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Elasto-Dipolar Magnons—A New Class of Nonexchange Spin-Wave Excitations

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Interference of phonon and magnetic dipolar spin–spin interaction mechanisms in a finite magnet can result in a previously unexplored class of nonexchange spin waves.

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A metamaterial is knowingly a composite medium of locally resonant structural elements with the wave properties qualitatively different in the long wavelength limit from the dynamic characteristics of the structural elements forming the composite. The number of objects regarded as metamaterials is constantly expanding. These include, in particular, artificial insulators and magnets, chiral and omega structures, biisotropic and bianisotropic media, photonic crystals, etc. [1]. Lapine et al. [2] attempted to take into account the acoustic continuity and mechanical degrees of freedom of electromagnetic composite structures in order to make magnetoelastic metamaterials based on such structures [2]. At the same time, making both electromagnetic and acoustic metamaterials with the characteristics that could be smoothly adjusted by external fields (magnetic, electric, or their combination) became a problem of recent years. In this respect, a magnetic medium is of particular interest because it already includes, in the case of a single crystal, a natural locally resonant structure represented by the magnetic moment of unit volume. Magnonics is one of the most quickly developing fields of modern physics of magnetic phenomena. It is based on the idea of using spin-wave excitations for making a new class of composite media—magnetic metamaterials [3–5]. In addition to the inhomogeneous exchange interaction, an important role among the interactions forming the dispersion properties of a magnetic medium is played by the magnetic dipole field. This long-range interaction induces indirect spin exchange, which leads, in particular, to the formation of "exchangeless" spin-wave excitations termed as magnetostatic spin waves in bounded spatially homogeneous magnets [6]. Taking into account the boundary conditions on the surface of a magnet, the dynamics of this class of magnetostatic spin-wave modes is determined by the set of equations

$$\operatorname{div}\mathbf{B} = 0, \operatorname{curl}\mathbf{H} = 0, \quad (1)$$

where **B** and **H** are the magnetic flux density and the magnetic field, respectively. Such magnetostatic spin-wave modes intensively studied in magnonics, primarily in ferromagnetic media [3–5], can be considered as analogs of magneto-inductive waves formed in composite electromagnetic metamaterials [7]. Magnetoelastic interaction also can be the mechanism of indirect spin exchange in real magnetic media. Its influence on spin dynamics in antiferromagnets is enhanced owing to the exchange effects, which leads to a phonon mechanism of indirect spin exchange in the case of a sufficiently weak inhomogeneous spin-exchange interaction (low-temperature antiferromagnets). As a consequence, another class of "exchangeless" spin-wave modes referred to as elastostatic spin waves is formed in bounded spatially homogeneous magnets taking into account the boundary conditions [8]. Their dynamics is determined by the elastostatic equations

$$\operatorname{div}\sigma_{ik} = 0, \quad (2)$$

where σ_{ik} is the elastic stress tensor.

In the general case, the magnetic dipolar and phonon mechanisms of the formation of magnetostatic magnons in a bounded magnet supplement each other. At the same time, according to calculations, neither magnetostatic nor elastostatic bulk spin waves with the wave vector $\mathbf{k} \in (XY)$ can be formed or propagate in a uniaxial ferromagnetic or antiferromagnetic plate with the easy axis directed along the high-order axis OZ and the sagittal plane (XY). Physically, this is associated with the fact that the indirect spin exchange via both the magnetic dipole field and the quasistatic elastic strain field is independent of the direction of the wave vector within the above sagittal plane.

It is well known, however, that, under certain symmetry conditions imposed on the magnetic structure, an antiferromagnet exhibits the piezomagnetic interaction [9]

$$W_{pm} = \gamma_{ijk} H_i u_{jk}\,, \tag{3}$$

where γ_{ijk} is the piezomagnetic tensor and u_{jk} is the elastic strain tensor. According to calculations, this interaction can lead to the impossibility of independent propagation of shear elastic and electromagnetic waves even within the plane with the normal along the high-order axis coinciding with the direction of the antiferromagnetic vector $\mathbf{l} \parallel OZ$. Consequently, the degeneracy with respect to the direction of $\mathbf{k} \in (XY)$ in such a piezomagnet can be lifted already in the electrostatic limit owing to anisotropic interaction (3) between the quasistatic magnetoelastic strain field and the magnetic dipole field. This allows expecting the possibility of the formation of a previously unexplored class of traveling nonexchange spin-wave modes near the magnetic resonance frequency. However, the spin dynamics of piezomagnetic antiferromagnetic plates with the inclusion of this issue has not been discussed so far.

This work is aimed at finding the conditions under which only the interference of the phonon and magnetic dipole indirect spin-exchange mechanisms leads to the formation of a new class of traveling nonexchange spin waves in a low-temperature piezomagnetic plate.

As an example, we consider the two-sublattice model of an easy-axis (the OZ axis) exchange-collinear antiferromagnet [8] ($\mathbf{M}_1$ and $\mathbf{M}_2$ are the magnetizations of the sublattices, $|\mathbf{M}_1| = |\mathbf{M}_2| = M_0$), the elastic properties of which are thought to be isotropic for simplicity and clarity of calculations. The respective thermodynamic potential density with the inclusion of the Dzyaloshinskii interaction (W_D) can be expressed in terms of the ferromagnetic ($\mathbf{m}$) and antiferromagnetic ($\mathbf{l}$) vectors as

$$\begin{aligned} W = {}& \frac{\delta}{2}\mathbf{m}^2 + \frac{b}{2}(l_x^2 + l_y^2) + W_\mathrm{D} - \mathbf{mh} \\ & + \gamma l_i l_k u_{ik} + \frac{\lambda}{2}u_{ii}^2 + \mu u_{ik}^2. \end{aligned} \tag{4}$$

Here, δ, b, and γ are the intersublattice exchange, easy-axis anisotropy ($b > 0$), and isotropic magnetoelastic interaction constants, respectively; λ is the compression modulus; μ is the shear modulus; $\mathbf{m} = (\mathbf{M}_1 + \mathbf{M}_2)/2M_0$; $\mathbf{l} = (\mathbf{M}_1 - \mathbf{M}_2)/2M_0$; and $\mathbf{h}$ is the reduced magnetic field. Below we restrict ourselves to the following structures of the Dzyaloshinskii–Moriya interaction (d is the interaction constant):

$$W_\mathrm{D} = d(m_x l_y \pm m_y l_x)\,, \tag{5}$$

$$W_\mathrm{D} = d(m_x l_x \pm m_y l_y)\,. \tag{6}$$

The magnetoelastic dynamics of model (4)–(6) neglecting the fact that the propagation velocity of the electromagnetic wave is finite is described by the closed set of equations including the Landau–Lifshitz equations for the vectors $\mathbf{m}$ and $\mathbf{l}$, the basic equation of the mechanics of a continuous medium, and the magnetostatic equations

$$\begin{aligned} \frac{1}{g}\frac{\partial \mathbf{m}}{\partial t} &= [\mathbf{m}H_\mathbf{m}] + [\mathbf{l}H_\mathbf{l}], \\ \frac{1}{g}\frac{\partial \mathbf{l}}{\partial t} &= [\mathbf{m}H_\mathbf{l}] + [\mathbf{l}H_m], \end{aligned} \tag{7}$$

$$\operatorname{div}\mathbf{h} = -8\pi \operatorname{div}\mathbf{m}, \quad \rho\frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k},$$

where ρ is the density; $\mathbf{u}$ is the elastic displacement vector; g is the gyromagnetic ratio, which we will assume to be equal for both sublattices [9]; and $H_r \equiv -\delta W/\delta r$ is the effective field with $r = \mathbf{m}, \mathbf{l}$.

In the case of a shear wave with $\mathbf{u} \parallel OZ$, $\mathbf{k} \in (XY)$, the material relations for the antiferromagnet with the structure $4_z^- 2_d^+$ (the plus sign in Eq. (5)) can be written as

$$\begin{cases} \sigma_{zx} = c_\perp \dfrac{\partial u_z}{\partial x} + (\beta_{14} - i\beta_*)h_y, \\ \sigma_{zy} = c_\perp \dfrac{\partial u_z}{\partial y} + (\beta_{41} + i\beta_*)h_x, \end{cases}$$

$$\begin{cases} B_x = \mu_\perp h_x - 4\pi(\beta_{41} - i\beta_*)\dfrac{\partial u_z}{\partial y}, \\ B_y = \mu_\perp h_y - 4\pi(\beta_{14} + i\beta_*)\dfrac{\partial u_z}{\partial x}, \end{cases} \tag{8}$$

$$\begin{cases} c_\perp \equiv \mu\dfrac{\omega_*^2 - \omega_\mathrm{me}^2 - \omega^2}{\omega_*^2 - \omega^2}, \quad \omega_\mathrm{me}^2 \equiv \dfrac{g^2 M_0^2 \delta\gamma^2}{\mu}, \\ \omega_*^2 \equiv \omega_0^2 + \omega_\mathrm{me}^2, \quad \mu_\perp \equiv 1 + 4\pi\chi, \\ \beta_{14} = \beta_{41} \equiv \dfrac{g^2 M_0^2 \gamma d}{\omega_*^2 - \omega^2}, \quad \beta_* \equiv \dfrac{g M_0 \gamma\omega}{\omega_*^2 - \omega^2}, \\ \chi \equiv \chi_\perp \dfrac{\omega_*^2}{\omega_*^2 - \omega^2}, \quad \chi_\perp \equiv \dfrac{4}{\delta}, \end{cases}$$

where $\mathbf{B}$ is the magnetic flux density; φ is the magnetostatic potential ($\mathbf{h} \equiv -\nabla\varphi$); $c_\perp$ is the effective elastic modulus; β_{14}, β_{41}, and β_* are the effective piezomagnetic moduli; $\mu_\perp$ are the components of the permeability tensor; $\omega_0^2 = g^2M_0^2(\delta b - d^2)$ is the activation energy of the spin wave induced by uniaxial anisotropy; and ω_{me}^2 is the magnetoelastic gap [9].

Since this work is aimed at studying the magnetoelastic dynamics of a piezomagnetic antiferromagnetic plate with the thickness $2L$ and the sagittal plane (XY), the set of dynamic equations under consideration must be supplemented by the respective elastic and electromagnetic boundary conditions. Below we will restrict ourselves to the case of $\mathbf{n} \parallel [100]$ assuming that the set of boundary conditions has the form [10]

$$\mathbf{u} = 0, \quad \mathbf{Bn} = 0, \quad x = \pm L\ . \tag{9}$$

According to Eq. (8), we find the following characteristic equation of the boundary-value problem specified by Eqs. (7)–(9) for the shear wave with $\mathbf{u} \parallel OZ$ propagating along the OY axis ($\omega_d^2 \equiv g^2M_0^2\delta d$, $s_t^2 \equiv \mu/\rho$, $\mathbf{k} = \{k_\parallel, k_\perp, 0\}$):

$$(k_\parallel^2 + k_\perp^2)\left[c_\perp + \varepsilon\frac{\omega_{me}^2\omega_d^2}{\mu_\perp(\omega_{me}^2 + \omega_0^2 - \omega^2)^2}\frac{k_\parallel^2k_\perp^2}{(k_\parallel^2 + k_\perp^2)^2}\right] = k_0^2,$$

$$k_0^2 \equiv \frac{\omega^2}{s_t^2}, \quad \varepsilon \equiv \frac{16\pi}{\delta}. \tag{10}$$

This implies that the spatial distribution of both the shear elastic displacement field and the magnetostatic potential in the antiferromagnetic plate under consideration correspond to the two-partial wave

$$u_z = \sum_{i=1}^{2}(A_i\cos q_ix + B_i\sin q_ix)\exp[i(k_\perp y - \omega t)], \tag{11}$$

$$\varphi = \sum_{i=1}^{2}(-A_i\Delta_i\sin q_ix + B_i\Delta_i\cos q_ix)\exp[i(k_\perp y - \omega t)],$$

$$\Delta_i \equiv \frac{q_ik_\perp}{q_i^2 + k_\perp^2}. \tag{12}$$

Thus, the calculation already in the elastostatic limit (for which one has to proceed to the $k_0^2 \longrightarrow 0$ limit on the right-hand side of Eq. (10)) yields the following expression for the spectrum of the normal bulk modes propagating along the piezomagnetic plate under consideration ($\kappa_\nu \equiv \pi\nu/2L$, $\omega_*^2 \equiv \omega_{me}^2 + \omega_0^2$):

$$[\omega_*^2(1+\varepsilon) - \omega^2][\omega_0^2 - \omega^2] + \varepsilon\omega_{me}^2\omega_d^2\frac{\kappa_\nu^2k_\perp^2}{(\kappa_\nu^2 + k_\perp^2)^2} \cong 0. \tag{13}$$

The derived relation determines the dispersion properties of nonexchange bulk spin-wave modes with $\mathbf{k}_\perp \parallel OY$, $\mathbf{n} \parallel [100]$ propagating along the piezomagnetic plate. In contrast to the previously known types of nonexchange (both dipolar and elastostatic) magnons, the present class of exchangeless spin waves can be termed elasto-dipolar magnons, since, as follows from Eq. (13), they possess dispersion only if $\varepsilon\omega_{me}^2\omega_d^2 \neq 0$ (i.e., only at the simultaneous inclusion of the magnetic dipole and phonon indirect spin-exchange mechanisms in the bounded piezomagnet under consideration).

The spectrum of nonexchange bulk elasto-dipolar magnons specified by relations (13) is two-band in the $\omega - k_\perp$ plane. Below we conditionally divide it into the low-frequency ($c_\perp < 0$) and high-frequency ($\mu_\perp < 0$) bands. Found dispersion relations (13) in each band for the chosen orientation of the normal $\mathbf{n}$ include both the long wavelength (formally at $k_\perp \longrightarrow 0$) and short wavelength (formally at $k_\perp \longrightarrow \infty$) spectrum condensation points degenerate in frequency: $\omega^2 = \omega_0^2$ if $c_\perp < 0$ and $\omega^2 = (\omega_0^2 + \omega_{me}^2)(1+\varepsilon)$ if $\mu_\perp < 0$. The spectrum of the nonexchange spin-wave mode with the given mode index ν at $k_\perp = k_* \neq 0$ has the critical point determined by the condition

$$k_*^2 = -\kappa_\nu^2\left(1 \pm \sqrt{-\frac{16\pi\beta_{14}^2}{c_\perp\mu_\perp}}\right). \tag{14}$$

Such a point in the low(high)-frequency band corresponds to the maximum (minimum) (at $\mathbf{n} \parallel [100]$) of the respective dispersion curve. Respective dispersion curve (13) changes the type of mode with the given mode index ν and the chosen band if the transverse wavenumber of the mode becomes greater than critical value (14). The type of mode in the low- and high-frequency bands changes with an increase in $k_\perp$ from forward ($k_\perp\partial\omega/\partial k_\perp > 0$) to backward ($k_\perp\partial\omega/\partial k_\perp < 0$) and vice versa, respectively. In addition, at $k_\perp \neq 0$, there exists a spectrum degeneracy point for each pair of modes with the given indices ν and τ belonging to the same band of spectrum (13). This point corresponds to the crossing of the respective dispersion curves (more specifically, one mode is necessarily forward and the other one is backward). From the physical point of view, this is the exchangeless elasto-dipolar mechanism of the formation of an inhomogeneous spin–spin resonance. It can be regarded as an analog of the well-known spin–spin resonance in magnetic plates with the participation of magnetostatic and exchange spin waves [6]. It is also worth mentioning that the number of both extreme and degeneracy points in the spectrum of spin-wave excitations under consideration forms an infinite countable set in the exchangeless limit.

Similar to the magnetostatic spin wave, which belongs to the slow branch of the spectrum of normal spin-electromagnetic modes in a bounded magnet [6], the discovered class of nonexchange elasto-dipolar magnons is a part of the slow branch of the spectrum of shear elastic waves in a bounded piezomagnetic of the type under consideration.

Let us study the basic dispersion effects caused by the finiteness of the propagation velocity of the shear elastic wave polarized along the equilibrium antiferromagnetic vector (**l** || *OZ*) in the sagittal plane (*XY*). If again **n** || [100], we can use for calculations characteristic equation (10) assuming now that its right-hand side is nonzero. The solution of the boundary-value problem can be structurally sought again in the form similar to Eqs. (11) and (12). As a result, the respective relation for the spectrum of normal shear elastic bulk modes propagating along the piezomagnetic plate with allowance for the magnetoelastic and magnetic dipole interactions can be expressed as ($\kappa_\nu \equiv \pi\nu/d$)

$$(\kappa_\nu^2 + k_\perp^2)\left[1 + K^2\frac{\kappa_\nu^2 k_\perp^2}{(\kappa_\nu^2 + k_\perp^2)^2}\right] = \frac{k_0^2}{c_\perp},$$

$$K^2 \equiv \varepsilon\frac{\omega_{\mathrm{me}}^2\omega_d^2}{(\omega_{\mathrm{me}}^2 + \omega_0^2 - \omega^2)^2\mu_\perp c_\perp}. \tag{15}$$

According to the analysis, the finiteness of the propagation velocity of the elastic waves leads to a finite number of extreme and degeneracy points. This number depends on the mode index and the plate thickness and decreases to zero with an increase in the thickness.

The point $c_\perp = 0$ for shear elastic bulk modes with $k_\perp \longrightarrow 0$ is a spectrum condensation point, but only at $\omega \longrightarrow \omega_0 - 0$. The long wavelength spectrum compression points at $k_\perp/\kappa_\nu \longrightarrow 0$ found in the elasto-dipolar approximation will be absent at $k_0 \neq 0$ in both high-frequency ($\mu_\perp < 0$) and low-frequency ($c_\perp < 0$) ranges. For the short wavelength spectrum condensation points at $k_\perp/\kappa_\nu \longrightarrow \infty$, one still has $\mu_\perp \longrightarrow 0$ if $\mu_\perp < 0$ and $c_\perp \longrightarrow 0$ if $c_\perp < 0$.

Taking into account the retardation, the relations for the spectrum degeneracy points become ($\kappa_\tau \equiv \pi\tau/2L$)

$$k_0^2 = c_\perp(\kappa_\nu^2 + k_\perp^2) + \frac{16\pi\beta_{14}^2}{\mu_\perp}\frac{\kappa_\nu^2 k_\perp^2}{\kappa_\nu^2 + k_\perp^2};$$

$$1 + K^2\frac{k_\perp^4}{(\kappa_\nu^2 + k_\perp^2)(\kappa_\tau^2 + k_\perp^2)} = 0. \tag{16}$$

According to the analysis, additional spectrum degeneracy points can arise (in the same frequency range $c_\perp\mu_\perp < 0$) for certain initial mode indices at a plate thickness below the critical one (and the points existing in the elastostatic approximation will be shifted in both frequency and wavenumber). These are the crossing points of two forward modes.

It should be mentioned that this "additional" type of spectrum degeneracy points associated with the inclusion of the finite propagation velocity of elastic waves is dissimilar to fast magnetoelastic modes studied quite thoroughly by the example of homogeneously magnetized ferromagnetic plates [11]. If we formally set to zero the magnetoelastic interaction in the case of [11], fast magnetoelastic waves are reduced to the emergence of degeneracy points on the (frequency–transverse wavenumber) plane between the dispersion curves corresponding to the spectrum of traveling magnetostatic modes and the dispersion curves specifying the spectrum of bulk elastic modes of the plate. In our case, the dispersion of elasto-dipolar magnons results from hybridization of the dipolar and magnetoelastic interactions; if the latter is set to zero, there is no crossing of the spectra of magnetostatic and elastic waves.

Bulk modes in a plate knowingly result from interference of plane elastic waves reflected from its surfaces. This gives us the opportunity to find correspondence between the geometry of the cross section of the surface of wave vectors of a normal shear elastic wave by the sagittal plane in the infinite piezomagnetic under consideration and the structure of the shear bulk mode found above and propagating along such a piezomagnetic plate. The shape of the cross section of the surface of wave vectors by the sagittal plane (*XY*) can be represented as

$$k^2 = \left(\frac{1}{c_\perp}\right)\frac{k_0^2}{1 + K^2\sin^2 2\varphi}, \quad \tan\varphi \equiv \frac{k_\perp}{k_\parallel}, \quad \mathbf{n} \parallel [100]. \tag{17}$$

At $\omega_0^2 < \omega^2 < (\omega_{\mathrm{me}}^2 + \omega_0^2)(1 + \varepsilon)$ (i.e., in the region of elasto-dipolar magnons), the surface of wave vectors transforms from a closed surface to an open one. There appear the angle intervals (sectors) of the width $\varphi = 2\varphi_c$ ($1 = |K_2|\sin^2 2\varphi_c$) in the sagittal plane, inside which only evanescent waves with the appropriate frequency can exist in the antiferromagnet. At $c_\perp < 0$, the bisectrices of such sectors are orthogonal to each other and directed along the coordinate axes ($\varphi = 0, \pm\pi/2$), whereas at $\mu_\perp < 0$ they coincide with the directions $\varphi = \pm\pi/4, \pm 3\pi/4$. The outward normal to curve (17) determines the direction of the average energy flux carried by the wave. As follows in particular from comparison of Eqs. (13) and (17), the backward elasto-dipolar bulk modes at $c_\perp\mu_\perp < 0$ correspond to the presence of regions with a negative Gaussian curvature in the cross section of the surface of wave vectors by the sagittal plane. In the opposite case, the bulk mode propagating in the antiferromagnetic plate is the forward one.

So far, we restricted ourselves to the analysis of the conditions of the formation of bulk elasto-dipolar magnons only in the antiferromagnetic plate, in which the Dzyaloshinskii–Moriya interaction is given by

Eq. (5) with the plus sign. At the same time, according to calculations, a similar type of excitations can take place also in the antiferromagnetic plates with invariant (6). One has to take into account only that invariants (5) with the plus sign and (6) with the minus sign transform to one another under the rotation of the used reference frame by an angle of $\pi/4$ with respect to the OZ axis. Thus, all relations found above hold also in the case of an antiferromagnetic plate with invariant (6) with the minus sign under boundary conditions (9) if $\mathbf{n} \parallel [110]$, (XY) is the sagittal plane, and $\mathbf{l} \parallel OZ$. All other combinations of signs in Eqs. (5) and (6) do not lead to the formation of the discussed type of exchangeless bulk elasto-dipolar magnons in an antiferromagnetic plate. Already a weak deviation from the above boundary conditions results in disappearance of spectrum degeneracy points (16) found above. Instead, the dispersion curves corresponding to different modes of the spectrum are pushed apart and exhibit bottlenecking in the vicinity of the degeneracy points. In this case, if the degeneracy point was formed by the forward and backward modes, the additional critical points $\partial\omega/\partial k_\perp = 0$ corresponding to the maximum and minimum appear in pairs on the dispersion curves when the degeneracy is lifted. Owing to a strong dependence of the dispersion properties of the normal shear wave with $\mathbf{u} \parallel OZ$ on the propagation direction in the (XY) plane, which follows from Eq. (10), the spectrum of exchangeless elasto-dipolar magnons investigated in this work appears to be ultimately sensitive to the orientation of the surface normal of the antiferromagnetic film within the sagittal plane.

The effect of the orientation of $\mathbf{n}$ within the sagittal plane on the structure of the spectrum of bulk elasto-dipolar magnons and the effects of inhomogeneous exchange interaction will be considered in more detail separately.

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