Electromagnetic Waves Reflectance of Graphene—Magnetic Semiconductor Superlattice in Magnetic Field

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Electrodynamic properties of the graphene–magnetic semiconductor–graphene superlattice placed in magnetic field have been investigated theoretically in Faraday geometry, while taking into account dissipation processes. Frequency and field dependences of the reflectance, transmittance, and absorbtance of electromagnetic waves by such superlattice have been calculated for different numbers of periods of the structure and different sizes of the periods with using a transfer matrix method. The possibility of efficient control of electrodynamic properties of graphene–magnetic semiconductor–graphene superlattice has been shown.

***Index Terms*— Composite materials, electromagnetic propagation in absorbing media, electromagnetic reflection, electromagnetic wave absorption, magnetic semiconductors, periodic structures, semiconductor superlattices, superlattices.**

I. INTRODUCTION

NOWADAYS, graphene, 2-D honeycomb-like lattice of carbon atoms, attracts researchers' attention with its special properties, including electronic and electrodynamic ones [1]–[4]: linear dispersion of carriers and room-temperature quantum Hall effect. The graphene layer can support highly localized surface electromagnetic waves—surface plasmon polaritons [5]–[8], the waveguide modes with the negative group speed can exist in the structure of two graphene layers with a layer of dielectric [9], the hyperbolic metamaterial based on graphene–dielectric multilayer structure may be created [10], and so on. Despite the large number of studies, the authors are usually limited themselves by investigation of a nonmagnetic dielectric medium, where graphene is placed. Recently, the dispersion properties of an anisotropic metamaterial composed of periodic stacking of graphene–liquid crystal layers have been investigated [11]. The switching between the elliptic and hyperbolic dispersion phases via control of the temperature, voltage, and external electric field has been studied. It has been shown that this switching can be used to control the transmission and reflection at the interface of the metamaterial and air. Thus, it is very interesting to study the dynamic characteristics of graphene structures with more complex materials. A magnetic semiconductor could be an example of such material. Frequency dispersion of the permittivity is one of the significant differences of semiconductor from dielectric; the plasma waves can excite in the semiconductor structures. When the semiconductor is placed in an external magnetic field, the helicons can to propagate in the material. Their properties depend on the magnetic field value. In its turn, the magnetic semiconductors, for example, may have a large magnetoresistance, magneto-optical properties, and so on. Thus, the electrodynamic properties of graphene–magnetic semiconductor-based structures can be quite interesting. This paper is devoted to investigation of the electrodynamical properties of graphene–magnetic semiconductor superlattice placed in an external magnetic field.

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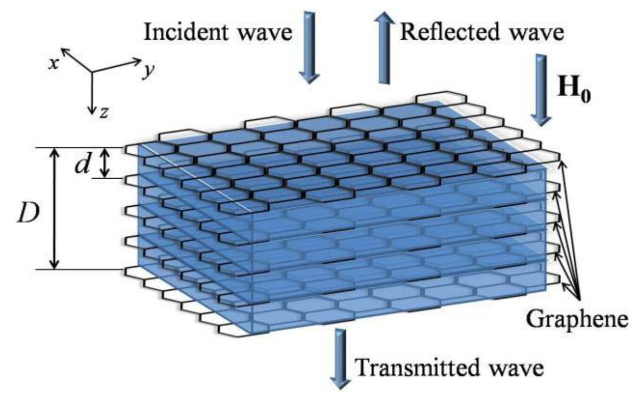

Fig. 1. Geometry of the structure with the thickness D composed of a periodic structure of graphene layers separated by magnetic semiconductor slabs with the thickness d placed in external magnetic field $\mathbf{H_0}$.

II. GEOMETRY OF THE PROBLEM

The geometry of the problem is shown in Fig. 1. The structure is a periodic stacking of graphene monolayers and magnetic semiconductor layers. Suppose that linearly polarized plane electromagnetic wave is normally incident in the surface of the structure and that an external magnetic field $\mathbf{H_0}$ is directed perpendicular to the structure surface (i.e., Faraday geometry). Due to the axial symmetry of the problem, it is sufficient to consider an electromagnetic wave polarized along only one axis. The coordinate axes are chosen so that the z-axis coincides with the direction of the external magnetic field. The thickness of the magnetic semiconductor is denoted as d; D is the thickness of all the structure.

For solving this problem, one has to know the characteristics of each component of the structure. For the magnetic semiconductor, such characteristics are the tensors of the permeability $\hat{\mu}$ and the permittivity $\hat{\varepsilon}$. The permeability tensor of the magnetic semiconductor placed in an external magnetic field can be described as

$$\hat{\mu} = \begin{pmatrix} \mu_\perp & i\mu_a & 0 \\ -i\mu_a & \mu_\perp & 0 \\ 0 & 0 & \mu_\parallel \end{pmatrix}$$
$$\mu_\perp = 1 + \frac{\omega_M (\omega_H - i\alpha\omega)}{\omega_H^2 - (1+\alpha^2)\omega^2 - 2i\alpha\omega\omega_H}$$
$$\mu_a = \frac{-\omega_M \omega}{\omega_H^2 - (1+\alpha^2)\omega^2 - 2i\alpha\omega\omega_H}$$
$$\mu_\parallel = 1 - \frac{i\alpha\omega_M}{\omega + i\alpha\omega_H}. \quad (1)$$

In (1), we used the following notation: $\omega_H = gH_0$, $\omega_M = 4\pi g M_0$, g is the gyromagnetic ratio, M_0 is the saturation magnetization, and α is the damping parameter.

The permittivity tensor has the usual form

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\perp & i\varepsilon_a & 0 \\ -i\varepsilon_a & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix} \varepsilon_\perp = \varepsilon_0\left(1 - \frac{\omega_p^2(\omega + i\nu)}{\omega\left[(\omega+i\nu)^2 - \omega_c^2\right]}\right)$$
$$\varepsilon_a = \varepsilon_0 \frac{\omega_p^2 \omega_c}{\omega\left[(\omega+i\nu)^2 - \omega_c^2\right]} \varepsilon_\parallel = \varepsilon_0\left(1 - \frac{\omega_p^2}{\omega(\omega+i\nu)}\right). \quad (2)$$

Here, ε_0 is the lattice caused part of the permittivity, $\omega_p = \sqrt{4\pi n_s e^2/m^*}$ and $\omega_c = eH_0/m^*c$ are the plasma and the cyclotron frequencies, consequently, e and m^* are the charge and the effective mass of carriers, n_s is the carriers density, and ν is the effective collision rate.

Note that for representation of magnetic semiconductor like a material with the tensors of the permeability (1) in the presented geometry, magnetization of semiconductor should be directed along an external magnetic field (perpendicular to the structure). Such scenario will take a place when an external magnetic field value is more than demagnetizing field, $H_0 > 4\pi M_0$; in (1) and (2), in place of H_0 should be written an effective magnetic field value $H_0^{\text{eff}} = H_0 - 4\pi M_0$. We will consider this fact, but will not to write superscript eff in further.

Graphene can be represented as a conductive surface [5] with the frequency-dependent tensor of conductivity $\hat{\sigma}$, the components of which have been obtained in [12], [13]

$$\hat{\sigma} = \begin{pmatrix} \sigma_0 & \sigma_H \\ -\sigma_H & \sigma_0 \end{pmatrix}$$
$$\sigma_0 = \frac{e^2 v_F^2 |eH_0| (\hbar\omega + 2i\Gamma)}{i\pi c} \times \sum_n \left\{ \begin{array}{l} \frac{[n_F(M_n) - n_F(M_{n+1})] - [n_F(-M_n) - n_F(-M_{n+1})]}{(M_{n+1}-M_n)^3 - (\hbar\omega+2i\Gamma)^2(M_{n+1}-M_n)} \\ + \frac{[n_F(-M_n) - n_F(M_{n+1})] - [n_F(M_n) - n_F(-M_{n+1})]}{(M_{n+1}+M_n)^3 - (\hbar\omega+2i\Gamma)^2(M_{n+1}+M_n)} \end{array} \right\}$$
$$\sigma_H = -\frac{e^2 v_F^2 eH_0}{\pi c} \times \sum_n \left\{ \begin{array}{l} \{[n_F(M_n) - n_F(M_{n+1})] \\ +[n_F(-M_n) - n_F(-M_{n+1})]\} \\ \times \frac{2[M_n^2 + M_{n+1}^2 - (\hbar\omega+2i\Gamma)^2]}{[M_n^2+M_{n+1}^2 - (\hbar\omega+2i\Gamma)^2]^2 - 4M_n^2 M_{n+1}^2} \end{array} \right\}. \quad (3)$$

Here, $M_n = v_F (2neH_0/c)^{1\ 2}$ is the energy of the corresponding Landau level, n_F is the function of Fermi–Dirac distribution, v_F is the velocity of electrons on the Fermi surface, and $\Gamma/\hbar$ is the scattering rate.

For solving the problem, one has to use the system of Maxwell's equations

$$rot\mathbf{E} = -c^{-1}\partial\mathbf{B}/\partial t; \quad rot\mathbf{H} = c^{-1}\partial\mathbf{D}/\partial t \quad (4)$$

with the material equations

$$\mathbf{D} = \hat{\varepsilon}\mathbf{E}; \quad \mathbf{B} = \hat{\mu}\mathbf{H}; \quad \mathbf{j} = \hat{\sigma}\mathbf{E} \quad (5)$$

and the boundary conditions

$$(\mathbf{E}_2 - \mathbf{E}_1) \times \mathbf{n}_{12} = 0; \quad (\mathbf{H}_2 - \mathbf{H}_1) \times \mathbf{n}_{12} = 4\pi\mathbf{j}/c \quad (6)$$

where indexes 1 and 2 mean the fields in the first and the second medium, $\mathbf{n}_{12}$ is the normal vector to the partition surface directed from the first medium to the second one, and $\mathbf{j}$ is the density of the surface current in graphene layer.

Dispersion equation of the magnetic semiconductor has a usual for bigyrotropic medium form [14]

$$k_\pm = k_0\sqrt{(\varepsilon_\perp \pm \varepsilon_a)(\mu_\perp \pm \mu_a)} \quad (7)$$

where $k_0 = \omega/c$ is the wavenumber of electromagnetic wave in vacuum, and indexes + and − correspond to the right- and left-polarized waves.

Solving the system of equations (4)–(6) with the dispersion equation (7) for each layer of magnetic semiconductor, we obtain the amplitudes of reflected and transmitted waves. Then, reflectance R and transmittance T can be found

$$R = \frac{|E_{xR}|^2 + |E_{yR}|^2}{|E_{x0}|^2 + |E_{y0}|^2}; \quad T = \frac{|E_{xT}|^2 + |E_{yT}|^2}{|E_{x0}|^2 + |E_{y0}|^2} \quad (8)$$

where indexes R and T denote the amplitudes of reflected and transmitted waves, and consequently, index 0 denotes the amplitude of incident wave. Obtaining their values, we define the absorptance $A = 1 - R - T$.

Such a method of solving the problem is not very convenient due to sharp increase in the number of equations with increasing periods of the structure. Using a transfer matrix method is more convenient for solving our problem. When the electromagnetic wave in the medium can be classified into TE and TM polarization, transfer matrixes 2 × 2 are usually used. In our case, due to gyrotropy of the medium, electromagnetic wave cannot to be classified into TE and TM polarization, and one has to use a transfer matrix 4 × 4. Transfer matrix $\hat{M}$ connects the amplitudes of the tangential components of the electric and magnetic fields before and after layer of the material

$$\mathbf{\Xi}_{\text{before}} = \hat{M}\mathbf{\Xi}_{\text{after}}; \quad \mathbf{\Xi} = \left(E_x,\ E_y,\ H_x,\ H_y\right)^T. \quad (9)$$

Will denote the transfer matrix of graphene layer $\hat{M}_g$ and the transfer matrix of magnetic semiconductor layer $\hat{M}_{\text{ms}}$. Thus, the transfer matrix of one period of the structure (graphene–magnetic semiconductor) is $\hat{M}_1 = \hat{M}_g\hat{M}_{\text{ms}}$, and the transfer matrix of all the structure is $\hat{M} = (\hat{M}_1)^N\hat{M}_g$, where $N = D/d$ is the number of the periods. When the matrix $\hat{M}$ is known, we have the following equations for obtaining the amplitudes of reflected and transmitted waves: $\Xi_0 + \Xi_R = \hat{M}\Xi_T$.

For the numerical simulation, we will use the characteristic parameters of magnetic semiconductor $CdCr_2Se_4$ with Curie temperature $T_C = 130$ K [15], [16]

$$M_0 = 350\,G, \alpha = 0.1, g = 1.75 \cdot 10^7 Oe^{-1}s^{-1},$$
$$\varepsilon_0 = 20, m^* = 0.15m_e, n_s = 10^{18} cm^{-3}, \nu = 10^{15} s^{-1}. \quad (10)$$

For simulation of the graphene properties, we will use parameters $\upsilon_F = 10^8$ cm/s and $\Gamma = 2 \times 10^{-15}$ erg, and the value of the chemical potential μ_{chem} from the Fermi–Dirac distribution function depends on temperature T and carrier density in graphene n_0. It has been shown in [17] that, for temperatures $T \sim 100$ K and carrier density $n_0 \sim 10^{11}$ cm^{-2}, the value of chemical potential is $\mu_{\text{chem}} \sim 3.5 \times 10^{-14}$ erg.

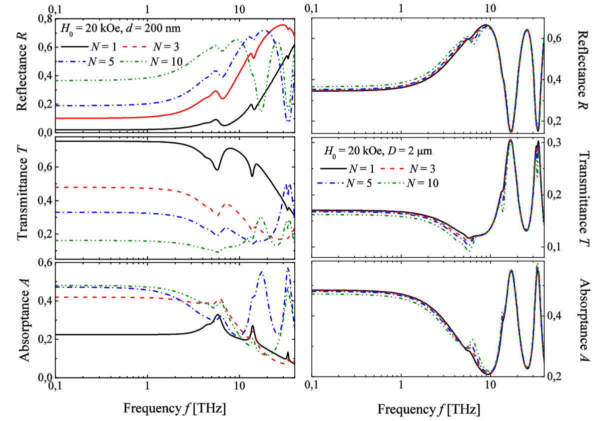

Fig. 2. Frequency dependencies of electromagnetic waves reflectance R, transmittance T, and absorptance A for different number of periods N. Left panel: fixed size of one period d. Right panel: fixed size of all structure D. An external magnetic field value $H_0 = 20$ kOe.

III. Results and Discussion

Due to resonant dependencies of components of permittivity, permeability tensors of magnetic semiconductor (1), (2), and conductivity tensor of graphene (3), it is clear that reflectance, transmittance, and absorptance will have some features near the resonant. At the frequencies near the ferromagnetic resonant frequency $\omega \cong \omega_H(1+\alpha^2)^{-1/2}$, spin oscillations (or magnons) will excite in magnetic semiconductor, and therefore, electromagnetic wave will be strongly damped in the medium. This leads to decrease in T and increase in R and A. Near the cyclotron resonance frequency $\omega \cong \omega_c(1+\nu^2)^{-1/2}$, the energy of the electromagnetic wave is actively converse to energy of the electrons rotating around the magnetic field lines. Thus, electromagnetic radiation of these frequencies will also strongly absorb by medium. The resonant frequencies are determined by the value of external magnetic field. The effect of graphene layers on the electromagnetic properties of the structure is more evident at the frequencies corresponding to carriers' transitions between Landau levels. At these frequencies, the conductivity of graphene sharply increases, which leads to increase in absorptance and reflectance (the structure becomes more metallic) and decrease in transmittance.

Fig. 2 shows the frequency dependencies of R, T, and A for the structure placed in magnetic field of 20 kOe. One can see that increasing number of periods with fixed size of one period leads to increasing reflectance and absorptance and decreasing transmittance. This also leads to shift in size resonance frequencies caused by interference of waves reflected from different borders of the layers. In the frequency dependence of transmittance shown in Fig. 2, there is practically no feature associated with a ferromagnetic resonance. This is due to insufficient thickness of the magnetic semiconductor layer to observe a significant effect. Increase of thickness of the structure leads to the fact that the electromagnetic wave runs longer path in magnetic semiconductor and, therefore, more energy will be transferred to magnetic subsystem. For the structure with the fixed size, increasing the number of graphene layers from two to 11 (see right panel of Fig. 2) leads to increasing reflectance and decreasing transmittance and absorptance about 3%–4%. Decreasing absorptance is caused by the following fact: some part of electromagnetic wave reflects from each graphene layer. So, an effective length electromagnetic wave goes into the material is less than without graphene layers in the structure. At frequencies of graphene electron transitions between Landau levels, electromagnetic power losses in graphene layers become more significant than an effect of reflection, and the features associated with such transitions become more evident with increasing number of graphene layers.

Resonant frequency values, energy of Landau levels in graphene, and distance between them depend on the value of external magnetic field. Fig. 3 shows the field dependencies of reflectance, transmittance, and absorptance of electromagnetic wave by graphene–magnetic semiconductor superlattice for frequencies $f = \omega/2\pi = 1$ THz and $f = 5$ THz. The field dependencies have a resonance form. Calculation shows that for $f < 3$–4 THz, there is mainly one resonance, which is associated with the cyclotron resonance in magnetic semiconductor layers. At frequencies $f > 3$–4 THz, there are two coupled resonances: the cyclotron resonance and the one that is associated with graphene electron transitions between Landau levels. Increase in the number of periods of the structure leads to increasing resonant value of reflectance and decreasing resonant values of transmittance and absorptance for $f < 3$–4 THz. The overall scenario of the change in reflectance, transmittance, and absorptance with increasing magnetic field in this frequency range is as follows: the sharp change in resonance (increase in R and decrease in T and A) and then slow relaxation. At frequencies $f > 3$–4 THz, the slump is replaced by slow decrease in R (increase in A) at magnetic field value about 10 kOe and takes a turning point at magnetic field about 15–20 kOe. Magnetic field value of this turning point depends from the number of periods N of the structure and goes to lower magnetic fields with increasing N. This is due to interaction of carriers oscillations in graphene layers. The behavior of transmittance T is different: at low magnetic fields, T increases and takes a maximum at magnetic field values about 10 kOe, and then it gradually decreases to a minimum at high magnetic fields about 25 kOe. The critical

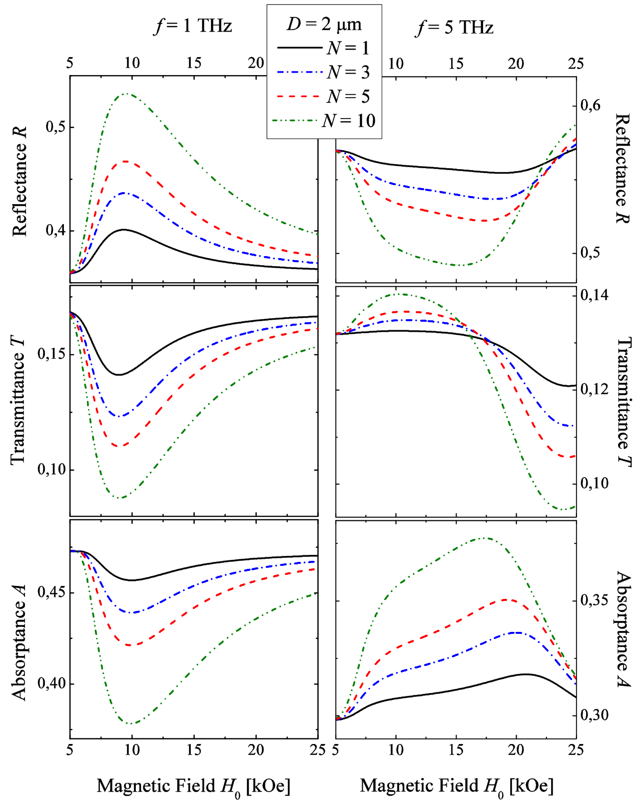

Fig. 3. Field dependencies of electromagnetic waves reflectance R, transmittance T, and absorptance A for different number of periods N. Left panel: $f = \omega/2\pi = 1$ THz. Right panel: $f = 5$ THz. The size of all structure $D = 2$ μm. (Color online.)

magnetic field values for the transmittance behavior almost not depend on the number of periods of the structure, but increasing N makes the change in transmittance to be greater.

IV. Conclusion

The investigation of electrodynamic characteristics of graphene–magnetic semiconductor superlattice placed in an external magnetic field showed that electrodynamic characteristics of such structure can be efficiently controlled. Reflectance, transmittance, and absorptance of electromagnetic waves can be changed with the change of external magnetic field and the number of periods of the structure. At frequencies $f < 3$–4 THz, this change may reach more than 10% in relatively weak magnetic fields about 10 kOe. At $f > 3$–4 THz, the change of R and A is not so great: a little about 5% at magnetic field values of 10 kOe and a little less than 10% at magnetic field values of 15–20 kOe. The change in T at this frequency range is even less: about 1% at 10 kOe and about 5% at 25 kOe.

For room-temperature applications, magnetic semiconductors with higher T_C should be used: for example, $Cd_{1-x}Mn_xGeP_2$ [18], $Co_xTi_{1-x}O_2$ [19] or $Zn_{1-x}Mn_xO$ [20]. However, the features of graphene conductivity at high temperature will not to be so sharp. Therefore, the effects discussed in this paper at room temperature may be less than that we have predicted.

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