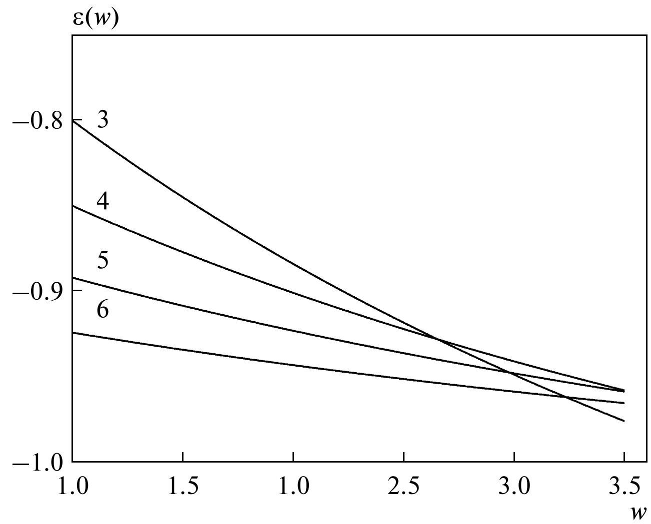

Fig. 2. Trajectories of the resonance parameters of the modes with the index m in the w–ε plane for $\frac{a}{b} = 0.81$; $kb = 0.2$, and $\mu = -1$.

3. DEGENERATION OF LOW-FREQUENCY RESONANCES

A case is possible when parameters ε, μ, w, $\frac{a}{b}$, and kb have resonance values simultaneously for two different azimuthal indices m. A set of dispersion curves describing the coupling between resonance values of w and ε for $\mu = -1$, $a/b = 0.81$, and $kb = 0.2$ is displayed in Fig. 2. The curves, corresponding to different values of index m, are plotted according to formula (15). As is seen from the figure, all of the curves intersect, a circumstance that indicates the degeneration phenomenon. The point on the parameter plane (w, ε) where the dispersion curves with different indices m intersect is below referred to as a degeneration point, for example, the 3–4 degeneration point.

Let us present results of calculation of the wave fields under the conditions of quasi-static resonance degeneration. In the subsequent calculation (with the exception of the last example for $w < 0$), it is assumed that $\mu = -1$, $a/b = 0.81$, and $r_0 = 1.2b$. The only variable parameters are ε, w, and kb.

We describe the behavior of the field within a frequency interval with the help of the amplitude–frequency characteristic (AFC) that is considered to mean the frequency dependence of the absolute value of the field on the boundary of the structure under study at the point $r = b$, $\varphi = \pi$.

Figure 3 depicts the AFC of the cylinder for the parameters

$$\varepsilon = 0.9478999730915, \quad w = 2.98, \quad (16)$$

which correspond to the degeneration of the resonances with azimuthal indices $m = 4$ and $m = 6$. The AFC is a resonance curve with the maximum at the

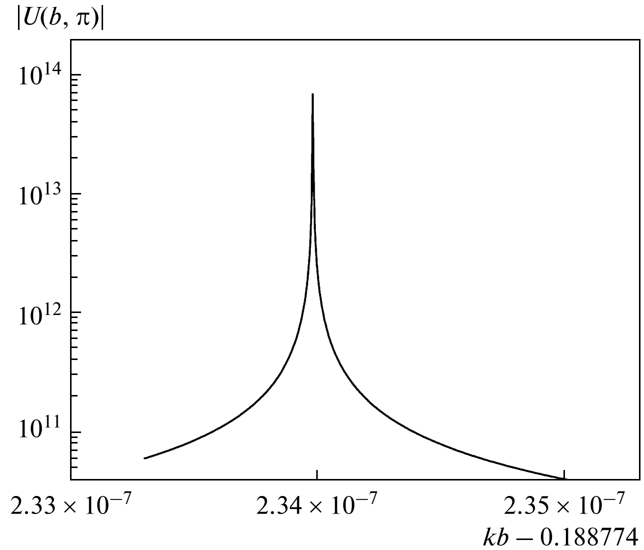

Fig. 3. The AFC of a cylinder covered by a metamaterial at the 4–6 degeneration point for $\varepsilon = -0.947899973091$; $\mu = -1$; $w = 2.98$; $\frac{a}{b} = 0.81$, and $r_0 = 1.2b$.

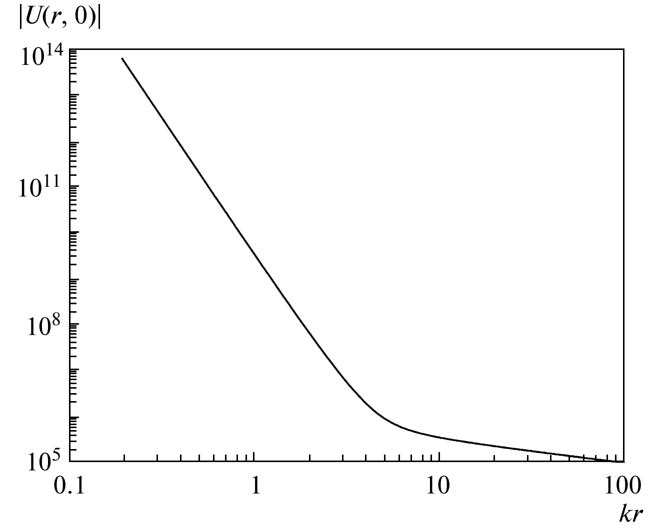

Fig. 4. Distribution of the absolute value of the field over the radial coordinate beyond an impedance cylinder covered by a metamaterial for $\varepsilon = -0.9478999730915$; $\mu = -1$; $w = 2.98$; $\frac{a}{b} = 0.81$; $r_0 = 1.2b$, and $kb_{4,6} = 0.18877423398$.

point $kb_{4,6} = 0.18877423...$. We find approximate values of the parameters responsible for oscillation degeneration with the help of the plot from Fig. 2. Next, we apply the numerical method of successive iterations to determine their exact values. We take into account that deviations of parameters ε and w from their actual values result in AFC splitting.

The computation shows that, at the resonance frequency $kb_{4,6}$, the field distribution over the boundary and the scattering pattern are described by one higher order harmonic: $U(b,\varphi) = A_6\cos(6\varphi)$ and $\Phi^S(\varphi) = B_6\cos(6\varphi)$, where $A_6 \sim 10^{12}$ and $B_6 \sim 10^4$. The distribution of the absolute value of the field over the radial coordinate along the direction $\varphi = 0$ is depicted in Fig. 4. The curve contains two sections—$kr < 6$ and $kr > 10$—where the field decreases according to the laws $|U| \sim (kr)^{-6}$ and $|U| \sim (kr)^{-1/2}$. These sections correspond to the near and far zones of the diffraction field, respectively.

The AFC of the considered structure is displayed in Fig. 5 for the case when the quantity $\varepsilon = -0.94789995$ differs from value (16) in the digit in the seventh decimal place. The curve contains two maxima spaced by a dip. The frequencies corresponding to the AFC maxima are rather close: $kb_6 = 0.18875964$ and $kb_4 = 0.18877076$. In other words, the degeneration disappears; and, in addition, a deep dip at the frequency $kb = 0.188767015$ is formed. The near and far fields at resonance frequencies kb_6 and kb_4 are described by one azimuthal harmonic: $\cos(6\varphi)$ or $\cos(4\varphi)$; in this case, $A_6 \sim 10^{11}$, $A_4 \sim 10^9$, $B_6 \sim 2\times10^5$, and $B_4 \sim 2\times10^4$.

The curve from Fig. 5 can be considered as the frequency characteristic of a complex resonance consisting of two coupled oscillations. Varying the frequency within narrow interval (kb_6, kb_4), we can obtain any relationship between the amplitudes of the azimuthal harmonics with the indices $m = 4$ and $m = 6$ forming resonance field $U(b, \varphi)$. The distribution of the absolute value of field $U(b, \varphi)$ at the frequency of the AFC dip is depicted in Fig. 6. The zero value of the field in the directions of the angles $\varphi = 0$ and $\varphi = \pi$ is explained by the fact that this distribution is formed as a result of an antiphase combination of two even harmonics having equal amplitudes:

$$U(b,\varphi) = A\left(\cos(4\varphi) - \cos(6\varphi)\right) = 2A\sin\varphi\sin(5\varphi). \quad (17)$$

In this situation, the scattering pattern is described by the lower order azimuthal harmonic: $\Phi^S(\varphi) = B\cos(4\varphi)$, where $B \sim 10^2$.

The wave fields of other pairs of degenerate resonances exhibit similar properties. Consider the case

$$\varepsilon = -0.932015347, \quad w = 2.3217, \quad (18)$$

which corresponds to degeneration of the oscillations with the azimuthal indices $m = 3$ and $m = 6$. Field $U(b, \varphi)$ and pattern $\Phi^S(\varphi)$ at the resonance frequency $kb_{3,6} = 0.18357102492$ are described by the higher order harmonic $\cos(6\varphi)$ with the amplitudes 10^{13} and 10^5, respectively. In the presence of a slight deviation of the permittivity ($\varepsilon = -0.932012$), the AFC resembles the curve depicted in Fig. 3; in this case, the resonance peaks are associated with the frequencies $kb_6 = 0.181185$ and $kb_3 = 0.183365$. Figure 7 shows the field

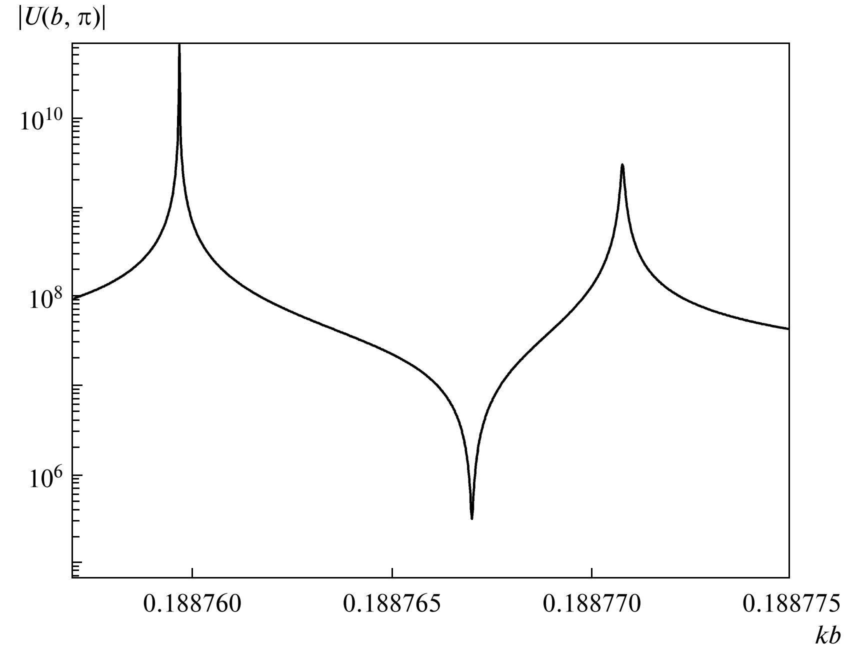

Fig. 5. The AFC of an impedance cylinder covered by a metamaterial in the neighborhood of the 4–6 degeneration point for $\varepsilon = -0.94789995$; $\mu = -1$; $w = 2.98$; $\frac{a}{b} = 0.81$, and $r_0 = 1.2b$.

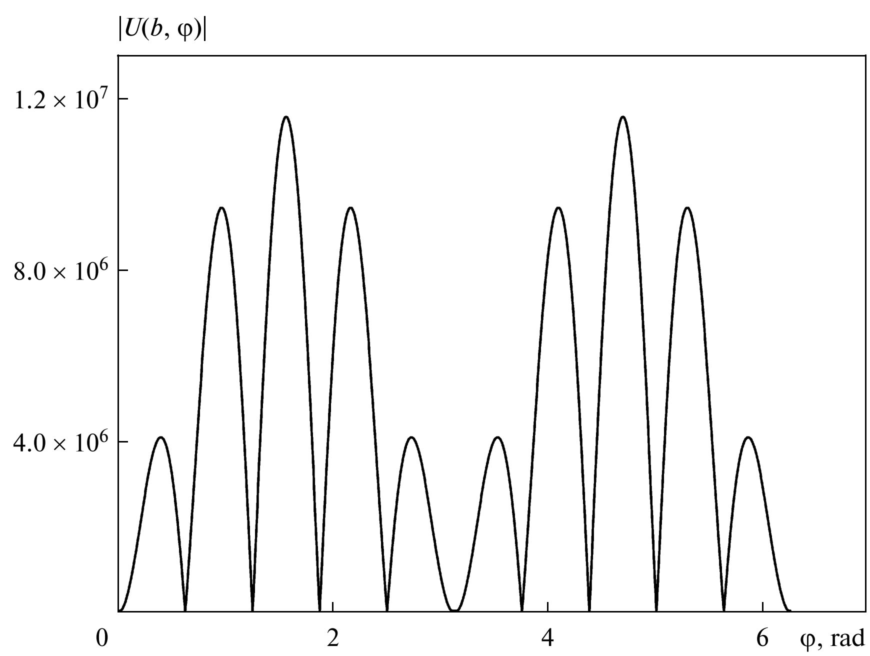

Fig. 6. Distribution of the absolute value of the field over the boundary $r = b$ of an impedance cylinder covered by a metamaterial in the neighborhood of the 4–6 degeneration point for $\varepsilon = -0.94789995$; $\mu = -1$; $w = 2.98$; $\frac{a}{b} = 0.81$; $r_0 = 1.2b$, and $kb = 0.188767015$.

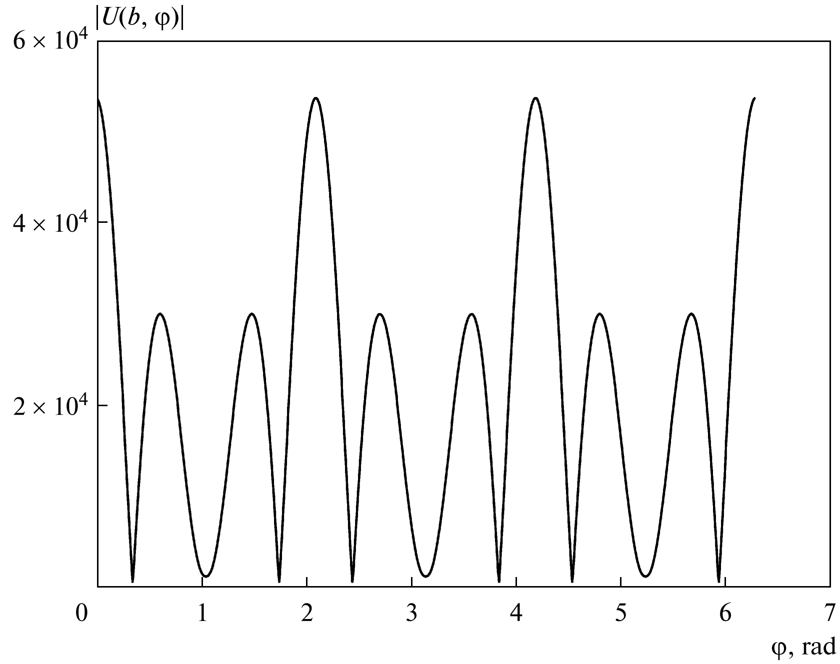

Fig. 7. Distribution of the absolute value of the field over the boundary $r = b$ of an impedance cylinder covered by a metamaterial in the neighborhood of the 3–6 degeneration point for $\varepsilon = -0.932012$; $\mu = -1$; $w = 2.3217$; $\frac{a}{b} = 0.81$; $r_0 = 1.2b$, and $kb = 0.182873$.

distribution over the cylinder's boundary at the frequency $kb = 0.182873$, which corresponds to the AFC dip. In this case, the dip is formed due to an in-phase combination of an even harmonic and an odd harmonic. This distribution can be approximated by the function

$$\begin{aligned} U(b,\varphi) &= A(\cos(3\varphi) + \cos(6\varphi)) \\ &= 2A\cos(3\varphi/2)\cos(9\varphi/2). \end{aligned} \tag{19}$$

As in the previous example, the scattering pattern at this frequency contains only a lower order harmonic: $\Phi^S(\varphi) = B_3\cos(3\varphi)$, where $B_3 \sim 30$.

Consider the field formation at the 3–4 point, where the oscillations with azimuthal indices $m = 3$ and 4 degenerate. For this point, we have

$$\varepsilon = -0.8687104, \quad w = 1.79449. \tag{20}$$

At the frequency point $kb_{3,4} = 0.1807376$, the field on the boundary $r = b$ has the form $A\cos(4\varphi)$ with $A \sim 10^7$, i.e., the higher order harmonic dominates. The scattering pattern in this case is $B\cos(4\varphi)$ with $B \sim 10^4$. In the presence of a slight deviation of the permittivity ($\varepsilon = -0.8688$) from the above point, the AFC is transformed into a curve with two resonance peaks at the points $kb_3 = 0.18672$ and $kb_4 = 0.1969$ and a deep dip at the frequency $kb = 0.190517$. The absolute value of field $U(b, \varphi)$ at the dip frequency is shown in Fig. 8. It is seen that this distribution is the superposition of two harmonics

$$\begin{aligned} U(b,\varphi) &= A(\cos(3\varphi) + \cos(4\varphi)) \\ &= 2A\cos\left(\frac{1}{2}\varphi\right)\cos\left(\frac{7}{2}\varphi\right). \end{aligned} \tag{21}$$

The degeneration effects are observed for negative values of w as well. Let us discuss the peculiarities of this case considering, as an example, a resonance containing the azimuthal harmonics with the indices $m = 3$ and $m = 4$. Assume that $a/b = 0.663$, $\mu = -1$, and $r_0 = 1.2b$. Then, the degeneration occurs for the values

$$w = -4.97519, \quad \varepsilon = -2.04579262 \tag{22}$$

at the frequency $kb_{3,4} = 0.1466443765$. The corresponding frequency characteristic is depicted in Fig. 9 (curve *1*). A slight deviation of ε from the value indicated in (22) results in formation of two resonance peaks in the AFC at the close frequencies $kb_3 = 0.1431388$ and $kb_4 = 0.15241413$ (see Fig. 10). The field distribution $|U|$ over the boundary $r = b$ is shown in Fig. 11 for the frequency $kb = 0.14775$, which corresponds to the AFC minimum from Fig. 10. It is seen that function $U(b, \varphi)$ is the sum of two harmonics

$$\begin{aligned} U(b,\varphi) &= A(\cos(3\varphi) - \cos(4\varphi)) \\ &= 2A\sin\left(\frac{1}{2}\varphi\right)\sin\left(\frac{7}{2}\varphi\right). \end{aligned} \tag{23}$$

In contrast to similar distributions (17), (19), and (21), obtained for $w > 0$, field $U(b, \varphi)$ in the considered case

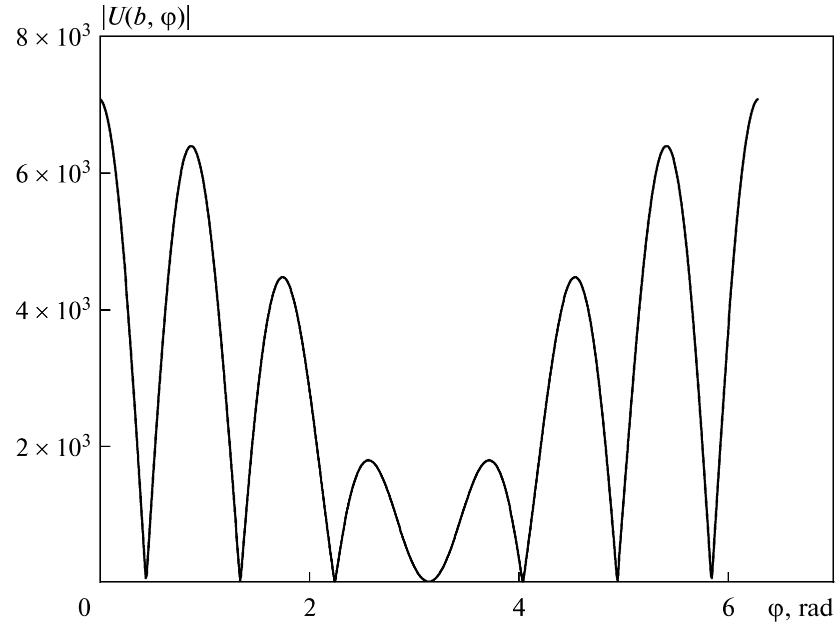

Fig. 8. Distribution of the absolute value of the field over the boundary $r = b$ of an impedance cylinder covered by a metamaterial in the neighborhood of the 3–4 degeneration point for $\varepsilon = -0.8688$; $\mu = -1$; $w = 1.79449$; $\frac{a}{b} = 0.81$; $r_0 = 1.2b$, and $kb = 0.190517$.

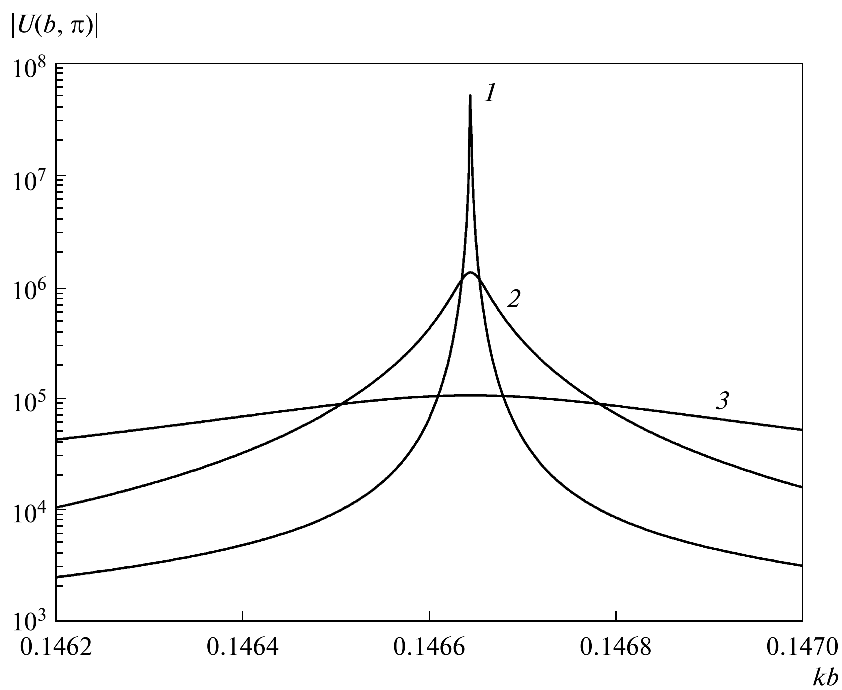

Fig. 9. The AFC of an impedance cylinder covered by a metamaterial at the 3–4 degeneration point for $\varepsilon = -2.04579262$; $\mu = -1$; $w' = -4.97519$; $\frac{a}{b} = 0.663$, and $r_0 = 1.2b$: w'' = (curve *1*) 0, (curve *2*) 10^{-7}, and (curve *3*) 10^{-5}.

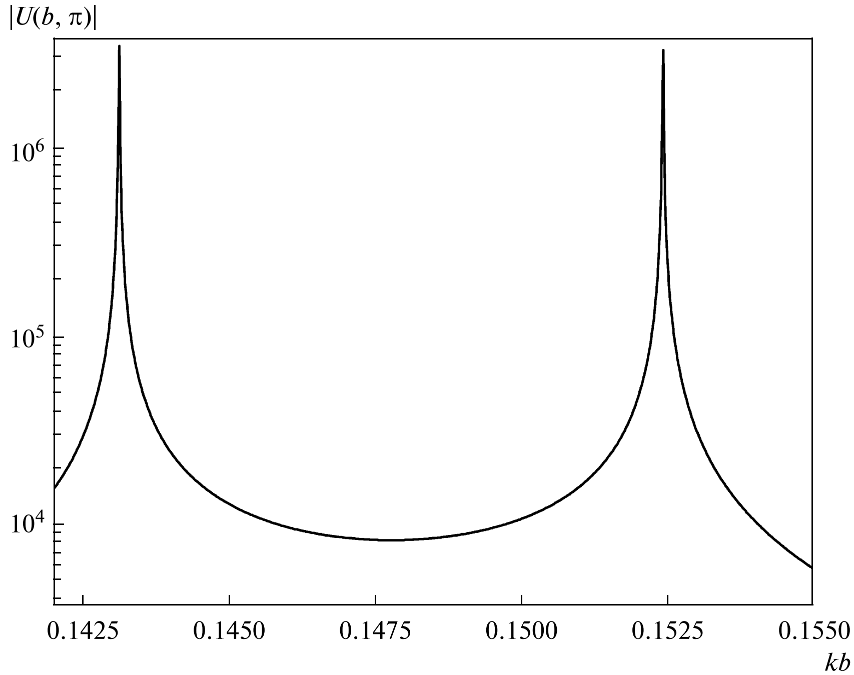

Fig. 10. The AFC of an impedance cylinder covered by a metamaterial in the neighborhood of the 3–4 degeneration point for $\varepsilon = -2.0457$; $\mu = -1$; $w = -4.97519$; $\frac{a}{b} = 0.663$, and $r_0 = 1.2b$.

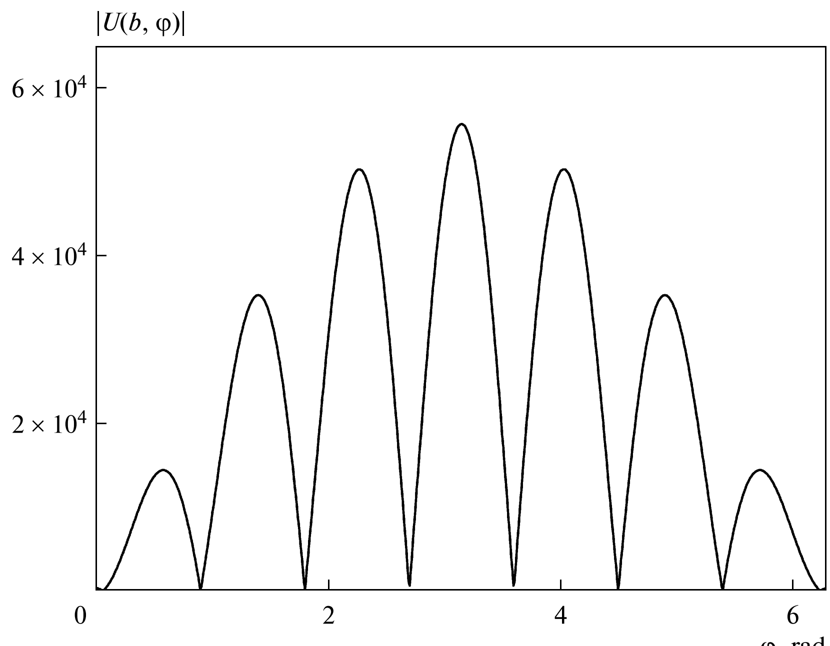

Fig. 11. Distribution of the absolute value of the field over the boundary $r = b$ of an impedance cylinder covered by a metamaterial in the neighborhood of the 3–4 degeneration point for $\varepsilon = -2.0457$; $\mu = -1$; $w = -4.97519$; $\frac{a}{b} = 0.663$; $r_0 = 1.2b$, and $kb = 0.14775$.

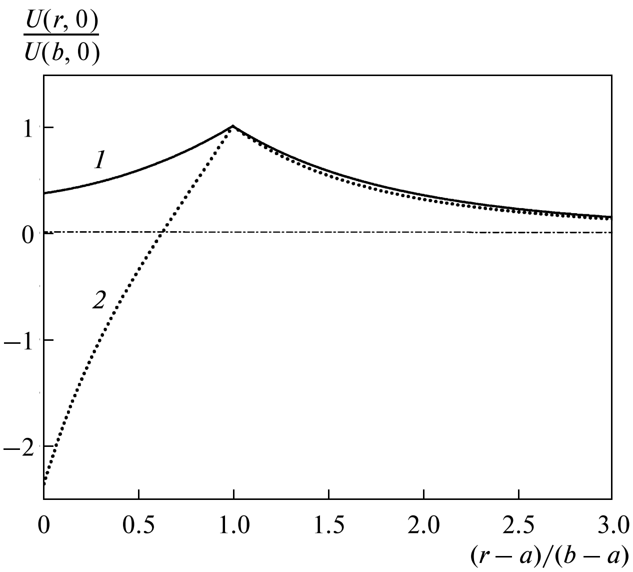

Fig. 12. Distribution of the absolute value of the field over the radial coordinate of an impedance cylinder covered by a metamaterial for $r_0 = 1.2b$, $\mu = -1$: (curve *1*) $\varepsilon = -0.947899973091$, $w = 2.98$, and $\frac{a}{b} = 0.81$ (the 4–6 degeneration point) and (curve *2*) $\varepsilon = -2.04579262$, $w = -4.97519$, and $\frac{a}{b} = 0.663$ (the 3–4 degeneration point).

does not vanish at $\varphi = \pi$. Therefore, the curve from Fig. 10 differs from the analogous characteristic from Fig. 5 by the absence of a sharp dip between resonance peaks.

It is interesting to compare the behaviors of the resonance fields as functions of the radial coordinate for positive and negative values of w. Curves *1* and *2* in Fig. 12 show the normalized radial dependences of the fields calculated for parameters (16) and (22), which cause strict degeneration of the oscillations at the points $m = 4$ and 6 and $m = 3$ and 4. It is seen that the fields monotonically decrease as the observation point moves from the boundary ($r = b$) deep into the layer. The function $\frac{U(r,0)}{U(b,0)}$ is positive when $w > 0$, and the field vanishes at a certain point when $w < 0$. The comparison of curves *1* and *2* shows that, when $w < 0$, field $U(r, 0)$ concentrates not only near the metamaterial boundary $r = b$ but also near the surface of the impedance cylinder $r = a$. This result can be attributed to the fact that an impedance boundary with $w < 0$ can support surface waves.

Beyond the layer ($r > b$) normalized fields $U(r, 0)$ in the static proximity to the boundary decrease according to the law $(r/a)^{-m}$, where m is the number of the higher order degenerate harmonic. When $kb \ll 1$ and $m \gg 1$, the standing wave field is characterized by oscillations along the metamaterial boundary that are frequent on the wavelength scale. This very slow surface wave is formed by both the medium interface and the impedance surface of the cylinder.

Let us assess the influence of the heat loss on the resonance Q factor. When ε, μ, and w are real quantities, the Q factor of resonances is determined only by the radiation loss, which turns out to be quite low, a circumstance that necessitates calculation of the resonance parameters with a high accuracy. The heat loss in impedance cylinders can be taken into account through setting $w = w' + iw''$, where $w'' > 0$. The influence of this loss on the resonance Q factor is illustrated by curves *2* and *3* from Fig. 9. It is seen that, even when $w'' = 10^{-7}$, the heat loss exceeds the radiation loss. When $w'' = 10^{-5}$ (curve *3* from Fig. 9), the Q factor of the resonance is $Q \approx 350$.

CONCLUSIONS

The effect of degeneration of high-Q quasi-static oscillations can be observed in an impedance cylinder covered by a metamaterial layer. Under the degeneration conditions, the wave field is a superposition of two azimuthal harmonics. Slight deviations of the problem parameters from the resonance values result in a substantial change in the relationship between of the amplitudes of these harmonics. The considered electrodynamic structure has a frequency response typical of a system of two coupled resonators.

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