ISSN 0021-3640, JETP Letters, 2014, Vol. 100, No. 11, pp. 737–741. © Pleiades Publishing, Inc., 2014.
Original Russian Text © O.V. Prikhod'ko, O.S. Sukhorukova, S.V. Tarasenko, V.G. Shavrov, 2014, published in Pis'ma v Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki, 2014, Vol. 100, No. 11, pp. 837–842.

# Magnetoelastic Rearrangement of the Spectrum of Surface Phonons in a Piezomagnetic Crystal

**O. V. Prikhod'ko[a], O. S. Sukhorukova[a], S. V. Tarasenko[a], and V. G. Shavrov[b]**

[a] *Donetsk Institute of Physics and Engineering, National Academy of Sciences of Ukraine, Donetsk, 83114 Ukraine*
*e-mail: s.v.tarasenko@mail.ru*

[b] *Kotel'nikov Institute of Radio Engineering and Electronics, Russian Academy of Sciences, ul. Mokhovaya 11-7, Moscow, 125009 Russia*

Received October 16, 2014; in final form, October 29, 2014

The inclusion of the magnetoelastic interaction, together with the linear magnetostriction, results in the fundamental rearrangement of the spectrum of shear elastic surface waves propagating along the mechanically free surface of a piezomagnetic crystal.

**DOI:** 10.1134/S002136401423009X

The necessity of creating new types of smart materials with dynamic properties easily varying in the acoustic frequency range by an external field has recently initiated a sharp increase in the number of studies of the dynamic properties of composite structures involving piezomagnetic media [1–3]. The wave characteristics of such structures are closely related to the conditions of localization of elastic oscillations and their transmission through the interface between contacting materials. The calculation of the conditions of the formation of shear elastic surface waves in magnetically ordered media is traditionally based on the use of one of two fundamentally different (for the essence of this work) theoretical approaches. One of them is applied in the case of an acoustically gyrotropic medium, e.g., an easy axis ferromagnet [4–7], whereas the second approach is used in the case of an acoustically nongyrotropic magnetic medium (compensated easy axis antiferromagnet in the absence of an external magnetic field) [8–11].

Beginning with pioneering works [4, 5], the spectrum of the shear elastic surface wave propagating along the mechanically free surface of an easy axis ferromagnet magnetized by a static external magnetic field along the tangent to the surface of the ferromagnet is traditionally calculated with the inclusion of only the magnetoelastic interaction:

$$W_{\mathrm{me}} = \gamma_{\mathrm{me}} M_i M_k u_{ik}, \tag{1}$$

where $\mathbf{M}$ is the magnetic moment per unit volume, $u_{ik}$ is the elastic strain tensor, and $\gamma_{\mathrm{me}}$ is the isotropic magnetoelastic coupling constant. However, in this approach, the interaction between the elastic subsystem and magnetic dipole field is taken into account indirectly (through the spin subsystem of the magnet). Consequently, in the high-frequency range, the effect of the indicated interaction on the condition of the localization of an SH wave almost vanishes with an increase in the frequency as compared to the effects of the quadratic magnetostriction interaction:

$$W_{\mathrm{ms2}} = \gamma_{\mathrm{ms2}} H_i H_k u_{ik}, \tag{2}$$

where $\gamma_{\mathrm{ms2}}$ is the isotropic quadratic magnetostriction constant and $\mathbf{H}$ is the magnetic field.

A fundamentally different (than the acoustic of magnetically gyrotropic media) approach is used to calculate the spectrum of shear elastic surface waves in a magnetically compensated medium (in particular, easy axis antiferromagnets in the collinear phase in the works cited above). As a mechanism of the localization of a shear elastic wave near the mechanically free surface of an antiferromagnet, the authors of [8–11] traditionally considered only the piezomagnetic interaction (also called linear magnetostriction) whose structure has the form

$$W_{\mathrm{ms1}} = \overline{\overline{\gamma}}_{\mathrm{ms1}} H_i u_{kl}, \tag{3}$$

where $\overline{\overline{\gamma}}_{\mathrm{ms1}}$ is the piezomagnetic constant tensor. This interaction can already exist in the absence of spontaneous magnetization of the medium and, therefore, quadratic magnetostriction (e.g., in the magnetically compensated phase of an easy axis antiferromagnet). However, it is known that the necessary conditions of the formation of the magnetostriction interaction under consideration in the antiferromagnetic crystal impose certain symmetry constraints on the allowable equilibrium spin configuration [12]. At the same time, in the magnetically compensated phase of the exchange collinear antiferromagnet at any spin configuration, there is the magnetoelastic interaction of the form

$$W_{\mathrm{me}} = \bar{\bar{\gamma}}_{\mathrm{me}} l_i l_k u_{ik}, \tag{4}$$

where $\bar{\bar{\gamma}}$ is the magnetoelastic constant tensor and **l** is the antiferromagnetism vector (for the two-sublattice model of the antiferromagnet). The analysis of the magnetoelastic dynamics of the unbounded easy axis antiferromagnet [13] indicates the possibility of resonance between the amplitude of small oscillations of the vector $l_i$ near the equilibrium orientation and individual components of the elastic strain tensor $u_{ik}$ (magnetoacoustic resonance). This allows the assumption that the inclusion of magnetoelastic interaction (4) in addition to linear magnetostriction (3) will result in the possibility of the fundamental transformation of the previously known spectrum of shear elastic waves in a semibounded piezomagnet [8–11]. However, such an analysis has not yet been performed.

The aim of this work is to study the anomalies of the spectrum of the shear elastic surface wave propagating along the mechanically free surface of a piezomagnetic crystal with the simultaneous inclusion of linear magnetostriction and magnetoelastic interactions induced by the spin system of a magnetic.

As an example, we consider a two-sublattice model of an easy axis (the $OZ$ axis) exchange collinear antiferromagnet (with the magnetizations of sublattices $\mathbf{M}_1$ and $\mathbf{M}_2$, $|\mathbf{M}_1| = |\mathbf{M}_2| = M_0$) [8]. For simplicity and clarity of calculations, we assume that its magnetoelastic and elastic properties are isotropic. As a result, the corresponding density of the thermodynamic potential with allowance for linear magnetostriction $W_{\mathrm{ms1}}$ and Dzyaloshinskii interaction $W_{\mathrm{D}}$ can be represented in terms of ferromagnetism vectors **l** in the form

$$\begin{gathered} W = \frac{\delta}{2}\mathbf{m}^2 + \frac{b}{2}(l_x^2 + l_y^2) + W_{\mathrm{D}} + W_{\mathrm{ms1}} \\ -\mathbf{mh} + \gamma l_i l_k u_{ik} + \frac{\lambda}{2}u_{ii}^2 + \mu_{ik}^2. \end{gathered} \tag{5}$$

Here, $\delta$, $b$, and $\gamma$ are the intersublattice exchange, easy axis magnetic anisotropy ($b > 0$), and isotropic magnetoelastic interaction constants, respectively; $\lambda$ and $\mu$ are the bulk modulus and shear modulus, respectively; $\mathbf{m} = (\mathbf{M}_1 + \mathbf{M}_2)/2M_0$ and $\mathbf{l} = (\mathbf{M}_1 - \mathbf{M}_2)/2M_0$ are the ferromagnetism and antiferromagnetism vectors, respectively; and **h** is the reduced magnetic field.

Similar to [8–11], the analysis below will be restricted to the surface magnetoelastic dynamics of the antiferromagnet under consideration in the collinear phase ($\mathbf{l} \parallel OZ$) under the assumption that the $(XY)$ plane is sagittal and the elastic displacement vector in a shear wave is directed along the $OZ$ axis. If the spin structure of the piezomagnet is $4_z^- 2_d^-$ according to [12], the following components of the linear (piezomagnetic) interaction are of the most interest for the type of propagating elastic waves under consideration:

$$W_{\mathrm{ms1}} = \gamma_{\mathrm{ms1}}(H_x u_{xz} - H_y u_{yz}). \tag{6}$$

The invariant corresponding to Eq. (6) that determines the Dzyaloshinskii interaction has the form

$$W_{\mathrm{D}} = d(m_x l_x - m_y l_y), \tag{7}$$

where $d$ is the coupling constant.

If the finiteness of the propagation velocity of an electromagnetic wave is disregarded, the magnetoelastic dynamics of the model given by Eqs. (5)–(7) is described by a closed system of equations including the Landau–Lifshitz equations for the vectors **m** and **l**, the basic equation of continuum mechanics, and equations of magnetostatics [13]:

$$\begin{gathered} \frac{1}{g}\frac{\partial \mathbf{m}}{\partial t} = [\mathbf{m}H_{\mathbf{m}}] + [\mathbf{l}H_{\mathbf{l}}], \quad \frac{1}{g}\frac{\partial \mathbf{l}}{\partial t} = [\mathbf{m}H_{\mathbf{l}}] + [\mathbf{l}H_{\mathbf{m}}], \\ \operatorname{div}\mathbf{h} = -8\pi \operatorname{div}\mathbf{m}, \quad \rho\frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k}. \end{gathered} \tag{8}$$

Here, $\rho$ is the density; **u** is the elastic shift vector; $g$ is the gyromagnetic ratio, which is taken the same for both sublattices; $H_r \equiv -\delta W/\delta r$ is the effective field; $r = \mathbf{m}, \mathbf{l}$; and $\sigma_{ik}$ is the elastic stress tensor.

The calculation shows that the material relations for the shear wave with $\mathbf{u} \parallel OZ$, $\mathbf{k} \in (XY)$ in the easy axis antiferromagnet characterized by Eq. (5) with the structure $4_z^- 2_d^-$ can be represented in the form

$$\begin{gathered} \begin{cases} \sigma_{zx} = c_\perp \dfrac{\partial u_z}{\partial x} + \beta_{15}h_x - i\beta_* h_y; \\ \sigma_{zy} = c_\perp \dfrac{\partial u_z}{\partial y} + \beta_{24}h_y - i\beta_* h_y; \end{cases} \\ \begin{cases} B_x = \mu_\perp h_x - 4\pi\beta_{15}\dfrac{\partial u_z}{\partial x} + 4\pi i\beta_*\dfrac{\partial u_z}{\partial y}; \\ B_y = \mu_\perp h_y - 4\pi\beta_{24}\dfrac{\partial u_z}{\partial y} - 4\pi i\beta_*\dfrac{\partial u_z}{\partial x}; \end{cases} \end{gathered} \tag{9}$$

$$\begin{cases} c_\perp \equiv \mu\dfrac{\omega_*^2 - \omega_{\mathrm{me}}^2 - \omega^2}{\omega_*^2 - \omega^2}; \quad \omega_{\mathrm{me}}^2 \equiv \dfrac{g^2 M_0^2 \delta\gamma^2}{\mu}; \\ \omega_*^2 \equiv \omega_0^2 + \omega_{\mathrm{me}}^2; \quad \mu_\perp \equiv 1 + 4\pi\chi; \\ \beta_{15} = \beta_{24} \equiv \gamma_{\mathrm{ms1}} + \dfrac{g^2 M_0^2 \gamma d}{\omega_*^2 - \omega^2}, \quad \beta_* \equiv \dfrac{g M_0 \gamma\omega}{\omega_*^2 - \omega}; \\ \chi \equiv \chi_\perp\dfrac{\omega_*^2}{\omega_*^2 - \omega^2}; \quad \chi_\perp \equiv \dfrac{4}{\delta}, \end{cases} \tag{10}$$

where **B** is the magnetic induction; $\varphi$ is the magnetostatic potential ($\mathbf{h} \equiv -\nabla\varphi$); $c_\perp$ is the effective elastic modulus; $\beta_{15}$, $\beta_{24}$, and $\beta_*$ are the effective piezomagnetic moduli; $\mu_\perp$ are the components of the magnetic permeability tensor; $\omega_0^2 = g^2 M_0^2(\delta b - d^2)$ is the uniaxial-anisotropy-induced active energy of the spin wave; and $\omega_{\rm me}^2$ is the magnetoelastic gap [9].

Since the aim of this work is to study the conditions of the formation and propagation of the shear elastic wave with $\mathbf{u} \parallel OZ$ and $\mathbf{k} \in (XY)$ near the mechanically free surface of the piezomagnet specified by Eqs. (9) and (10) with the normal $\mathbf{n}$, the system of dynamic equations under consideration should be supplemented with the corresponding elastic and electrodynamic boundary conditions. Below, we will assume that the boundary conditions on the outer surface of the semibounded easy axis antiferromagnet have the form [10]

$$\sigma_{zi}n_i = 0, \quad \mathbf{Bn} = \tilde{\mu}k_\perp\phi, \quad \zeta = 0 \qquad (11)$$

(where $\zeta$ is the current coordinate along $\mathbf{n}$) and represent the requirement of localization of the shear wave near the outer surface of the easy axis antiferromagnet under consideration in the form

$$u_z(\zeta \to -\infty) \to 0, \quad \phi(\zeta \to -\infty) \to 0. \qquad (12)$$

The calculation shows that, owing to Eqs. (9) and (10), for the propagation of the shear wave with $\mathbf{u} \parallel OZ$ along the $OY$ axis for the boundary value problem specified by Eqs. (11) and (12) at $\mathbf{n} \parallel OX$, the following characteristic equation is valid:

$$k_\parallel^4 - k_\parallel^2 k_\perp^2\left[\frac{k^2}{2\bar{c}_\perp(1+\kappa^2)} - k_\perp^2\frac{1-\kappa^2}{1+\kappa^2}\right] + k_\perp^2\left[k_\perp^2 - \frac{k_0^2}{\bar{c}_\perp(1+\kappa^2)}\right] = 0, \quad \kappa^3 \equiv \frac{4\pi\beta_{15}^2}{\mu_\perp c_\perp}, \qquad (13)$$

where $s_t^2 \equiv \mu/\rho$, $k_0^2 \equiv \omega^2/s_t^2$, $\bar{c}_\perp \equiv c_\perp/\mu$, and $\mathbf{k} = \{k_\parallel, k_\perp, 0\}$. (For convenient comparison of the relations obtained with [8–11] when determining the magnetomechanical coupling constant, we retain the notation $\kappa^2$. However, it can be negative in our case owing to dispersion.)

For such an elastic wave to be surface in the piezomagnetic medium specified by Eqs. (9) and (10) at given external parameters of the frequency and transverse wavenumber, the following system of inequalities should be satisfied:

$$k_\perp^2 > \frac{k_0^2}{\bar{c}_\perp(1+\kappa^2)}, \qquad \frac{k_0^2}{2\bar{c}_\perp(1+\kappa^2)} < k_\perp^2\frac{1-\kappa^2}{1+\kappa^2} \qquad (14)$$

or

$$\frac{k_0^2}{2\bar{c}_\perp(1+\kappa^2)} < k_\perp^2\frac{1-\kappa^2}{1+\kappa^2};$$
$$\left[\frac{k_0^2}{2\bar{c}_\perp(1+\kappa^2)} - k_\perp^2\frac{1-\kappa^2}{1+\kappa^2}\right] < k_\perp^2\left[k_\perp^2 - \frac{k_0^2}{\bar{c}_\perp(1+\kappa^2)}\right]. \qquad (15)$$

In both cases, the spatial distribution of the field, shear elastic displacements, and magnetostatic potential in a piezomagnet that satisfies condition (12) corresponds to the two-partial wave ($k_\parallel^2 \equiv -q^2$)

$$U_z = \sum_{i=1}^{2} A_i \exp(q_i x)\exp[i(k_\perp y - \omega t)],$$
$$\varphi - \sum_{i=1}^{2} A_i\Delta_i \exp(q_i x)\exp[i(k_\perp y - \omega t)]; \qquad (16)$$
$$\Delta_i \equiv \frac{4\pi\beta_{15}}{\mu\mu_\perp}\frac{k_\perp^2 + q_i^2}{k_\perp^2 - q_i^2}.$$

The solution of the boundary value problem specified by Eqs. (11) and (12) gives the following expression for the spectrum of the shear surface wave with the elastic displacement vector $\mathbf{u} \parallel OZ$ that propagates along the $OY$ axis in the semibounded piezomagnet specified by Eqs. (9) and (10) with the mechanically free surface ($\mathbf{n} \parallel OX$):

$$\sqrt{1-x}\left[\frac{2}{\eta(\alpha+1)}\sqrt{2\alpha - x + 2\sqrt{1-x}} + 1\right] = \frac{2x}{\alpha+1} - 1 - \frac{\beta^4}{\eta}\sqrt{2\alpha - x + 2\sqrt{1-x}}, \qquad (17)$$

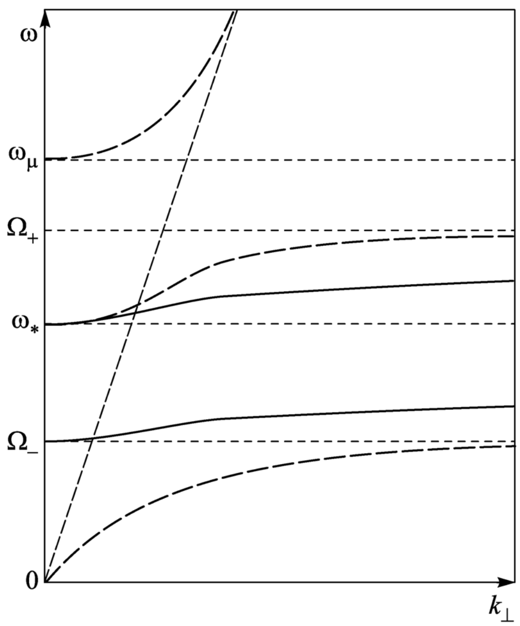
where

$$\beta^4 \equiv \frac{4\pi\beta_*^2}{\mu_\perp c_\perp}; \quad x \equiv \frac{k_0^2}{\bar{c}_\perp(1+\kappa^2)k_\perp^2};$$
$$\alpha \equiv \frac{1-\kappa^2}{1+\kappa^2}; \quad \eta \equiv \frac{\tilde{\mu}}{\mu_\perp}.$$

The exactly solvable particular case of Eq. (17) at $\eta = 0$ corresponds to the spectrum of the shear elastic surface wave propagating along the (piezomagnet–ideal diamagnet) sliding interface:

$$k_\perp^2 = \frac{k_0^2\mu\mu_\perp(\mu_\perp c_\perp + 4\pi\beta_{15}^2)}{(\mu_\perp c_\perp + 4\pi\beta_{15}^2)^2 - (4\pi\beta_*^2)^2}. \qquad (18)$$

According to Eq. (18), such a surface SH wave in the piezomagnet specified by Eqs. (9) and (10) does not exist in the absence of magnetoelastic interaction ($\gamma = 0$). Spectrum (18) consists of two branches. These branches lie in the frequency range determined by the conditions $x < 1$ and $\mu_\perp c_\perp + 4\pi\beta_{15}^2 < 0$ (see figure).




Structure of the spectrum of a shear elastic surface wave at the piezomagnet–ideal diamagnet given by Eq. (18). The dashed line corresponds to $x(\omega \to \infty) = 1$, $\mu_\perp(\omega = \omega_\mu) = 0$. The frequencies $\Omega_\pm$ are the roots of the equation $1 + \kappa^2 = 0$.

In the limit $\gamma \to 0$ (i.e., the magnetoelastic interaction in Eq. (5) is neglected), Eq. (17) is transformed into the "piezomagnetic" variant of the Tseng equation for the spectrum of the shear two-partial elastic wave [11]:

$$\sqrt{1-x}\left[\frac{2}{\eta(\alpha+1)}\sqrt{2\alpha - x + 2\sqrt{1-x}} + 1\right] = \frac{2x}{\alpha+1} - 1. \tag{19}$$

The joint analysis of Eqs. (9), (10), and (17) shows that the following relation in both the low- and high-frequency limits is valid:

$$\beta_*^2 \ll \beta_{15}^2. \tag{20}$$

As a result, the dispersion curve of the shear surface wave given by Eq. (17) in these frequency ranges almost coincides with Eq. (19), but the coefficients in Eq. (19) are determined from material relations (10), in contrast to [11].

If Eq. (20) is not satisfied, according to Eq. (17), end points can appear in the spectrum of the discussed shear SH elastic surface wave. According to Eq. (17), the frequency and wavenumber $k_\perp \neq 0$ at these points are determined by the relations

$$\beta_{15}^2 = \frac{\beta_*^2}{\eta}\sqrt{\frac{1-3\kappa^2}{1+\kappa^2}}; \quad x = 1. \tag{21}$$

Since $q_1^2 = 0$ and $q_2^2 = k_\perp^2$ in Eqs. (13)–(16) at $x = 1$, $x = 1$ determines the spectrum of the special wave of the second type in the piezomagnet under consideration, according to the classification introduced in [14]. In this case, it is a bulk wave with the energy flux parallel to a given surface that satisfies boundary conditions (11) and (12) at a certain value $k_\perp \neq 0$ only together with the inhomogeneous wave ($q_1^2 = 0$ and $q_2^2 = k_\perp^2$). Thus, Eq. (21) can be considered as a result of the intersection of the spectra of the shear elastic surface wave (17) and the special wave of the second type in the semibounded piezomagnet specified by Eqs. (9)–(12).

If $k_\perp = 0$, the long-wavelength end points of the spectrum given by (17) correspond to the frequencies $\omega = 0$, $\omega_*$, and $\mu_\perp(\omega = \omega_\mu) = 0$. Furthermore, analysis shows that the spectrum given by Eq. (17) also has the long-wavelength end point with $k_\perp \neq 0$. Its frequency is determined by the condition $1 + \kappa^2 = 0$.

The fundamentally new effect associated with the magnetoelastic interaction is the possibility of the formation the SH elastic surface wave specified by Eq. (17) in the short-wavelength (elastostatic, $x \to 0$) limit:

$$1 + \eta\sqrt{1+\kappa^2} = -\frac{4\pi(\beta_*^2 + \beta_{15}^2)}{\mu_\perp c_\perp}. \tag{22}$$

Thus, the resulting relations indicate that, when the formation of the shear elastic surface wave in the piezomagnet is possible only owing to linear magnetostriction, the magnetoelastic interaction can result in the significant transformation of both dispersion properties and the conditions of localization of transverse SH phonons.

It is noteworthy that the formation of the shear surface acoustic wave specified by Eq. (17) with $\mathbf{k} \in (XY)$ owing to the hybridization of linear magnetostriction and magnetoelastic interaction is also possible when the easy axis antiferromagnet specified by Eq. (5) has a spin structure of $4_z^- 2_d^-$, boundary conditions (11) and (12) are satisfied, but $\mathbf{n} \parallel [110]$. In this geometry, the material relations have the form similar to Eqs. (9) and (10).

According to [12], contributions in Eq. (5) from linear magnetostriction and Dzyaloshinskii interaction are given by the expressions

$$W_{\mathrm{ms1}} = \gamma_{\mathrm{ms1}}(H_x u_{yz} + H_y u_{xz}),$$
$$W_{\mathrm{D}} = d(m_x l_y + m_y l_x). \tag{23}$$

The analysis shows that the magnetoelastic interaction can lead to the formation of the SH elastic surface wave in the piezomagnet specified by Eq. (5) even when its localization near the mechanically free surface specified by Eqs. (11) and (12) only owing to the linear magnetostriction mechanism is impossible. As an example, we consider the easy axis antiferromagnet specified by Eq. (5), where the linear magnetostriction and Dzyaloshinskii interactions have the form

$$\begin{aligned} W_{\mathrm{ms1}} &= \gamma_{\mathrm{ms1}}(H_x u_{yz} - H_y u_{xz}), \\ W_{\mathrm{D}} &= d(m_x l_y - m_y l_x). \end{aligned} \tag{24}$$

According to [12], this is possible if the magnet has a spin structure of $4_z^+ 2_d^-$ or $6_z^+ 2_x^-$. The calculation shows that, in view of Eq. (10), the material relations for the shear wave with $\mathbf{u} \parallel OZ$, $\mathbf{k} \in (ZY)$ in this case can be represented in the form ($\beta_{14} = \beta_{25} = \beta_{15} = \beta_{24}$):

$$\begin{cases} \sigma_{zx} = c_\perp \dfrac{\partial u_z}{\partial x} - \beta_{14} h_y - i\beta_* h_y; \\ \sigma_{zy} = c_\perp \dfrac{\partial u_z}{\partial y} + \beta_{25} h_x + i\beta_* h_x; \end{cases}$$
$$\begin{cases} B_x = \mu_\perp h_x - 4\pi\beta_{25}\dfrac{\partial u_z}{\partial y} + 4\pi i\beta_* \dfrac{\partial u_z}{\partial y}; \\ B_y = \mu_\perp h_y - 4\pi\beta_{14}\dfrac{\partial u_z}{\partial x} - 4\pi i\beta_* \dfrac{\partial u_z}{\partial x}. \end{cases} \tag{25}$$

As a result, for the above geometry of propagation of the shear wave ($\mathbf{u} \parallel OZ$, $\mathbf{k} = \{k_\parallel, k_\perp, 0\}$), by analogy with [15], we obtain the following characteristic equation for the unbounded piezomagnet model:

$$(k_\parallel^2 + k_\perp^2)\left(k_\parallel^2 + k_\perp^2 - \frac{k_0^2}{\bar{c}_\perp}\right) = 0. \tag{26}$$

This means that the spatial distribution of the fields of shear elastic displacements and magnetostatic potential in the semibounded piezomagnet ($x < 0$) corresponds to a single partial surface wave:

$$\begin{aligned} u_z &= A\exp(qx)\exp[i(k_\perp y - \omega t)], \\ k_\parallel^2 \equiv -q^2 &= k_\perp^2 - \frac{k_0^2}{\bar{c}_\perp}, \\ \varphi &= B\exp(k_\perp x)\exp[i(k_\perp y - \omega t)]. \end{aligned} \tag{27}$$

The solution of the boundary value problem specified by Eqs. (11) and (12) with allowance for Eqs. (10) and (25) gives the following expression for the spectrum of the shear surface wave that is polarized along the $OZ$ axis and propagates along the $OY$ axis in the semibounded piezomagnet ($\mathbf{n} \parallel OX$):

$$k_\perp^2 = \frac{k_0^2 \mu c_\perp [\mu_\perp(1+\eta)]^2}{[\mu_\perp c_\perp(1+\eta)]^2 - [4\pi(\beta_*^2 + \beta_{14}^2)]^2}. \tag{28}$$

The analysis shows that the necessary condition of the existence of this surface wave is the simultaneous satisfaction of the inequalities

$$k_\perp^2 > \frac{k_0^2}{\bar{c}_\perp}, \quad c_\perp(\mu_\perp + \tilde{\mu}) < 0. \tag{29}$$

Thus, the magnetoelastic interaction is of significant importance for this type of piezomagnetic crystal in addition to linear magnetostriction. This interaction is responsible for the possibility of the localization of shear phonons both at the piezomagnet–vacuum interface and piezomagnet–ideal diamagnet sliding interface.

The effects caused by electrostriction and quadratic magnetostriction on the localization conditions and dispersion properties of the shear elastic surface wave for the types of piezomagnetic crystals under consideration will be considered in the future.

This work was supported in part jointly by the National Academy of Sciences of Ukraine (project no. 29-02-14(U)) and the Russian Foundation for Basic Research (project no. 14-02-90416(R)) and by the Russian Science Foundation (project no. 14-22-00279).

## REFERENCES


1. L. Li and P. J. Wei, J. Sound Vibrat. **333**, 2312 (2014).
2. D. G. Piliposyan, K. B. Ghazaryan, and G. T. Piliposian, J. Appl. Phys. **116**, 044107 (2014).
3. Y. Pang', J.-S. Gao, and J.-X. Liu, Ultrasonics **54**, 1341 (2014).
4. J. P. Parekh, Electron. Lett. **6**, 322 (1969).
5. H. Matthews and H. van de Vaart, Appl. Phys. Lett. **15**, 373 (1969).
6. G. A. Maugin, *Continuum Mechanics of Electromagnetic Solids* (Elsevier Science, Amsterdam, New York, Oxford, Tokyo, 1988; Mir, Moscow, 1991).
7. Yu. V. Gulyaev, I. E. Dikshtein, and V. G. Shavrov, Phys. Usp. **40**, 701 (1997).
8. Yu. V. Gulyaev, Yu. A. Kuzavko, I. N. Oleinik, and V. G. Shavrov, Sov. Phys. JETP **60**, 386 (1984).
9. M. I. Kaganov and Yu. A. Kosevich, Poverkhnost' **5**, 148 (1986).
10. V. I. Alshits, A. N. Darinskii, and J. Lothe, Wave Motion **16**, 265 (1992).
11. Y. A. Kosevich, E. S. Syrkin, and A. M. Kosevich, Prog. Surf. Sci. **55**, 59 (1997).
12. E. A. Turov, *Physical Properties of Magnetically Ordered Crystals* (Nauka, Moscow, 1963; Academic, New York, 1965).
13. E. A. Turov and V. G. Shavrov, Sov. Phys. Usp. **26**, 593 (1983).
14. V. I. Alshits, J. Lothe, and V. N. Lyubimov, Sov. Phys. Crystallogr. **28**, 374 (1983).
15. M. K. Balakirev and I. A. Gilinskii, *Waves in Piezoelectric Crystals* (Nauka, Novosibirsk, 1982) [in Russian].


*Translated by R. Tyapaev*