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Reflection of Electromagnetic Waves from the Layered Structure of a High-Temperature Superconductor-Multiferroic with Cycloidal Antiferromagnetic Structure

I. V. Bychkov[a], D. A. Kuzmin[a], and V. G. Shavrov[b]

[a]*Chelyabinsk State University, Chelyabinsk, 454001 Russia*

[b]*Kotelnikov Institute of Radio Engineering and Electronics, Russian Academy of Sciences, Moscow, 125009 Russia*

e-mail: bychkov@csu.ru

Abstract—The reflection of electromagnetic waves from the layered structure of a high-temperature superconductor-multiferroic with cycloidal antiferromagnetic structure in an external magnetic field is studied. The frequency dependence of the reflection coefficient at different values of an external magnetic field is calculated. The possibility of effectively controlling the reflective properties of the structure is established.

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INTRODUCTION

Multiferroics—materials in which the magnetic and electric ordering coexist—are now attracting increased attention from researchers. One reason for this interest is the discovery of a new class of multiferroics in which ferroelectric regularity is induced by magnetic ordering [1]. There is spontaneous polarization in such materials, owing to their modulated cycloidal spin structure. The modulated magnetic structure is itself responsible for many features in the spectra of electromagnetic, spin and elastic waves [2, 3]. However, the low temperatures of magnetic ordering hamper the practical use of these materials. Being promising in practical applications as well, superconductors suffer from the same problem associated with the low temperature of this phenomenon. High-temperature superconductor-based layered structures have long been an object of study. The possibility of an increased magnetostatic wave being generated by the moving flux of Abrikosov vortexes in a ferromagnetic superconductor structure was demonstrated in [4]. The authors of [5] studied the impact of vortex pinning on the propagation of surface magnetostatic waves. Features of electromagnetic wave propagation in layered structures like dielectric and metamaterial superconductors with negative refraction indices were studied in [6, 7]. The properties of polaritons at superconductor–dielectric and superconductor–ferromagnetic interfaces were studied in [8, 9]. However, the electrodynamic properties of a layered structure of superconductor-multiferroic remain to be investigated. In this work, we study the reflection of electromagnetic waves by the layered structure of a high-temperature superconductor and a multiferroic with cycloidal antiferromagnetic structure. The superconductor $Y_1Ba_2Cu_3O_7$ and the multiferroic $TbMnO_3$ were chosen as examples.

ANALYTICAL APPROACH

Figure 1 shows the geometry of the problem. The electromagnetic wave falls onto the multifferroic superconductor's layered structure at angle φ to the normal. A layer of superconductor (*1*) with thickness l lies on a layer of multiferroic (*2*) with thickness d and cycloidal antiferromagnetic structure. The multiferroic is oriented such that the axis of modulation coincides with the y axis and the spontaneous polarization vector is directed along the z axis. Transport current J flows inside the superconducting layer. External magnetic field H_0 is directed along the y axis. We assume that the thickness of the superconducting layer is considerably less than wavelength λ of the incident radiation; i.e., $l \ll \lambda$. We consider the presence of the superconducting layer by introducing special boundary conditions [4]. One of these is the continuity of the normal component of magnetic induction

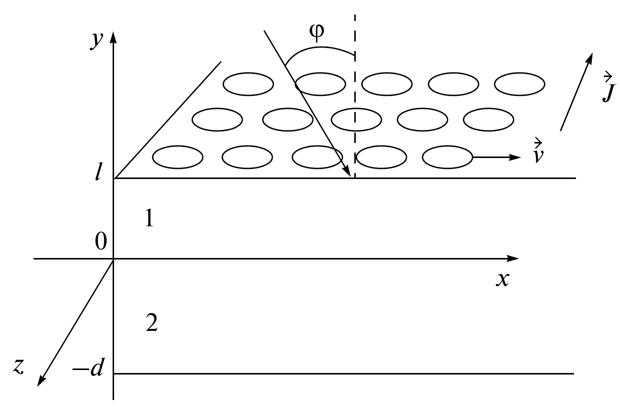

Fig. 1. Geometry of the problem: (*1*) superconductor, (*2*) multiferroic.

$B_y\big|_{y=0,l} = n\Phi_0$, where n is the magnetic vortex density in the superconductor and Φ_0 is a magnetic flux quantum. The second boundary condition is obtained from the law of conservation for the vortex flux during the time of motion,

$$\partial n/\partial t = -\upsilon\,\partial n/\partial x - n\,\partial\upsilon/\partial x, \quad (1)$$

where υ is the velocity of the Abrikosov vortex grid. In an inertialess approximation and without allowing for the elastic stiffness of the vortex grid (which is valid in a linear approximation), this velocity is determined by the local value of the density current in the superconductor,

$$\upsilon = -j_z\,\Phi_0/\eta, \quad (2)$$

where $\eta = H_{c2}\Phi_0/\rho_n$ is the viscosity coefficient of the magnetic vortex, H_{c2} is the second critical superconductor magnetic field value, and ρ_n is the specific resistance in normal state. Integrating the equation $\mathrm{rot}\vec{H} = 4\pi\vec{j}/c$ over the thickness of superconductor, we obtain the relation between the current and the jump in the tangential magnetic field component:

$$4\pi c^{-1} j_z l = H_x\big|_{y=0} - H_x\big|_{y=l}. \quad (3)$$

From (1)–(3), we obtain the boundary condition in the linear approximation

$$\begin{gathered}\partial B_y/\partial t\big|_{y=l} - \Phi_0 J\eta^{-1}\partial B_y/\partial x\big|_{y=l} \\ = (4\pi\eta l)^{-1} c\Phi_0 H_0\,\partial\left[H_x\big|_{y=0} - H_x\big|_{y=l}\right]/\partial x.\end{gathered} \quad (4)$$

The material equations for the multiferroic can be written in the form

$$\vec{M} = \hat{\chi}\vec{H} + \hat{\kappa}^{me}\vec{E}, \quad \vec{P} = \hat{\alpha}\vec{E} + \hat{\kappa}^{me}\vec{H}, \quad (5)$$

where $\vec{P}$ and $\vec{M}$ are the polarization and magnetization vectors, $\hat{\alpha}$ and $\hat{\chi}$ are the tensors of electric and magnetic susceptibility, $\kappa_{ij}^{me} = (\kappa_{ij}^{em})^*$ is a tensor of magnetoelectric susceptibility (operation * denotes complex conjugation). Lagrange's method is used to obtain the tensors. The ground state can be obtained by minimizing the Ginzburg–Landau functional. To obtain the susceptibility tensors of the multiferroic, we must solve the set of Lagrange equations for quantities $\vec{A}$, $\vec{M}$ and $\vec{P}$. Solving the system of Lagrange equations by means of small oscillations, and by linearizing it and expanding the unknown quantities in a harmonic series, we can obtain the components of the susceptibility tensor. The components of the susceptibility tensors for the multiferroic $TbMnO_3$ were obtained in [10] with various directions of the magnetic field. The spectra of coupled waves in the multiferroic $TbMnO_3$ were analyzed in [10]. When the external magnetic field was directed along the axis of modulation, the tensors had the form

$$\hat{\alpha} = \begin{pmatrix}\alpha_{xx} & 0 & 0\\ 0 & \alpha_{yy} & 0\\ 0 & 0 & \alpha_{zz}\end{pmatrix}; \quad \hat{\chi} = \begin{pmatrix}\chi_{xx} & 0 & 0\\ 0 & \chi_{yy} & 0\\ 0 & 0 & \chi_{zz}\end{pmatrix};$$

$$\hat{\kappa}^{me} = \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & \kappa_{yz}^{me}\\ 0 & 0 & 0\end{pmatrix};$$

$$\begin{gathered}\alpha_{xx} = \lambda^{-1}\left(\omega_p^2 - \omega^2\right)^{-1}; \quad \alpha_{yy} = \lambda^{-1}\Delta_{mz}\omega^2 \\ \times\left(\Delta_{ay}^{-}\Delta_{az}^{+} + \left[\Delta_{az}^{+ay}\right]^2\right)\left(\Delta_{mz}\tilde{\Delta}_{py}\right)^{-1}; \\ \alpha_{zz} = \frac{\Delta_{my}\omega^2\left(\Delta_{ay}^{+}\Delta_{az}^{-} + \left[\Delta_{ay}^{+az}\right]^2\right)}{\lambda\left(\Delta_{mz}\tilde{\Delta}_{pz} - \Delta_{my}^{pz}\Delta_{pz}^{my}\right)}; \quad \chi_{xx} = \Delta_{ax}^{+}\tilde{\mu}^{-1} \\ \times\left\{\left(\omega_{mx}^2 - \omega^2\right)\Delta_{ax}^{+} + \frac{1}{2}\omega_{1yy}^4\omega^2\right\}^{-1}; \\ \chi_{yy} = \lambda\alpha_{zz}\Delta_p\tilde{\mu}^{-1}\Delta_{my}{}^{-1}\omega^{-2}; \quad \chi_{zz} = \lambda\alpha_{yy}\Delta_p\tilde{\mu}^{-1}\Delta_{mz}{}^{-1}\omega^{-2}; \\ \kappa_{yz}^{me} = -\alpha_{zz}\Delta_{my}^{pz}\Delta_{my}{}^{-1}.\end{gathered} \quad (6)$$

In (6), we used the notation $\Delta_{ai}^{\pm} = -\omega^2(\omega^2 - \omega_{ai}^{\pm\,2})$, $\Delta_{ay}^{+az} = i\Omega_{-}^2\omega^2$, $\Delta_{az}^{+ay} = -\omega^2(\omega^2 - \omega_p^2)$, $\omega_{i\,j(y,z)}^2 = 2\mu^{-1}\nu A_{(1,2)}k$ $\Omega_{\pm}^2 = \mu^{-1}\{uA_1A_2 \pm 2\nu P_0 k\}$, $\omega_p^2 = \lambda^{-1}b$, for the frequencies of the characteristic polarization oscillations; $\omega_{mx}^2 = \tilde{\mu}^{-1}[\beta + \lambda_2(A_1^2 + A_2^2)/2]$, $\omega_{m(y,z)}^2 = \tilde{\mu}^{-1}[\beta + \lambda_2(A_1^2 + A_2^2)/2 + \lambda_1 A_{(1,2)}^2/2]$ for the frequencies of ferromagnetic resonance; $\omega_{ai}^{\pm\,2} = \mu^{-1}[a + \lambda_1 M_{0i} + \lambda_2\vec{M}^2 + \gamma k^2 + \alpha k^4 + u\{A_1^2 + A_2^2\}/2 \pm u\{A_1^2 - A_2^2\}/4]$ for the eigen frequencies of oscillation of the antiferromagnetism vector; $\tilde{\Delta}_{py} = \Delta_{py}(\Delta_{ay}^{-}\Delta_{az}^{+} + [\Delta_{az}^{+ay}]^2)$, $\tilde{\Delta}_{pz} = \Delta_p(\Delta_{ay}^{+}\Delta_{az}^{-} + [\Delta_{ay}^{+az}]^2) + \omega^2\{i\omega_{me}^{Ay\,4}\Delta_{ay}^{+} - \omega_{me}^{Az\,4}\Delta_{az}^{-}\}$, $\tilde{\Delta}_{pi} = 0$ for the spectra of coupled oscillations of the corresponding components of polarization and the antiferomagnetism vector; $\Delta_{mz} = 2^{-1}\omega^2[\omega_{1yy}^4\Delta_{ay}^{-} + \omega_{1yz}^4\Delta_{az}^{+}] - (\omega^2 - \omega_{mz}^2) \times (\Delta_{ay}^{-}\Delta_{az}^{+} + [\Delta_{ay}^{+az}]^2)$, $\Delta_{my} = -2\omega^2[(\omega_{1yy}^2 + \omega_{2yy}^2)(\omega_{1yy}^2 + \omega_{2yy}^2 + 2i\omega_{2yz}^2\Delta_{ay}^{+az})\Delta_{az}^{-} + \omega_{2yz}^4\Delta_{ay}^{+}] - (\omega^2 - \omega_{my}^2) \times \tilde{\Delta}_{py}\Delta_{py}^{-1}$, $\Delta_{mi} = 0$ for the spectra of coupled oscillations of the magnetization and the antiferromagnetism vector; $\Delta_{my}^{pz} = \omega^2\sqrt{\lambda/\tilde{\mu}}\{(\omega_{me}^{Ay\,2}\Delta_{ay}^{+az} + \omega_{me}^{Az\,2}\Delta_{az}^{-})(\omega_{1yy}^2 + \omega_{2yy}^2) - i\omega_{2yz}^2(\Delta_{ay}^{+}\omega_{me}^{Ay\,2} + \Delta_{ay}^{+az}\omega_{me}^{Az\,2})\}$, $\Delta_{pz}^{my} = 2\omega^2\sqrt{\tilde{\mu}/\lambda}[(\omega_{1yy}^2 + \omega_{2yy}^2)(\omega_{me}^{Az\,2}\Delta_{az}^{-} + i\omega_{me}^{Ay\,2}\Delta_{ay}^{+az}) + \omega_{2yz}^2(\omega_{me}^{Ay\,2}\Delta_{ay}^{+} + i\omega_{me}^{Az\,2}\Delta_{ay}^{+az})]$, w for the anisotropy constant; a, u, β, λ_1, λ_2 for the constants of homogeneous exchange; α, γ for the constants of inhomogeneous exchange; and b, ν for the parameters of electric and

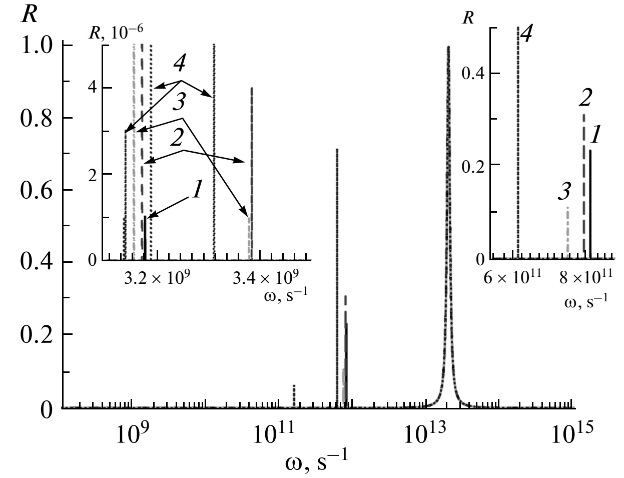

Fig. 2. Frequency dependence of the reflection coefficient of electromagnetic waves from the $Y_1Ba_2Cu_3O_7/TbMnO_3$ structure at different values of external magnetic field H_0: (*1*) 1–10 kOe, (*2*) 2–30 kOe, (*3*) 4–70 kOe.

magnetoelectric interaction. $\lambda = m\upsilon_c/z^2$, where z and m are the charge and the reduced mass of an elementary cell with volume υ_c; $\mu = \chi_\perp/8g^2M_0^2$, where $\chi_\perp$ is the static transverse magnetic susceptibility; g is the gyromagnetic ratio; and M_0 is the saturation magnetization. In contrast to works [10, 11], interaction with an elastic subsystem is not considered in the above expressions.

To solve the problem of the reflection of an electromagnetic wave from a multiferroic superconductor structure, we must solve a set of Maxwell equations with allowance for boundary conditions (4) and material equations (5) with tensor components (6). We used the following parameters of the superconductor $Y_1Ba_2Cu_3O_7$ ($J = 10^6$ A/cm^2, $\eta = 10^{-8}$ H s/cm^2) and the multiferroic $TbMnO_3$ ($\gamma \sim -10^{-14}$ cm^2, $\alpha \sim 10^{-28}$ cm^4, $a \sim -100$, $u \sim 0.1$, $w \sim 10$, $\beta \sim 100$, $b \sim 0.4$, $\nu \sim 10^{-9}$, $\lambda_{1,2} \sim 10^{-4}$). The layer thicknesses were $l = 50$ nm, $d = 100$ nm, and the angle of incidence was $\varphi = 0.01$ rad. The calculation results are shown in Fig. 2. We can see that the frequency dependences of the reflection coefficient for electromagnetic waves from the system under consideration have resonances that coincide with the frequencies of characteristic oscillations of the subsystems. The low-frequency resonances correspond to the excitation of spin waves in the structure. Resonances with frequencies on the order of 10^{11} s^{-1} correspond to antiferromagnetic resonances. The sharp resonance with a frequency on the order of 10^{13} s^{-1} was caused by the excitation of the electric dipole oscillations. The resonance position depends on the external magnetic field applied to the structure. Our investigations also show that the system under consideration has weak amplifying properties, so the transmission coefficient slightly exceeds a unit in all frequency ranges where the reflection coefficient equals zero. The amplification of a reflected wave is observed in the region of resonances, due to the transfer of energy from the moving lattice of Abrikosov vortices to the electromagnetic wave. Since an electromagnetic wave easily passes through the layer of multiferroic in the range far from resonances when the layer thickness of a superconductor is not sufficient for reflection, the transmitted wave is amplified. At the frequencies of resonances, an electromagnetic wave does not pass through the multiferroic. Even though the observed amplification is negligible, it could be important in superlattices based on the considered structure. The possibility of effective reflectivity control using a superconductor–multiferroic structure is thus established.

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