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Peculiarities of the resonant transmission of a TM (TE) wave through an antiferromagnet plate in crossed dc magnetic and electric fields

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We studied the relation between the topological characteristics of a refraction surface and the characteristics of the transmission of a TM or TE bulk electromagnetic wave through a transparent half-wave antiferromagnet plate in crossed dc magnetic and electric fields. It was shown that the conditions for resonant transmission correspond to the spectrum of escaping bulk magnetic polaritons of the layer as well as the spectrum of electromagnetic waves in the plate with extreme values of the surface impedance. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4862472]

1. Introduction

As well-known from the theory of wave propagation in layered media, for a plane electromagnetic wave with polarization $\alpha = p, s$, the Fresnel (amplitude) reflection and refraction coefficients, V_α and T_α, at the interface between two half-spaces can be represented as[1]

$$T_\alpha = \frac{2\tilde{Z}_\alpha}{\tilde{Z}_\alpha + Z_\alpha}, \quad V_\alpha = \frac{\tilde{Z}_\alpha - Z_\alpha}{\tilde{Z}_\alpha + Z_\alpha}, \tag{1}$$

where $\tilde{Z}_\alpha$ is the surface impedance (admittance) of the medium from which a TM (TE) wave is incident, and Z_α is the input surface impedance (admittance) of the conjugate environment. Recently, metasurfaces—electrodynamic structures, the input impedance of which depends in a specified way on the frequency, polarization, and propagation direction of the incident plane electromagnetic wave—have attracted a significant interest due to potential antenna applications.[2] Particular attention is paid to the use of metamaterials—composite media containing locally resonating structural elements, the wave properties of which in the long-wavelength limit are qualitatively different from the properties of the structural elements forming this metamaterial.[3] However, in an electromagnetic metamaterial, both the relative orientation of the electromagnetic particles forming the material and their operating frequency are strictly defined. Furthermore, with increasing frequency of the incident wave and due to lack of a well-defined surface in such a composite medium, spatial dispersion effects associated with the relatively large size of the metamaterial unit cell quickly become significant.[4]

In this respect, of great interest would be to employ as a metasurface a spatially homogeneous medium the wave properties of which can be selectively adjusted by easily implementable external parameters (e.g., external dc magnetic $\mathbf{H}_0$ or electric $\mathbf{E}_0$ fields). Prominent among these media are antiferromagnetic (AFM) structures. Their electrodynamic characteristics can be significantly modified by the external parameters mentioned above, whereas the frequency of the antiferromagnetic resonance (AFMR) can lie in the terahertz range.[5] In particular, it is well-known that already in the collinear phase of an exchange-collinear AFM, the application of an external magnetic field $\mathbf{H}_0$ perpendicular to the easy magnetic axis transforms this material, according to Ref. 6, into a gyrotropic medium. However, despite a large number of publications devoted to studying the effects of a constant external magnetic field on the transmission of a plane TE or TM wave through the interface between a nonmagnetic insulator and AFM,[7,8] this issue has not lost its relevance.[9] Considerably smaller number of publications is devoted to the effects of an external dc electric field on the transmission of a TE or TM plane wave through such an interface. Furthermore, as follows from the results of Refs. 10–12, a compensated easy-axis (EA) AFM in a field $\mathbf{E}_0$ orthogonal to the easy axis is characterized by the material equations analogous to those of a particular type of bianisotropic medium—a planar lattice of omega particles.[3] On the other hand, if $\mathbf{E}_0$ is directed along the easy magnetic axis of a compensated EA AFM, the corresponding constraint equations become analogous to the case of mutually orthogonal planar lattices of omega particles[3] ($\mathbf{E}_0/|\mathbf{E}_0|$ is the intersection line of the planes of such lattices). In the case where the AFM is uncompensated, for $\mathbf{E}_0 \neq 0$ the AFM acquires not only the gyrotropic and pseudo-gyrotropic properties but also the magnetoelectric ones.[11,13] Moreover, nonzero tensor components involved in the constrain equations for such a medium exhibit resonant features depending on the frequency of the propagating wave. Thus, we can assume that the change in the direction of $\mathbf{E}_0$ can significantly affect the transmission of a bulk electromagnetic (EM) wave of TM or TE type through the interface between magnetic and nonmagnetic media. It should be noted that quadratic magneto-optical interaction (QMOI) has been considered in Refs. 10–12 as the mechanism responsible for the interaction of the spin subsystem of an exchange-collinear centrosymmetric AFM with an electric field. This interaction becomes exchange-reinforced in the spin-wave electrodynamics of the AFM environment. Moreover, there also appears an additional magnetic anisotropy associated with the orientation of $\mathbf{E}_0$. This effect is quite small, however if the own magnetic anisotropy in the sagittal plane under study is also small, we can expect that by changing the direction of $\mathbf{E}_0$ in such a plane, it is possible to smoothly change the equilibrium orientation of the AFM vector and therefore the orientation of the principal axes of the tensors involved in the constrain equations for such AFM. Conditions for the implementation of this possibility can be expected, in particular, in the case of EA AFM in the "spin-flop" phase with $\mathbf{H}_0$ aligned along the easy magnetic axis* assuming that $\mathbf{E}_0$ lies in the sagittal plane with the normal along $\mathbf{H}_0$.

In this regard, the aim of this work is to analyze in the Voigt geometry the influence of the orientation of a constant external electric field $\mathbf{E}_0$ in the sagittal plane on the conditions of resonant transmission of bulk TM or TE EM waves incident on a transparent plate of a uniaxial AFM in the "spin-flop" phase.

2. Basic equations

Let us consider, as an example, a two-sublattice model ($\mathbf{M}_{1,2}$ are the sublattice magnetizations, $|\mathbf{M}_1| = |\mathbf{M}_2| = M_0$) of the magnetically compensated exchange-collinear uniaxial (OZ) AFM.[5] In this case, in terms of the ferromagnetic ($\mathbf{m} = (\mathbf{M}_1 + \mathbf{M}_2)/2M_0$) and antiferromagnetic ($\mathbf{l} = (\mathbf{M}_1 - \mathbf{M}_2)/2M_0$) vectors, the density of the thermodynamic potential of the AFM under consideration takes the form

$$F = M_0^2\left(\frac{\delta}{2}\mathbf{m}^2 - \frac{b}{2}l_z^2 + \frac{b_1}{2}l_x^2 l_y^2 - 2\mathbf{mh} - \frac{r_m}{2}(\mathbf{mP})^2 - \frac{r_l}{2}(\mathbf{lP})^2 - \frac{s_m}{2}\mathbf{m}^2\mathbf{P}^2 - \frac{s_l}{2}\mathbf{l}^2\mathbf{P}^2\right) + \left(\frac{P_x^2 + P_y^2}{2\kappa_\perp} + \frac{P_z^2}{2\kappa_\parallel} - \mathbf{PE}\right), \tag{2}$$

where δ and b, b_1 are the homogeneous exchange and magnetic anisotropy constants, respectively, $\mathbf{h}$ is the renormalized magnetic field, $\mathbf{E}$ and $\mathbf{P}$ are the vectors of the electric field and polarization, respectively, $\kappa_\parallel$ and $\kappa_\perp$ are the longitudinal and transverse dielectric susceptibilities; r_m, r_l, s_m, and s_l are the QMOI coefficients. For $b > 0$, the relation (2) corresponds to the collinear phase with an easy magnetic axis OZ. If $\mathbf{E}_0 \perp \mathbf{H}_0 \parallel \mathbf{m}_0 \parallel OZ$, then, depending on the sign of the QMOI constant, $\mathbf{l}_0 \parallel \mathbf{E}_0$ or $(\mathbf{l}_0\mathbf{E}_0) = 0$ ($\mathbf{m}_0$ and $\mathbf{l}_0$ are the equilibrium ferromagnetic and antiferromagnetic vectors, respectively). The case of a uniaxial antiferromagnet with an easy plane perpendicular to OZ corresponds to $b < 0$.

In the particular case of $\mathbf{l}_0 \parallel \mathbf{E}_0 \parallel OY$, the material relations for the AFM under consideration in the linear approximation with regard to the small oscillation amplitude take the form

$$\mathbf{B} = \hat{\mu}\mathbf{H} + \hat{A}^*\mathbf{E}, \qquad \mathbf{D} = \hat{\varepsilon}\mathbf{E} + \hat{A}^T\mathbf{H}, \tag{3}$$

$$\mu_{ik} = \begin{pmatrix} \mu_{xx} & -\mu_* i & 0 \\ \mu_* i & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{pmatrix}; \quad \varepsilon_{ik} = \begin{pmatrix} \varepsilon_{xx} & -\varepsilon_* i & 0 \\ \varepsilon_* i & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix};$$

$$A_{ik} = \begin{pmatrix} 0 & 0 & i\beta_2 \\ 0 & 0 & \beta_3 \\ -i\beta_1 & \beta_4 & 0 \end{pmatrix}.$$

Here $\hat{A}^*$ and $\hat{A}^T$ correspond to complex conjugation and transposition of the matrix $\hat{A}$, respectively.

Calculation shows that for the frequencies ω small compared with the eigenfrequencies of the electron subsystem and taking into account Eq. (2), the nonzero tensor components appearing in Eq. (3), can be represented as follows:

$$\left.\begin{aligned}
&\mu_{xx} = 1 + 4\pi T_x\frac{\omega_F^2}{\Delta_F}, \quad \mu_{yy} = 1 + 4\pi T_y\frac{\omega_F^2}{\Delta_F}, \quad \mu_{zz} = 1 + 4\pi T_z\frac{\omega_{AF}^2}{\Delta_{AF}}, \quad \mu_* = 4\pi\sqrt{T_xT_y}\frac{\omega_F\omega}{\Delta_F}, \\
&\varepsilon_{xx} = 1 + 4\pi\alpha_{x0} + 4\pi R_x\frac{\omega_{AF}^2}{\Delta_{AF}}, \quad \varepsilon_{yy} = 1 + 4\pi\alpha_{y0} + 4\pi R_y\frac{\omega_{AF}^2}{\Delta_{AF}}, \quad \varepsilon_* = 4\pi\sqrt{R_xR_y}\frac{\omega_{AF}\omega}{\Delta_{AF}}, \\
&\varepsilon_{zz} = 1 + 4\pi\alpha_{z0} + 4\pi R_z\frac{\omega_F^2}{\Delta_F}, \quad \beta_1 = 4\pi\sqrt{R_xT_z}\frac{\omega_{AF}\omega}{\Delta_{AF}}, \quad \beta_2 = 4\pi\sqrt{R_zT_x}\frac{\omega_F\omega}{\Delta_F}, \\
&\beta_3 = 4\pi\sqrt{R_zT_y}\frac{\omega_F^2}{\Delta_F}, \quad \beta_4 = 4\pi\sqrt{R_yT_z}\frac{\omega_{AF}^2}{\Delta_{AF}}, \quad \Delta_F = \omega_F^2 - \omega^2, \quad \Delta_{AF} = \omega_{AF}^2 - \omega^2.
\end{aligned}\right\} \tag{4}$$

If $|\mathbf{H}_0| = 0$, $|\mathbf{E}_0| \neq 0$, then in Eqs. (3) and (4) $\beta_3 = \beta_4 = \mu_* = \varepsilon_* = 0$, whereas for $|\mathbf{E}_0| = 0$, $|\mathbf{H}_0| \neq 0$, $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$. For $\mathbf{k} \in XY$ in the AFM insulator

*Or, in the case of an easy-plane AFM (EP AFM), for $\mathbf{H}_0$ aligned along the hard magnetic axis.

model under consideration (Eqs. (3) and (4)), independent propagation of normal TM or TE polaritons is possible. If, following Ref. 6, we introduce the unit vectors **q** and **b** along the normal to the interface and along the line of intersection of the sagittal plane and the boundary plane between the media ($\mathbf{qb}=0$), then the wave vector **k** in the sagittal plane can be represented as $\mathbf{k}=k_\perp\mathbf{b}+k_\parallel\mathbf{q}$. Based on the covariant method[6] for $\mathbf{I}_0 \parallel \mathbf{E}_0 \in XY$ ($\mathbf{H}_0 \parallel \mathbf{m}_0 \parallel OZ$), the corresponding dispersion relations take the form ($k_0 \equiv \omega/c$, φ is the misorientation angle of the vectors $\mathbf{I}_0$, $\mathbf{E}_0$, such that $\cos\varphi=1$ for $\mathbf{E}_0 \parallel OY$)

For a TM wave:

$$\left\{k_\parallel + \left[\frac{\varepsilon_{xx}c_p k_o + (\varepsilon_{yy}-\varepsilon_{xx})k_\perp\cos\varphi}{\varepsilon_\parallel}\right]\sin\varphi\right\}^2 - \left[\frac{k_0^2\varepsilon_\parallel\Delta_p - \varepsilon_{yy}\varepsilon_{xx}^2(c_p\cos\varphi - k_\perp)^2}{\varepsilon_{xx}\varepsilon_\parallel^2}\right] = 0,$$

for a TE wave:

$$\left\{k_\parallel + \left[\frac{\mu_{xx}c_s k_o + (\mu_{yy}-\mu_{xx})k_\perp\cos\varphi}{\mu_\parallel}\right]\sin\varphi\right\}^2 - \left[\frac{k_0^2\mu_\parallel\Delta_s - \mu_{yy}\mu_{xx}^2(c_s\cos\varphi - k_\perp)^2}{\mu_{xx}\mu_\parallel^2}\right] = 0. \quad (5)$$

Here

$$c_p \equiv k_o\frac{\varepsilon_{xx}\beta_4 - \varepsilon_*\beta_1}{\varepsilon_{xx}};\quad \Delta_p \equiv (\varepsilon_{xx}\varepsilon_{yy}-\varepsilon_*^2)(\varepsilon_{xx}\mu_{zz}-\beta_1^2);$$
$$\varepsilon_\parallel \equiv \varepsilon_{yy}\cos^2\varphi + \varepsilon_{xx}\sin^2\varphi; \quad (6)$$

$$c_s \equiv k_o\frac{\mu_*\beta_2 - \mu_{xx}\beta_3}{\mu_{xx}};\quad \Delta_s \equiv (\mu_{xx}\mu_{yy}-\mu_*^2)(\mu_{xx}\varepsilon_{zz}-\beta_2^2);$$
$$\mu_\parallel \equiv \mu_{yy}\cos^2\varphi + \mu_{xx}\sin^2\varphi. \quad (7)$$

Assuming the *XY*-plane as a sagittal, let us further consider a three-layer structure in which two identical nonmagnetic optically isotropic half-spaces are separated by a layer of the AFM under consideration of thickness *d*. Material equations for a nonmagnetic medium have the form

$$\mathbf{D} = \tilde{\varepsilon}\mathbf{E}, \qquad \mathbf{B} = \mathbf{H}. \quad (8)$$

In this case, due to Eqs. (3) and (4), the refractive properties of the interface between magnetic and nonmagnetic media depend on the orientation of $\mathbf{E}_0$ in the sagittal plane. Calculation based on the covariant method[6] shows that in the considered case of an AFM layer in an infinite isotropic medium, the amplitude transmission coefficient for a TM or TE wave takes the form

$$W_\alpha = \frac{2\tilde{Z}_\alpha}{(T_{11}^\alpha + T_{22}^\alpha)\tilde{Z}_\alpha - (T_{21}^\alpha + T_{12}^\alpha\tilde{Z}_\alpha^2)}, \quad (9)$$

where T_{ik}^α is the transition matrix that relates the tangential components of the electric and magnetic fields in a TM or TE wave on the surfaces of the AFM layer of thickness d ($\alpha = p, s$)

$$T_{ik}^\alpha = \exp(\pm i\tilde{c}_\alpha d)\begin{pmatrix} \cosh(\eta_\alpha d) + \dfrac{Z_{\alpha+}+Z_{\alpha-}}{Z_{\alpha+}-Z_{\alpha-}}\sinh(\eta_\alpha d) & -\dfrac{2}{Z_{\alpha+}-Z_{\alpha-}}\sinh(\eta_\alpha d) \\ \dfrac{2Z_{\alpha+}Z_{\alpha-}}{Z_{\alpha+}-Z_{\alpha-}}\sinh(\eta_\alpha d) & \cosh(\eta_\alpha d) - \dfrac{Z_{\alpha+}+Z_{\alpha-}}{Z_{\alpha+}-Z_{\alpha-}}\sinh(\eta_\alpha d)\end{pmatrix}, \quad (10)$$

$$\eta_p^2 \equiv \frac{\varepsilon_{yy}\varepsilon_{xx}^2(c_p\cos\varphi - k_\perp)^2 - k_0^2\varepsilon_\parallel\Delta_p}{\varepsilon_{xx}\varepsilon_\parallel^2},$$
$$\eta_s^2 \equiv \frac{\mu_{yy}\mu_{xx}^2(c_s\cos\varphi - k_\perp)^2 - k_0^2\mu_\parallel\Delta_s}{\mu_{xx}\mu_\parallel^2}, \quad (11)$$

$$\tilde{c}_p = \left[-\frac{\varepsilon_{xx}c_p k_0 + (\varepsilon_{yy}-\varepsilon_{xx})k_\perp\cos\varphi}{\varepsilon_\parallel}\right]\sin\varphi,$$
$$\tilde{c}_s = \left[-\frac{\mu_{xx}c_s k_0 + (\mu_{yy}-\mu_{xx})k_\perp\cos\varphi}{\mu_\parallel}\right]\sin\varphi. \quad (12)$$

Here, $Z_{\alpha+}$ and $Z_{\alpha-}$ are the surface impedances (admittances) for a normal TM (TE) polariton wave on the upper and lower boundaries of the AFM layer,[10–13] $\eta_\alpha \equiv \mathrm{Im}(k_{\parallel\alpha})$, $k_{\parallel\alpha}$ is the solution of Eqs. (5)–(7) with respect to $k_\parallel$ for given ω and $k_\perp$.

Thus, if simultaneously $\mathbf{E}_0 \neq 0$ and $\mathbf{E}_0 \perp \mathbf{H}_0$, then, as follows from Eqs. (3), (4), (11), and (12), in the case of the AFM plate, the transmittance for a TM or TE wave is not only nonreciprocal with respect to the sign inversion of the incidence angle (like in the case of $\mathbf{E}_0 = 0$ and $\mathbf{H}_0 \neq 0$), but also with respect to the sign inversion of ($\mathbf{E}_0\mathbf{q}$). The selection of sign for the wave transmitted through the plate (wave vector projection on the direction of **q**) with polarization $\alpha = p, s$

in the AFM medium is carried out by taking into account the Sommerfeld–Mandelstam radiation principles.[14]

3. Relation between the conditions for half-wave transmission through the layer and the spectrum of the escaping waveguide magnetic polaritons of the plate

On the external parameter plane, frequency ω—transverse wave number $k_\perp$, the boundaries separating the regions of bulk and evanescent waves of TM (TE) type in the uncompensated antiferromagnet under consideration, Eqs. (2)–(4), are determined from Eq. (11) as $\eta_\alpha = 0$. Based on Eq. (9), the condition for the complete transmission of a TM (TE) wave through the AFM plate ($|W_\alpha| = 1$) can be written as

$$(T_{11}^\alpha + T_{22}^\alpha - 2)\tilde{Z}_\alpha = T_{21}^\alpha + T_{12}^\alpha \tilde{Z}_\alpha^2. \tag{13}$$

From Eqs. (10) and (13) it follows that the complete transmission takes place in the case when an electromagnetic wave incident from outside has such values of the external parameters ω and $k_\perp$ that the layer is half-wave thick

$$\tilde{k}_{||\alpha} d = \pi\nu, \quad \nu = 1, 2\ldots, \quad \eta_\alpha^2 \equiv -\tilde{k}_{||\alpha}^2. \tag{14}$$

Since for the ω and $k_\perp$ under consideration the media external to the plate is optically denser than the AFM, the vanishing of the denominator in the transmission coefficient Eq. (9),

$$(T_{11}^\alpha - T_{22}^\alpha)\tilde{Z}_\alpha - (T_{21}^\alpha + T_{12}^\alpha \tilde{Z}_\alpha^2) = 0, \tag{15}$$

determines the spectrum of the bulk polariton TM (TE) wave, escaping into both the upper and lower half-spaces bordering the plate (in this case, there is no bulk wave incident from outside (see also Ref. 15)). If, on the other hand, we consider the case when in the upper medium there is only a bulk wave with polarization $\alpha = p$, s incident without reflection on the transparent plate and then transmitted to the lower half-space, the solution of this boundary value problem for the AFM layer under study is, instead of Eq. (15),

$$(T_{11}^\alpha + T_{22}^\alpha)\tilde{Z}_\alpha + (T_{21}^\alpha - T_{12}^\alpha \tilde{Z}_\alpha^2) = 0, \tag{16}$$

i.e., for every $\tilde{Z}_\alpha$, the condition for half-wave transmission (14) is fulfilled.

This means that Eq. (16) determines the spectrum of normal magnetic TM (TE) polaritons in a plate with a symmetrical environment and a special type of boundary conditions: For given ω and $k_\perp$, a bulk electromagnetic wave of the same polarization ($\alpha = p$ or $\alpha = s$) is incident on one surface of the plate and emitted from its other surface. Moreover, despite the fact that such a polariton wave in the layer (Eq. (16)) is leaky, it is not attenuated during its propagation along the plate due to the compensation (in contrast to Eq. (15)) by the energy fluxes associated with the electromagnetic waves incident on the plate (source) and radiated from the plate (drain). It is appropriate to note that despite the presence of the incident and transmitted waves, the number of independent amplitudes in this case is formally equal to the number of boundary conditions, as it should be, when calculating the spectrum of normal oscillations.[16]

On the plane of external parameters ω and $k_\perp$, the condition for the half-wave transmission of bulk TM or TE polaritons through the AFM plate, Eqs. (14), (11), and (12), in mutually orthogonal crossed magnetic and electric fields can be represented in the following form for the Voigt geometry ($\mathbf{I}_0 \| \mathbf{E}_0$)

$$(k_\perp - c_p \cos\varphi)^2 + \left(\frac{\pi\nu}{d}\right)^2 \frac{\varepsilon_{||}^2}{\varepsilon_{yy}\varepsilon_{xx}} = k_0^2 \frac{\varepsilon_{||}\Delta_{\mathbf{p}}}{\varepsilon_{yy}\varepsilon_{xx}^2}; \tag{17}$$

$$(k_\perp - c_s \cos\varphi)^2 + \left(\frac{\pi\nu}{d}\right)^2 \frac{\mu_{||}^2}{\mu_{yy}\mu_{xx}} = k_0^2 \frac{\mu_{||}\Delta_s}{\mu_{yy}\mu_{xx}^2}. \tag{18}$$

From these relations it follows that on the plane ω–$k_\perp$, depending on the polarization (TM or TE type), the regions of existence of bulk waves are limited by Eqs. (17) and (18) for $\nu = 0$. From the analysis of Eqs. (5)–(7) it follows that, since $c_\alpha(H_0) = -c_\alpha(-H_0)$ and $c_\alpha(\mathbf{E}_0\mathbf{q}) = -c_\alpha(-\mathbf{E}_0\mathbf{q})$, the spectrum of normal magnetic polaritons of TM and TE type, Eqs. (17) and (18), is not symmetric under the inversion of the direction of wave propagation. In case where simultaneously $|\mathbf{E}_0| = |\mathbf{H}_0| = 0$, the effects of non-reciprocity of the polariton spectrum, Eqs. (17) and (18), are absent.

Let us introduce for a given type of polarization $\alpha = s, p$ the characteristic frequencies $\omega_{\alpha x}$, $\omega_{\alpha y}$, $\Omega_{\alpha A}$, $\Omega_{\alpha B}$, $\omega_{\alpha||}$, and the wave number $k_{\perp\alpha}^*$ by using the following relations:

$$\begin{gathered} k_{\perp\alpha}^* = k_0 c_\alpha \cos\varphi\,; \quad \Omega_{\alpha B} \equiv \max\{\Omega_{\alpha 1}, \Omega_{\alpha 2}\}, \\ \Omega_{\alpha A} \equiv \min\{\Omega_{\alpha 1}, \Omega_{\alpha 2}\}, \quad \mu_{yy}(\omega_{sy}) = 0, \\ \mu_{xx}(\omega_{sx}) = 0, \quad \varepsilon_{yy}(\omega_{py}) = 0, \quad \varepsilon_{xx}(\omega_{px}) = 0, \\ \varepsilon_{||}(\omega_{p||}) = 0, \quad \mu_{||}(\omega_{s||}) = 0, \end{gathered} \tag{19}$$

$$\begin{gathered} \mu_{yy}(\Omega_{s1})\mu_{xx}(\Omega_{s1}) - \mu_*^2(\Omega_{s1}) = 0, \\ \varepsilon_{yy}(\Omega_{p1})\varepsilon_{xx}(\Omega_{p1}) - \varepsilon_*^2(\Omega_{p1}) = 0, \\ \mu_{xx}(\Omega_{s2})\varepsilon_{zz}(\Omega_{s2}) - \beta_2^2(\Omega_{s2}) = 0, \\ \varepsilon_{xx}(\Omega_{p2})\mu_{zz}(\Omega_{p2}) - \beta_1^2(\Omega_{p2}) = 0. \end{gathered} \tag{20}$$

As follows from Eqs. (17) and (18), the frequencies $\omega_{\alpha x}$ and $\omega_{\alpha y}$ ($\alpha = s$, p) correspond to the short-wavelength ($k \to \infty$) points, while $\omega = \omega_{\alpha||}$ (for $k_\perp = k_{\perp\alpha}^*$) corresponds to the long-wavelength points of accumulation of the spectrum. The location of these points in the plane of external parameters frequency–wave number for a given value of $\mathbf{H}_0$ significantly depends not only on the magnitude but also on the orientation of $\mathbf{E}_0$ in the sagittal plane. As a result, depending on the mode number ν, frequency interval, and value of the wave number, a bulk polariton wave propagating in the AFM plate, Eqs. (17) and (18), may be either the forward ($\mathbf{k}_\perp \partial\omega/\partial\mathbf{k}_\perp > 0$) or the backward ($\mathbf{k}_\perp \partial\omega/\partial\mathbf{k}_\perp < 0$) one. Thus, the direction of energy transfer in the plane of the plate can be changed by changing the relative orientation of $\mathbf{E}_0$ in the plane of incidence. For the wavenumbers corresponding to $k_{\perp\alpha}^*(\omega)$, the dispersion curves of the normal magnetic polaritons of TE and TM types, Eqs. (17) and (18), respectively, exhibit the extremum points (maximum or minimum) (Fig. 1). In particular, for $0 < \omega \le \omega_{\alpha x}$, the bulk polaritons of TM or TE type propagating along the AFM plate have short-wavelength accumulation point of the spectrum at $\omega_{\alpha x}$, while for $k_{\perp\alpha}^*(\omega)$ the

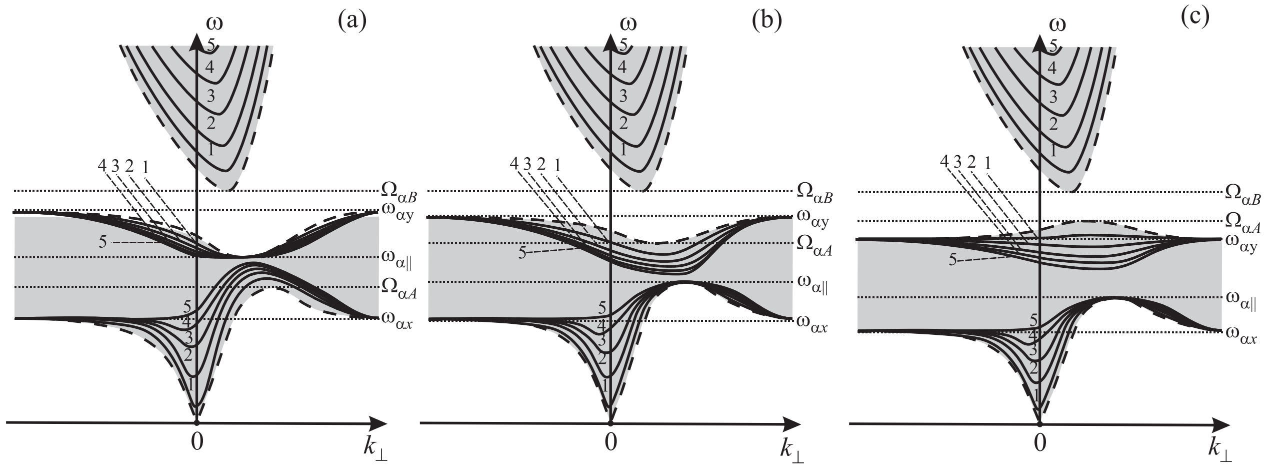

FIG. 1. Spectra of the normal bulk magnetic polaritons (solid numbered lines, $\nu = \overline{1,5}$) of the half-wave AFM layer for $\mathbf{I}_0 \parallel \mathbf{E}_0 \in XY$, $\mathbf{k} \in XY$, $\mathbf{H}_0 \parallel OZ$ which correspond to the conditions: Eq. (21) (a), Eq. (22) (b), and Eq. (23) (c).

wave with a given mode number ν has a minimum. In other words, depending on the magnitude and sign of $k_\perp$, the regions of the considered dispersion curve may correspond to forward as well as backward wave.

As the conducted analysis shows, in the frequency range $\omega_{\alpha x} < \omega < \max\{\omega_{\alpha y}; \Omega_{\alpha A}\}$, the characteristics of the considered bulk waves of TM and TE type significantly depend on the relative magnitude of the long-wavelength accumulation point of the spectrum $\omega_{\alpha\|}$ and the characteristic frequencies of $\Omega_{\alpha 1}$, $\Omega_{\alpha 2}$, and $\omega_{\alpha y}$. For a given magnitude and orientation of $\mathbf{H}_0$ and $\mathbf{E}_0$, several qualitatively different cases for the polariton wave with polarization $\alpha = s, p$ propagating along the AFM plate are observed under the conditions (Figs. 1(a)–1(c)

$$\Omega_{\alpha B} > \omega_{\alpha y} > \omega_{\alpha\|} > \Omega_{\alpha A} > \omega_{\alpha x}, \tag{21}$$

$$\Omega_{\alpha B} > \omega_{\alpha y} > \Omega_{\alpha A} > \omega_{\alpha\|} > \omega_{\alpha x}, \tag{22}$$

$$\Omega_{\alpha B} > \Omega_{\alpha A} > \omega_{\alpha y} > \omega_{\alpha\|} > \omega_{\alpha x}. \tag{23}$$

Moreover, while for the wave of TE type, the consecutive application of conditions (21)–(23) corresponds to an increase in the magnitude of $\mathbf{E}_0$ (for $|\mathbf{H}_0|$ fixed), for the TM wave this corresponds to a decrease in the magnitude of $\mathbf{E}_0$. Let us consider the peculiarities of the propagation of bulk magnetic polaritons, Eqs. (17) and (18) subject to Eqs. (21)–(23), assuming that for a TE wave $(\mathbf{E}_0\mathbf{q}) < 0$, whereas in the case of TM wave**, $(\mathbf{E}_0\mathbf{q}) < 0$. The filled areas in Figs. 1(a)–1(c) correspond to bulk waves, whereas the blank areas correspond to evanescent waves. The boundary between the regions (dashed line in Fig. 1) is determined from Eqs. (17) and (18) as $\nu = 0$.

4. Relation between the conditions for resonant transmission and the local geometry of the surface of wave vectors

As follows from Eq. (9), in the case when the half-spaces are identical and condition (14) is fulfilled, the amplitude of the reflected TM (TE) wave vanishes, while the electromagnetic wave transmitted through the AFM plate has the absolute value equal to the incident one and differs from it only by its phase (taking into account, whether it is incident on the top or bottom surface of the plate). For a given magnitude and orientation of $\mathbf{H}_0$ and $\mathbf{E}_0$ and the wave number $k_\perp$, such a non-reciprocity effect is related to the condition

$$T_{\alpha\pm} = \exp(i[\tilde{c}_\alpha \pm \tilde{k}_{\|\alpha}]d), \tag{24}$$

i.e., without changing the angle of incidence, the phase shift of the wave transmitted through the plate can be controlled by external magnetic and electric fields ($W_{\alpha+}W_{\alpha-} = \exp(2i\tilde{c}_\alpha d)$). The magnitude and sign of the phase of the TM or TE wave transmitted through the plate are determined by the location of the surface of wave vectors (SWV) in the $\mathbf{k}$-space for an infinite AFM in the same geometry. Note that the discussed phase shift and its non-reciprocity with respect to "top-bottom" inversion of the incidence angle also occur in the case of $|\mathbf{H}_0| = 0$ when the tilt angle of $\mathbf{I}_0$ (caused by the tilt of $\mathbf{E}_0$) in the sagittal plane differs from 0 and $\pi/2$.

It is well known (see, e.g., Ref. 16) that, for given ω and $k_\perp$, the normal to the cross-section of the SWV by the sagittal plane determines the direction of the energy flux carried by the wave. Thus, to analyze the direction of the energy flux carried by the TM (TE) wave along the plate (and hence the type of the wave, forward or backward) under the conditions (14), (17), and (18), it is practical to proceed from the cross-section of the SWV of a polariton of given polarization by the sagittal plane (in this case, $\mathbf{k} \in XY$).

For an optically isotropic nonmagnetic insulator surrounding the plate, the cross-section of the SWV of a normal TE or TM wave by the incidence plane is defined by the expression

$$k_\perp^2 + k_\|^2 = k_0^2\tilde{\varepsilon}. \tag{25}$$

In this case, the refractive properties of the AFM interface depend on the misorientation angle φ of the vector $\mathbf{I}_0$ (the direction of $\mathbf{E}_0$) with respect to the positive direction of the outer normal $\mathbf{q}$ in the sagittal plane XY. Assuming that the wave frequency ω is fixed, from Eqs. (5)–(7), (19), and (20) it follows that for an infinite (as well as a semi-infinite) easy-axis AFM with a center of symmetry and the selected magneto-optical configuration, the corresponding ratio for the cross-section of the SWV of both TM and TE waves by

**The sign change of $(\mathbf{E}_0\mathbf{q})$ corresponds to the substitution $k_\perp \to -k_\perp$ in the spectra shown in Fig. 1.

the sagittal plane in the particular case of $\mathbf{I} \parallel \mathbf{E}_0 \parallel \mathbf{n} \parallel OY$ has the form

$$\frac{(k_\perp - c_\alpha)^2}{a_\alpha^2} + \frac{k_\parallel^2}{b_\alpha^2} = 1; \quad a_s^2 \equiv \frac{\Delta_s k_0^2}{\mu_{xx}^2}; \quad b_s^2 \equiv \frac{\Delta_s k_0^2}{\mu_{xx}\mu_{yy}};$$
$$a_p^2 \equiv \frac{\Delta_p k_0^2}{\varepsilon_{xx}^2}; \quad b_p^2 \equiv \frac{\Delta_p k_0^2}{\varepsilon_{xx}\varepsilon_{yy}}; \tag{26}$$

where, taking into account Eqs. (4) and (5)

$$a_s^2 = \varepsilon_{z0} k_0^2 \frac{(\Omega_{s1}^2 - \omega^2)(\Omega_{s2}^2 - \omega^2)}{(\omega_{sx}^2 - \omega^2)^2},$$
$$a_p^2 = \varepsilon_{y0} k_0^2 \frac{(\Omega_{p1}^2 - \omega^2)(\Omega_{p2}^2 - \omega^2)}{(\omega_{px}^2 - \omega^2)^2},$$
$$b_s^2 = \varepsilon_{z0} k_0^2 \frac{(\Omega_{s1}^2 - \omega^2)(\Omega_{s2}^2 - \omega^2)}{(\omega_{sy}^2 - \omega^2)(\omega_{sx}^2 - \omega^2)}, \tag{27}$$
$$b_p^2 = \varepsilon_{x0} k_0^2 \frac{(\Omega_{p1}^2 - \omega^2)(\Omega_{p2}^2 - \omega^2)}{(\omega_{py}^2 - \omega^2)(\omega_{px}^2 - \omega^2)},$$
$$c_s = -k_0 \frac{R\omega_{sx}^2}{\omega_{sx}^2 - \omega^2}, \quad c_p = k_0 \frac{T\omega_{px}^2}{\omega_{px}^2 - \omega^2},$$

where

$$T \equiv 4\pi\sqrt{R_y T_z}, \quad R \equiv 4\pi\sqrt{T_y R_z},$$
$$\varepsilon_{x0} = 1 + 4\pi\alpha_{x0}, \quad \varepsilon_{y0} = 1 + 4\pi\alpha_{y0}, \quad \varepsilon_{z0} = 1 + 4\pi\alpha_{z0}.$$

In our case, the sagittal plane coincides with the easy magnetic plane for the equilibrium antiferromagnetic vector. Moreover, in this model, for any orientation of the external electric field $\mathbf{E}_0$ within this plane, it holds that $\mathbf{I}_0 \parallel \mathbf{E}_0$. Thus, a change in the relative orientation of the vectors $\mathbf{I}_0$ ($\mathbf{E}_0$) and $\mathbf{q}$ in the sagittal plane by an angle φ ($\cos\varphi \equiv \mathbf{E}_0\mathbf{q}/|\mathbf{E}_0 \parallel \mathbf{q}|$) in the $\mathbf{k}$-space corresponds to the rotation of curves (26) and (27) about the origin by the same angle. As a result, the cross-section is described by the full equation of a second order curve. In other words, if $\mathbf{H}_0 \perp \mathbf{E}_0$ and $(\mathbf{E}_0\mathbf{q}) \neq 0$, then inside the AFM plate under consideration, for given ω and $k_\perp$, subject to Eqs. (17) and (18), the bulk TM (TE) waves incident and reflected from the same boundary have a different period of spatial oscillations along the direction of the outward normal.

It is straightforward to see that in this case, for $(\mathbf{E}_0\mathbf{q}) \neq 0$ and $|\mathbf{H}_0| \neq 0$, for a wave with the polarization α, the direction and changes in the energy flux carried by a TM (TE) wave along the half-wave AFM plate are determined not by the structure of the SWV for the semi-infinite AFM, Eqs. (26) and (27), but by the SWV of the layer in the same geometry. This SWV differs by the fact that in it, for given ω and $k_\perp$, the projection of the inverse phase velocity on the direction $\mathbf{q}$ ($\tilde{k}_{\parallel\alpha}(k_\perp, \omega)$) is equal to the half-difference of the projections on the same direction of the inverse phase velocities corresponding to the SWV for a semi-infinite AFM in the same geometry

$$\frac{[k_\perp - k_0 c_\alpha \cos\varphi]^2}{\tilde{a}_\alpha^2} + \frac{k_\parallel^2}{\tilde{b}_\alpha^2} = 1, \quad \tilde{b}_s^2 \equiv b_s^2 \frac{\mu_{yy}}{\mu_{\parallel}}, \quad \tilde{a}_s^2 \equiv a_s^2 \frac{\mu_{\parallel}}{\mu_{yy}},$$
$$\tilde{b}_{\mathbf{p}}^2 \equiv b_p^2 \frac{\varepsilon_{yy}}{\varepsilon_{\parallel}}, \quad \tilde{a}_p^2 \equiv a_p^2 \frac{\varepsilon_{\parallel}}{\varepsilon_{yy}}. \tag{28}$$

Thus, the cross-sections of the SWV of a TM (TE) wave by the sagittal plane of the SWV for the half-space, Eq. (26), and the layer, Eq. (28), are different (Fig. 2). In particular, for given ω and $k_\perp$ on this cross-section for a TM (TE) wave propagating in the AFM layer, the points at which $(\mathbf{b}\partial\omega/\partial\mathbf{k}) = 0$ can move, disappear and reappear. Moreover, the points in the plane ω–$k_\perp$ for which on the SWV of the half-space $(\mathbf{q}\partial\omega/\partial\mathbf{k}) = 0$ (limiting wave) are conserved also for the SWV of the layer. If, for a wave of given polarization, there is a $k_\perp$ at which the direction of the group velocity vector differs by π at the corresponding points on the cross-section of the SWV of the half-space, Eq. (26), then, for a given ω, same value of $k_\perp$ determines the position of the center of the SWV of the layer, Eq. (28) on the corresponding axis in the $\mathbf{k}$-space.

Using the Sommerfeld–Mandelstam radiation principle for the calculation of the transmission coefficient (see Ref. 14), one should consider the possibility that the energy flux carried by such a wave is directed along the inner normal to the curve, which determines the cross-section of the SWV by the sagittal plane. In this model of the AFM media, such an effect is, in particular, possible when the frequency of the

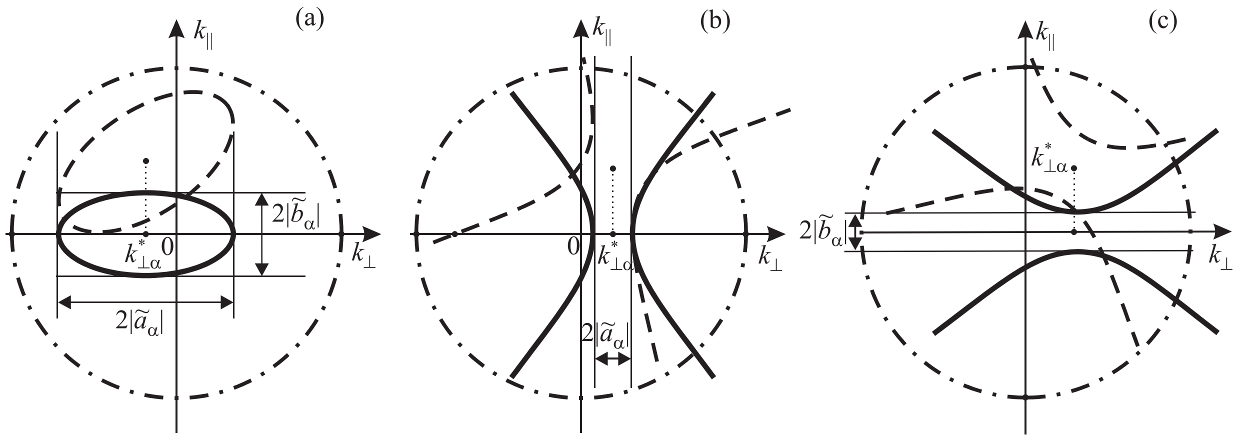

FIG. 2. Cross-sections of the SWV for the AFM half-space, Eq. (26), (dashed line) and for the AFM layer, Eq. (28), (solid line) in the case of $\mathbf{I}_0 \parallel \mathbf{E}_0 \in XY$, $\mathbf{k} \in XY$, $\mathbf{H}_0 \parallel OZ$ ($\tilde{a}_\alpha^2 a_\alpha^2 > 0$, $\tilde{b}_\alpha^2 b_\alpha^2 > 0$). The cross-section of the SWV of a non-magnetic insulator is shown as a dash-dotted line. $\tilde{b}_\alpha^2 > 0$; $\tilde{a}_\alpha^2 > 0$ (a), $\tilde{b}_\alpha^2 < 0$; $\tilde{a}_\alpha^2 > 0$ (b), and $\tilde{b}_\alpha^2 > 0$; $\tilde{a}_\alpha^2 < 0$ (c).

TM (TE) wave incident from outside satisfies the condition (see Fig. 1(c))

$$\omega_{\alpha y} < \omega < \Omega_{\alpha A}. \tag{29}$$

If, taking into account the Sommerfeld–Mandelstam radiation principle for given ω and $k_\perp$, the projection of the group velocity vector on the direction of propagation **b** is positive for the SWV of the layer (Eqs. (27) and (28)), then the corresponding bulk electromagnetic wave propagating along the plate (Eqs. (17) and (18)) is a forward wave ($\mathbf{k}_\perp \partial\omega/\partial\mathbf{k}_\perp > 0$). Otherwise the bulk electromagnetic wave in the plate is of the backward type ($\mathbf{k}_\perp \partial\omega/\partial\mathbf{k}_\perp < 0$). There is no one-to-one correspondence between the type of the wave in the AFM plate (Eqs. (17) and (18)) and the sign of the projection of the group velocity vector for the cross-section of the SWV of the half-space (Eqs. (26) and (27)) for given ω and $k_\perp$.

It should be noted that in all the cases considered in the present work, the out-of-the-sagittal-plane component of the energy flux is exactly zero.

5. Condition for the half-wave transmission through the plate and the polariton spectrum of the layer with extreme values of the surface impedance

From Eq. (16) it follows that, for ω and $k_\perp$ satisfying Eqs. (14), (17), and (18), it is unimportant, whether the TM (TE) wave is bulk or evanescent in the medium in which the plate is embedded. This means that the relations (14), (17), and (18) can be fulfilled also in the case when, for given ω and $k_\perp$, the medium outside of the plate is optically less dense than the material of the plate. Let the values of the external parameters of the problem, ω and $k_\perp$, be such that, for the wave in the layer, the conditions for total internal reflection (TIR) are simultaneously satisfied on both surfaces of the AFM plate of thickness $2d$. In this case, as follows from Ref. 15, for the waveguide propagation of a TM (TE) wave, within the geometric-optic approach, the following relation should be fulfilled

$$2(\varphi^\alpha_{A+} + \varphi^\alpha_{A-}) + 2\tilde{k}_{||\alpha} d = \pm 2\nu\pi, \qquad \nu = 1, 2\ldots, \tag{30}$$

where $\varphi^\alpha_{A\pm}$ is the phase jump upon reflection for a wave incident from the AFM (denoted as "A") to the upper ("+"sign) or lower ("$-$" sign) half-space. "$\pm$" in the right-hand side of Eq. (30) is chosen depending on the presence or absence of the effect of negative phase velocity upon reflection. Thus, if, within the AFM layer under consideration, the phase shift for a bulk TM (TE) wave upon sequential reflection from the surface under TIR condition satisfies the relation

$$\varphi^\alpha_{A+} + \varphi^\alpha_{A-} = 0, \tag{31}$$

then the bulk wave (30) will have the same polariton spectrum as a half-wave plate of the same thickness under reflectionless transmission (Eqs. (14), (17), and (18)). A special case of Eq. (31) is $|\varphi^\alpha_{A+}| = |\varphi^\alpha_{A-}| = 0$. According to the terminology commonly used in the theory of metasurfaces,[2] this condition corresponds to the magnet–perfect conductor interface for a TM wave ($\tilde{Z}_p = 0$) and to the magnet–perfect magnet interface ($\tilde{Z}_s = 0$) for a TE wave. Thus, the condition for the reflectionless transmission of a wave with polarization α through a half-wave layer formally determines the spectrum of normal TM (TE) polaritons of the plate of the same thickness, provided that the boundary conditions on its both surfaces correspond to the perfect metal (in the case of a TM wave) or the perfect ferromagnet (in the case of a TE wave). It should be noted that in this case, for the wave in the layer, on both surfaces of the plate the instantaneous energy flux through the magnet–non-magnet interface is zero at any time. As a result, neglecting dissipation, the condition of the reflectionless transmission of the wave with polarization $\alpha = p, s$ through a half-wave layer can be summarized as follows: *The plate is transparent to a TM (TE) wave incident from outside if the frequency and tilt angle match the polariton spectrum of the layer, on both surfaces of which either the wave impedance (for a TM wave) or the wave admittance (for a TE wave) vanishes.* In this form, this condition can be regarded as an electromagnetic analogue of the "matching rule" that is valid for the transmission of a longitudinal elastic wave through a solid plate in a fluid.[17]

6. Fano resonance for bulk magnetic polaritons

As follows from Eq. (9) and (10), the transmission coefficient squared has the following structure outside of the TIR region:

$$|W_\alpha|^2 = \frac{1}{N_\alpha \sin^2(\tilde{k}_{||\alpha} d) + 1}, \qquad \tilde{k}_{||\alpha} d = \pi\nu, \quad \nu = 1, 2. \tag{32}$$

As a result, in the case of Eq. (14) and $N_\alpha \gg 1$, we expect to observe narrow peaks corresponding to the complete transmission of a bulk electromagnetic wave with polarization $\alpha = p, s$ on the background of almost total reflection. One of the additional mechanisms of formation of the total reflection of a bulk TM (TE) electromagnetic wave incident from outside on the AFM plate is the Fano resonance.[18,19] Physical basis for the occurrence of this effect in the considered model is the possibility of degeneracy between the modes of the spectrum of bulk magnetic TM and TE polaritons propagating along the AFM plate for the specific values of the ω–$k_\perp$ pair (see, e.g., Eqs. (17) and (18)). This degeneracy is removed in the presence of some additional disturbance either in the bulk of the plate or on its surface, which due to its symmetry will prevent independent transmission of bulk TM (TE) waves through the AFM plate in the considered magneto-optical configuration.

$$\left.\begin{pmatrix} E_z \\ H_\perp \\ H_z \\ E_\perp \end{pmatrix}\right|_d = \bar{\bar{M}} \left.\begin{pmatrix} E_z \\ H_\perp \\ H_z \\ E_\perp \end{pmatrix}\right|_0, \quad \bar{\bar{M}} \equiv \bar{\bar{M}}_0 + \delta\bar{\bar{M}}, \qquad \bar{\bar{M}}_0 \equiv \begin{pmatrix} T^s_{ik} & 0 \\ 0 & T^p_{ik} \end{pmatrix}. \tag{33}$$

Here $\bar{\bar{M}}$ and $\delta\bar{\bar{M}}$ are the full transition matrix 4×4 and the perturbation matrix which relate the components of magnetic and electric fields tangential to the interface. The vector-columns refer to the upper and lower surfaces of the plate of thickness d, $A_\perp$ is the component of the vector A collinear to the normal to the sagittal plane.

Thus, it follows from Eq. (33) that the condition to which the frequency ω and the transverse wave number $k_\perp$ of the bulk TM ($W_p = 0$) or TE ($W_s = 0$) wave incident from outside on the AFM plate should simultaneously satisfy under the discussed type of Fano resonance is defined by Eq. (15) for $\alpha = s$ or $\alpha = p$, respectively (the influence of additional perturbation is thus neglected).

7. Conclusion

In summary, if a transparent plate of a centrosymmetric AFM in the "spin-flop" phase (**q** is the normal to the plate surface) is placed into the mutually orthogonal magnetic $\mathbf{H}_0$ and electric $\mathbf{E}_0$ fields ($\mathbf{E}_0$ lies in the sagittal plane), then for a bulk TM (TE) electromagnetic wave incident on the plate from outside in the Voigt geometry the following conditions hold.

1. For a given wave frequency ω the half-wave transmission condition is non-reciprocal with respect to the sign inversion of the angle of incidence ($k_\perp \leftrightarrow -k_\perp$). In addition, for given ω and $k_\perp$, the absolute value of the phase of the transmission coefficient W_α is non-reciprocal with respect to the sign inversion of $\mathbf{E}_0\mathbf{q}$.
2. The kinematic properties of the transmission of a bulk TM (TE) wave through a half-wave layer are uniquely related to the local geometry of the cross-section by the sagittal plane of not only the SWV of the half-space, but also the SWV of the layer. For given ω and $k_\perp$ the cross-section of the SWV of the layer is defined as the half-difference of the corresponding values of (**kq**) on the cross-section of the SWV of the half-space.
3. For a given frequency ω, the transverse wave number $k_\perp$ corresponding to the center of the SWV of the layer has the property that the projection of the energy flux on the wave propagation direction is zero. At this value of $k_\perp$, the points on the SWV of the half-space have the property that the projection of the energy flux on the direction of the wave propagation along the plate is an odd function of the sign of the normal **q** to the interface.
4. For the wave transmitted through the half-wave plate, the sign and magnitude of the phase shift in W_α are determined by the Sommerfeld–Mandelstam radiation conditions and the local geometry of the cross-section of the SWV of the half-space by the sagittal plane. The sign of the phase shift upon multiple reflections in the plate is defined by the cross-section of the SWV of the layer and may change in the presence of the negative phase velocity effect (or left-handed medium conditions are realized).
5. The sign change of the group velocity (and hence the change in the wave type, forward or backward, for a wave of TM (TE) polarization in the half-wave plate) is determined by the points on the SWV of the layer for which, in the considered sagittal plane, the projection of the group velocity on the interface plane is zero. In this case, such points might not exist in the cross-section of the SWV of the half-space by the sagittal plane, or they might correspond to different values of $k_\perp$.
6. The condition of the half-wave transmission is equivalent to the dispersion relation for bulk polaritons in the considered AFM plate which has either zero surface wave impedance (for TM waves) or zero surface wave admittance (for TE waves). This condition is an electromagnetic analogue of the matching rule for an elastic wave incident from outside on a solid plate in liquid.
7. The condition for the half-wave transmission of a bulk TM (TE) wave through the plate is identical to the spectrum of a particular type of escaping bulk polaritons. For these values of the ω–$k_\perp$ pair, when a bulk TM (TE) wave is incident on one surface of the plate embedded into symmetrical surrounding, there is no reflected wave, however, there exist a bulk wave of the same polarization emanating from another surface of the plate and carrying the energy into the lower half-space. Because in this case the wave is not damped in the plate and the number of independent amplitudes is equal to the number of boundary conditions, it can be argued that the condition for the half-wave transmission through the plate corresponds to the spectrum of the same type as that of normal polaritons in the plate for the aforementioned specific type of boundary conditions.
8. There exist frequency intervals in which, for a bulk TM (TE) wave with given ω and $k_\perp$, the direction of the energy flux transported along the plate is reversed upon sign inversion of $\mathbf{H}_0$ orthogonal to the sagittal plane or upon sign inversion of $\mathbf{E}_0\mathbf{q}$.
9. For a bulk TM (TE) wave incident from outside on the AFM plate, conditions for the Fano resonance can occur: The total reflection of the incident TM (TE) wave from the plate accompanied by the resonant excitation of a propagating wave of a TE (TM) type, respectively, in the AFM plate. The required prerequisite is the presence of a weak perturbation which disturbs the condition for the independent propagation of normal magnetic polaritons of *s*- and *p*-polarization in the AFM environment.

The obtained results can be applied to plates of bianisotropic media the constraint equations of which are structurally similar to Eq. (3), while the magneto-optical configuration allows independent propagation of waves of TM and TE types. This primarily relates to magnetic and ferroelectric structures, as well as other types of "field-transforming" environments (in particular, specific types of electromagnetic metamaterials) according to the classification in Ref. 20.

In the case of EP AFM ($b < 0$ in Eq. (2)) similar features should be observed in the spectra of the considered bulk waves. Their detailed analysis (as well as the theoretical study of the Fano resonance in the AFM environment involving surface or bulk polaritons, which is controlled by external magnetic and electric fields) will be presented in a separate paper.

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