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ELECTRICAL AND MAGNETIC PROPERTIES

Fractal Model of Complex Near-Surface-Domain Structure of Highly Anisotropic Uniaxial Single Crystals

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This work proposes a new fractal model of a complex near-surface domain structure that assembles itself in highly anisotropic uniaxial single crystals, and which is based on a previously unknown modification of the Sierpinski carpet. The simulation algorithm is described and an example of its application is given.

Keywords: domain structure, self-organization, self-assembly, fractal

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INTRODUCTION

Processes observed in magnetically ordered media of purposeful (spontaneous or induced) change of symmetry, shape, or type of domain structure (DS) under the action of external factors or even in their absence are impressive examples of the self-assembly of inanimate nature. These processes can be either static, or dynamic (in this case, they are attributed to the self-organization).

HISTORY

The existence of the static self-assembly of the DS was discovered immediately after the elaboration of the first approach to domain visualization in the 1930s. This approach was called the powder-figure method or the magnetic suspension method [1–3]. The domain arrangement in magnetic materials is not random and tends to the formation of certain configurations. Numerous photographs of the DSs obtained by this method can be found in articles and subject collections of that time. Growing interest in the observation, description, and classification of various manifestations of static self-assembly brought about other methods of domain visualization in the 1960s (magnetooptical, i.e., the Kerr effect [4], the Faraday effect [5], magnetic birefringence [6], electron microscopy [7, 8], and Hall detectors [9], etc.). Additional developments in the study of this problem were the experiments on dynamic self-organization carried out by Urals physicists Kandaurova G.S. and Svidersky A.E. They revealed the transformation of the labyrinthic DS into spiral and ring domains under the action of external low-frequency magnetic field on thin quasi-uniaxial epitaxial films of garnet ferrite [10, 11]. They later showed that under similar conditions, other types of dynamically induced DSs can be observed [12, 13]. The formation of the strictly ordered arrays of bubble magnetic domains with a generating line in the form of ellipse, boomerang, figure of eight, dumbbell, and etc. was observed by authors of [14–17] via the special choice of the parameters of the oscillating magnetic field.

An analysis of numerous patterns which occur in the course of the static and dynamic self-assembly of the DS, demonstrates that a number of cases are characterized by the presence of fragments with hierarchical structure and approximate (topological) self-similarity, which allows their description within the fractal approach. The formation of such units, which can be observed in various structured media in the case of the diffusive character of motion of their parts, is called the fractal clustering (see, e.g., [18]). The size of fragments, which is affected by numerous factors including the degree of uniformity of the magnetic material and the quality of surface on which the DS is observed, can vary within a significant range. The search for the complete analogy with the known types of geometrical fractals is usually unsuccessful; therefore, in all the cases under consideration, the term “fractal-like DSs” should be used as in [19].

If the classification of abstract fractals (geometrical, algebraic, and stochastic) uses the principle of personification (the Koch snowflake, the Sierpinski carpet, the Mandelbrot set, etc.); usually, in the case of the fractal-like DSs, the descriptive approach is employed, upon which they are classified as dendrite, mosaic, netlike, rhombic, tapered, zigzag, branching, and other configurations. Adequate models and algorithms of construction are necessary to study the practical usage of these structures.

MODEL DESCRIPTION

The configuration which is developed in the course of the self-assembly of the DS can be rather complicated, and so these models are usually created on the basis of well-known geometrical fractals, e.g., the Sierpinski carpet [20, 21]. Different modifications of Sierpinski carpet are often employed in this case, which is greatly favored by the fact that these fractal structures are applied in magnon, phonon, and photon crystals for micro- and nanoelectronics [22–24]. Models of clusters with the structure of the Sierpinski carpet were used to estimate the effect of fractal dimension on magnetization curves of materials composed of exchange-coupled nanoparticles with randomly oriented easy axes of magnetization [25].

To describe the perfect rhombic shape of closure domains on the (010) plane of FeB single crystals, the authors of [26] employed the model shown in Fig. 1. Essentially, this model is the parquet-type tiling in the chesslike order of two types of subfractals which may be regarded as a modification of the Sierpinski carpet. The adjacent subfractals of the two types transform into each other upon the color inversion.

For the simulation of the complex near-surface DS with “stars”, which is often observed in highly anisotropic uniaxial single crystals (Fig. 2), we suggest using the square-shaped Sierpinski carpet with a single type of deformation of boundaries of all the square elements in each generation of a fractal. Other examples can be found in review [12] and the works of Pastushenkov et al. [28].

The creation of the model can be divided into two stages. At the first stage, an intermediate modification of the common carpet is constructed by the algorithm shown in Fig. 3. The stage sequence of the subfractal formation of the first three generations of the common Sierpinski carpet is shown in the upper row. At the first step, the large black square with a dimension of L is divided into nine equal ones with dimensions of $L/3$, and the central square is removed; further, this procedure is performed for each of the eight black squares which form the boundary of the central “white” square which remains unchanged. The procedure is repeated for 64 black squares with dimensions of $L/9$, and etc.

The formation of the intermediate modification of the carpet (see the bottom row of Fig. 3) is also started from the large black square with a dimension of L , and the central square with a dimension of $L/3$ is removed; however, at the second step, contrary to the common Sierpinski carpet, the black square with a dimension of $L/9$ is inserted at the center of the geometrical figure. At the third step (last image at the bottom row), the square is divided by a mesh with the step of $L/9$ into 81 squares with the subsequent positioning at the center of each of them of a square with a dimension of $L/27$ and opposite color (black into white and vice

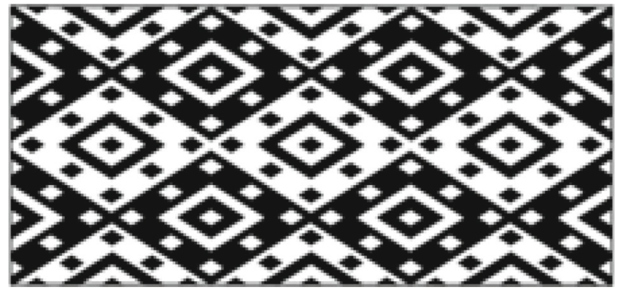

Fig. 1. Model of the structure of the closure domains on the (010) plane of the rhombic FeB single crystal [26].

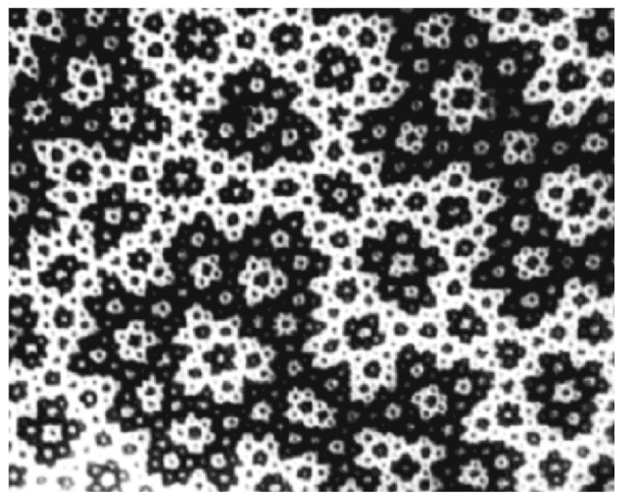

Fig. 2. Complex near-surface DS with “stars” on the basis (001) plane of the $ErFe_{11}Ti$ single crystal [27].

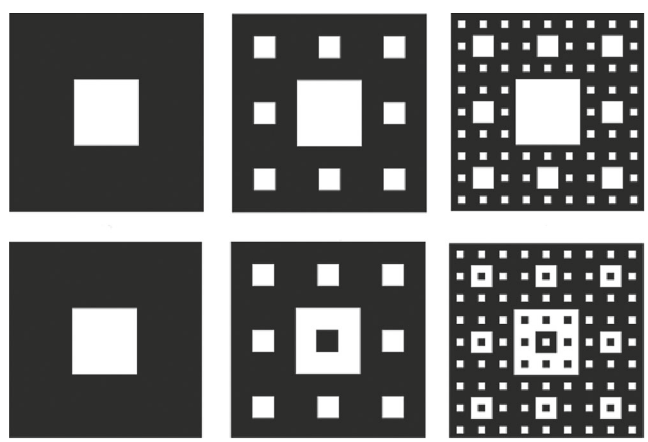

Fig. 3. Subsequent stages of the formation of the initial generations of the subfractals of the classical (top row) and modified (bottom row) Sierpinski carpet.

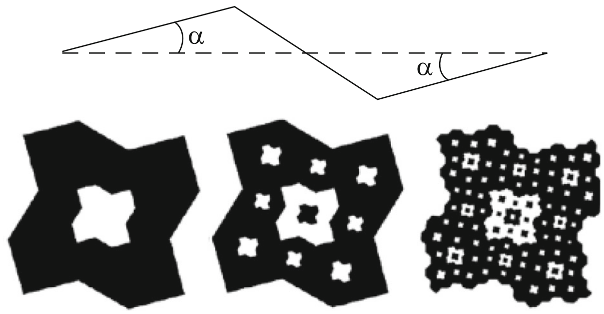

Fig. 4. Results of the replacement of the lines by the triple-portion broken lines for three generations of subfractals of the modified Sierpinski carpet.

versa). This process can be infinitely repeated. To iterate the suggested simulation algorithm of the chosen DS, the generations of the described modified Sierpinski carpet should be counted from the second step.

This carpet has all the characteristics of the fractal, i.e. upon changing its scale by a factor of three, both its shape and the diffraction pattern in the distant region remain the same. Contrary to the common Sierpinski carpet, which is a connected topologic set, this modified carpet is not connected. Its Hausdorff dimension is equal to 2 (for the Sierpinski carpet, it is $\ln 8/\ln 3 = 1.89$), and the scaling coefficients of both fractals are equal to 3.

At the second stage of model formation, to imitate the real shape of the domains, at each step of the construction of the modified Sierpinski carpet, sides of all squares (white and black) are replaced by symmetrical triple-portion broken lines with equal length of portions (top of Fig. 4). It is easy to show that the *k* ratio of the length of the portions of the broken line to the length of the replaced side of the square is determined as follows:

$$k = \left(4\cos\alpha - \sqrt{16\cos^2\alpha - 12}\right)\Big/6.$$

Here, α is an angle between the outer portions of the broken curve and the side of the square, which cannot exceed 30°. Further, we used the following value $\alpha = 15°$. By application of the described procedure to the intermediate modification of the carpet (bottom row in Fig. 3), for the anticlockwise treatment of the square sides, we obtained the structures depicted in the bottom row of Fig. 4.

The replacement of all portions of the lines obtained at the first stage of the modified Sierpinski carpet by triple-portion broken lines results in the transformation of both the outer border and all boundaries between "black" and "white" inner elements of the topologic set into fractal curves in the shape of closed multi-portion broken lines (see Fig. 4). It is necessary to note that this modification of the Sierpinski carpet is new and has never been described in literature before now.

CONCLUSIONS

The set of modified carpets connected into a chain or forming a chesslike order with the carpets of opposite "polarity" can be regarded as a rough but sufficiently adequate model of the complex DSs. The completely new fractal model suggested here is flexible and helps to achieve the maximal resemblance of the simulated image to the real shape of the simulated DSs by varying the α angle and generation number of the modified Sierpinski carpet. In this case, it is necessary to take into account that any geometrical model which uses non-interacting objects in an "empty" space cannot completely reproduce the behavior of the real fractals the constituents of which usually interact and affect each other. That is why the usage of the higher generations of subfractals for the simulation shifts the inner regions and changes their sizes.

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