Josephson effect in superconducting constrictions with hybrid SF electrodes: peculiar properties determined by the misorientation of magnetizations

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Josephson current in SFcFS junctions with arbitrary transparency of the constriction (c) is investigated. The emphasis is done on the analysis of the supercurrent dependencies on the misorientation angle θ between the in-plane magnetizations of diffusive ferromagnetic layers (F). It is found that the current-phase relation $I(\varphi)$ may be radically modified with the θ variation: the harmonic $I_1 \sin \varphi$ vanishes for definite value of θ provided for identical orientation of the magnetizations ($\theta = 0$) the junction is in the "π" state. The Josephson current may exhibit a nonmonotonic dependence on the misorientation angle both for realization of "0" and "π" state at $\theta = 0$. We also analyze the effect of exchange field induced enhancement of the critical current which may occur in definite range of θ.

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The dc Josephson effect in junctions with ferromagnetic interlayers (F) exhibit exchange field induced remarkable features, namely, the transition from a "0" state to a "π" state of junctions [1, 2], enhancement of the critical Josephson current [3, 4] and others. These features have been intensively studied in recent years theoretically and experimentally (see Reviews [5 − 7] and Refs. therein). In spite of a lot of works devoted to this problem there are several important ones which require additional investigation. The purpose of the present work is the study of one of these problems. Namely we investigate the Josephson current and peculiar properties of SFcFS junctions (c denotes a constriction) with arbitrary misorientation angle θ between the in-plane oriented magnetizations of the ferromagnetic metal (F) layers. This investigation is important because the supercurrent is very sensitive to the mutual orientation of magnetizations and, besides, because the variation of θ gives additional opportunity to switch experimentally between the "0" and "π" states. In the considered junctions the constriction is an insulator characterized by the arbitrary transparency or short (in comparison with the coherence length) diffusive metal channel. For the F layers the dirty limit, i.e. diffusive regime of electron transport is assumed. Note that for the case of ballistic transport through the F layers with arbitrary misorientation angle θ between their magnetizations the Josephson current for full transparent S-FNF-S junctions was analyzed by Waintal and Brouwer [8] and

for quantum S-FIF-S contacts this problem was studied by Barash et al. [9]. In the present paper we investigate the angle θ-dependence of different peculiar properties of the supercurrent in SFcFS junctions. We show that variation of θ may result in the transition between the "0" and "π" states and the disappearance of $I_1 \sin \varphi$ harmonic of the supercurrent for certain value of θ. Effect of exchange field induced enhancement of the critical current and its nonmonotonic dependence on the angle θ are also studied.

As a model of the constriction, we consider an aperture of a small radius in a thin impenetrable screen dividing two different electrodes. The constriction is supposed to include a barrier with the transparency $D = D(\vartheta)$, where ϑ is the angle between the electronic trajectory and normal to the junction plane. Consider first the case of ballistic constriction of small size assuming that its radius is small in comparison with $l_F, v_F/\Delta, d_F$, where l_F and v_F, are the mean free path and Fermi velocity in the F layers, d_F is their thickness, Δ is the energy gap in the superconductors. Under these assumptions the current may be expressed through the Matsubara Green's functions of the F layers $\hat{G}_{1,2}$ as follows [9 − 11]:

$$I(\varphi) = \frac{8\pi T}{eR_0} \sum_{n=0}^{\infty} \langle J(\omega_n) \rangle, \tag{1}$$

$$J = \frac{D}{4} \mathrm{Tr} \hat{\sigma}_0 \otimes \hat{\tau}_3 \frac{\hat{G}_- \hat{G}_+}{(1 - D)\hat{1} + D\hat{G}_+^2} =$$

$$= \frac{D}{4} \mathrm{Tr} \hat{\sigma}_0 \otimes \hat{\tau}_3 \frac{[\hat{G}_2, \hat{G}_1]_-}{2(2 - D)\hat{1} + D[\hat{G}_2, \hat{G}_1]_+},$$

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where $\omega_n = \pi T(2n+1)$ is the Matsubara frequency, $\hat{G}_\pm = (\hat{G}_2 \pm \hat{G}_1)/2$, $[\hat{G}_2, \hat{G}_1]_\pm = \hat{G}_2\hat{G}_1 \pm \hat{G}_1\hat{G}_2$, $R_0 = = 4\pi^2\hbar^3/e^2 p_F^2 A$ is the Sharvin resistance of the contact, A being the area of the contact, angular brackets denote angular-averaging $\langle(...)\rangle = 2\int_0^1 (...)\cos\vartheta \, d\cos\vartheta$, $\hat{\sigma}_k$ and $\hat{\tau}_k$ are the Pauli ($k=1,2,3$) or unite ($k=0$) matrices in the spin and particle-hole spaces, respectively, $\hat{1} = = \hat{\tau}_0 \otimes \hat{\sigma}_0$. As was noted above we consider the case of dirty thin F layers, i.e. their thickness is supposed to satisfy the conditions $l_F \ll d_F \ll (\hbar D_F/T_c)^{1/2}$ where D_F is the diffusive coefficient in the F layers, T_c is the critical temperature of the superconductors. For such layers the space variation of the Green's functions is negligible therefore they are given by the expression $\hat{G}_j = \exp(i\varphi_j\hat{\rho}_3/2)\hat{g}_j\exp(-i\varphi_j\hat{\rho}_3/2)$, where $\hat{\rho}_3 = \hat{\tau}_3 \otimes \hat{\sigma}_0$, φ_j is the phase of the order parameter of the jth S-electrode and $\hat{g}_j$ is determined by the following equation (for details of the derivation see, e.g., [13]):

$$[\hat{g}_j, \omega\hat{\rho}_3 + i\mathbf{h}_j\hat{\mathbf{S}} + \varepsilon_b\hat{g}_S]_- = \hat{0} \qquad (2)$$

where $\varepsilon_b = \hbar\langle D_b\rangle v_F/4d_F = D_F/2R_b\sigma_F d_F$, D_b and R_b are, respectively, the transparency and the resistance per unite area of the S-F interface, σ_F is the conductivity of the F metal, $\hat{g}_S$ is the Green's function of the superconductors at the S-F interfaces, $\mathbf{h}_{1,2}$ are the exchange fields in the F layers, the matrix vector $\hat{\mathbf{S}}$ is defined as [14, 15] $\hat{\mathbf{S}} = \hat{\sigma} \otimes \hat{\tau}_+ - \hat{\sigma}_2\hat{\sigma}\hat{\sigma}_2 \otimes \hat{\tau}_-$, where $\hat{\sigma} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$, $\hat{\tau}_\pm = (\hat{\tau}_0 \pm \hat{\tau}_3)/2$. Note that the value of the exchange fields, $|\mathbf{h}_{1,2}| = h$ is supposed to be small with respect to $\hbar/\tau_F$ and the Fermi energy of the F layers, where $\tau_F = l_F/v_F$; for considered here dirty layers $\hbar/\tau_F \gg \Delta$. We assume for simplicity that the S-F interfaces are identical and that their transparency D_b is small, therefore the effect of the F layers on the superconductors is negligible. The only difference between the F layers is related with the orientations of the exchange fields, $\mathbf{h}_{1,2}$, which being parallel to the layers, make an angle θ with each other. In this paper we confine ourselves to the case of conventional s-wave superconductors. Therefore to carry out further calculations it is convenient to represent the current with the use of the transformed Green's functions $\hat{G}_j = \exp(i\varphi_j\hat{\rho}_3/2)\hat{g}_j\exp(-i\varphi_j\hat{\rho}_3/2)$ where

$$\hat{g}_j = \hat{U}\hat{g}_j\hat{U}^{-1}, \qquad (3)$$

$\hat{U} = (\hat{\sigma}_0 \otimes \hat{\tau}_+ + \hat{\sigma}_2 \otimes \hat{\tau}_-)$. Taking into account that particle (hole) projection matrices, $\hat{\tau}_{+(-)}$ obey the relation $\hat{\tau}_\alpha\hat{\tau}_\beta = \hat{\tau}_\alpha\delta_{\alpha\beta}$ it is easy to check that $\hat{U}$ is the unitary Hermitian matrix, $\hat{U} = \hat{U}^{-1} = \hat{U}^+$. From (3), (2) one can find the representation for $\hat{g}_j$ with the help of the projection matrices in the spin space, $\hat{P}_\pm(\mathbf{n}_j) = \frac{1}{2}(\hat{\sigma}_0 \pm \mathbf{n}_j\hat{\sigma})$,

where the unite vectors $\mathbf{n}_j = \mathbf{h}_j/|\mathbf{h}_j|$. Taking into account the relation $\hat{P}_\alpha(\mathbf{n})\hat{P}_\beta(\mathbf{n}) = \hat{P}_\alpha(\mathbf{n})\delta_{\alpha\beta}$, we get

$$\hat{g}_j = \sum_{\alpha=\pm} \hat{g}_\alpha \otimes \hat{P}_\alpha(\mathbf{n}_j) =$$
$$= \frac{1}{2}(\hat{g}_+ + \hat{g}_-) \otimes \hat{\sigma}_0 + \frac{1}{2}(\hat{g}_+ - \hat{g}_-) \otimes \mathbf{n}_j\hat{\sigma}, \qquad (4)$$

where $\hat{g}_\alpha = g_\alpha\hat{\tau}_3 + f_\alpha\hat{\tau}_2$, $g_\alpha^2 + f_\alpha^2 = 1$,

$$g_\alpha = \bar{\omega}_\alpha/\xi_\alpha, \quad f_\alpha = \Delta_F/\xi_\alpha, \qquad (5)$$

$\xi_\alpha = \sqrt{\bar{\omega}_\alpha^2 + \Delta_F^2}$, $\bar{\omega}_\alpha = \omega + i\alpha h + \varepsilon_b g_S(\omega)$, $\Delta_F = = \varepsilon_b f_S(\omega)$. The form of the Green's function representation (4) does not depend on the specific expressions for $\hat{g}_\alpha$; it is valid for spatially-homogeneous orientation of the exchange fields in the F layers. Note that being written via the transformed matrices $\hat{G}_j$, the expression for the current coincides with (1). The advantage of the representation (4) is related with significant simplification of the matrix structure of the Green's functions which are given by sum of two terms determined by the direct product of matrices in the particle-hole and spin spaces. With the help of (4) we get

$$[\hat{G}_2, \ \hat{G}_1]_\pm = \frac{1}{2}\sum_{\alpha,\beta=\pm} [\hat{G}_{2\alpha}, \hat{G}_{1\beta}]_+ \otimes [\hat{P}_\alpha(\mathbf{n}_2), \hat{P}_\beta(\mathbf{n}_1)]_\pm +$$
$$+ [\hat{G}_{2\alpha}, \hat{G}_{1\beta}]_- \otimes [\hat{P}_\alpha(\mathbf{n}_2), \hat{P}_\beta(\mathbf{n}_1)]_\mp, \qquad (6)$$

where $\hat{G}_{j\alpha} = g_\alpha\hat{\tau}_3 + \exp(i\varphi_j\hat{\tau}_3)f_\alpha\hat{\tau}_2$. Taking into account that

$$[\hat{P}_\alpha(\mathbf{n}_2), \hat{P}_\beta(\mathbf{n}_1)]_+ = \frac{1}{2}[1 + \alpha\beta\mathbf{n}_2\mathbf{n}_1 + (\alpha\mathbf{n}_2 + \beta\mathbf{n}_1)\hat{\sigma}],$$
$$[\hat{P}_\alpha(\mathbf{n}_2), \hat{P}_\beta(\mathbf{n}_1)]_- = \frac{i\alpha\beta}{2}[\mathbf{n}_2 \times \mathbf{n}_1]\hat{\sigma}, \qquad (7)$$

after some calculations we obtain the following expression for the current

$$I(\varphi) = \frac{8\pi T\sin\varphi}{eR_0} \times$$
$$\times \sum_{n=0}^\infty \left\langle \frac{A(\varphi)}{B_0 - [A_0 + A(\varphi)]\sin^2\left(\frac{\varphi}{2}\right)} \right\rangle \equiv \langle \mathrm{I}(\varphi, D)\rangle, \qquad (8)$$

where $\varphi = \varphi_2 - \varphi_1$, $A(\varphi) = A_0 + A_1\sin^2\left(\frac{\varphi}{2}\right)$,

$$A_0 = 2D\left[2\cos^2\left(\frac{\theta}{2}\right)\mathrm{Re}\,f^2 + \sin^2\left(\frac{\theta}{2}\right)|f|^2(2 + Dq)\right],$$
$$A_1 = -4D^2|f|^4, \quad B_0 = \left[2 + D\sin^2\left(\frac{\theta}{2}\right)q\right]^2,$$
$$q = |g|^2 + |f|^2 - 1,$$

here $g \equiv g_+(\omega_n)$, $f \equiv f_+(\omega_n)$. Eq. (8) is significantly simplified for the cases of parallel (p) and antiparallel (a) magnetizations:

$$I = \frac{8\pi T \sin\varphi}{eR_0} \times$$

$$\times \sum_{n=0}^{\infty} \begin{cases} \mathrm{Re}\left\langle Df^2/[1 - Df^2 \sin^2\left(\tfrac{\varphi}{2}\right)]\right\rangle, \ p \\ \left\langle 2D\left|f\right|^2/[2 + D(|g|^2 + |f|^2 \cos\varphi - 1)]\right\rangle, \ a \end{cases} \cdot (9)$$

Note that for the case $\theta = 0$ Eq.(9) reduces to the result obtained in Ref.[16]. In general case the misorientation angle dependence of the current is rather nontrivial if the transparency D is not small: it does not reduce to the expression (if $\mathbf{h}_1 \not\parallel \mathbf{h}_2$) [17]

$$I(\varphi) = I^{(p)}(\varphi)\cos^2\left(\frac{\theta}{2}\right) + I^{(a)}(\varphi)\sin^2\left(\frac{\theta}{2}\right). \quad (10)$$

Simple form for the θ-dependence (10) is valid only in the limit of small transparencies (when $I^{(p,a)}(\varphi) = I^{(p,a)}\sin\varphi$); for this case it was first obtained in Ref.[3].

For numerical analysis of the Josephson current we apply the obtained results assuming that superconductors are described by the BCS theory therefore $g_S(\omega) = \omega/(\omega^2 + \Delta^2)^{1/2}$, $f_S(\omega) = \Delta/(\omega^2 + \Delta^2)^{1/2}$. The results of numerical calculations of $I(\varphi)$-dependences for a junction with the transparency $D = 0.5$ are presented in Fig.1 for a set of values of θ. It shows that the function
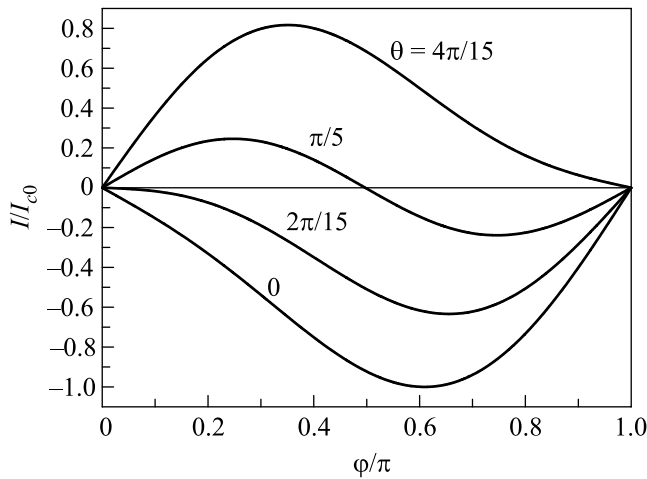

Fig.1. Current-phase relations at various misorientation angle θ for a SF-I-FS junction with the transparency of the barrier $D = 0.5$ at $h/\Delta = 1.5$, $T/\Delta = 0.15$, $\varepsilon_b = \Delta$. For $\theta \leq \theta_{0\pi} = \pi/4$ ($\theta_{0\pi} < \theta \leq \pi$) the junction is in the "π" state ("0" state). For θ in the vicinity of $\theta_{0\pi}$ second harmonic $I_2\sin(2\varphi)$ of the current dominates, first harmonics $I_1\sin\varphi$ vanishes at $\theta = \theta^* \approx \theta_{0\pi}$

$I(\varphi)$ may strongly deviate from the $\sin\varphi$-dependence and is strongly modified with increasing angle θ, especially in the vicinity of misorientation angle $\theta = \theta_{0\pi}$ (temperature $T_{0\pi}$) which corresponds to the transition

between the "0" and "π" states of junction. Note that the model with the angle-independent transparency corresponds to the case of quantum single-mode constriction with the resistance $R_0 = \pi\hbar/e^2$. At definite $\theta = \theta^*$ (temperature T^*) which with the accuracy higher than 5 percents coincides with the value $\theta_{0\pi}$ ($T_{0\pi}$) the harmonic $I_1\sin\varphi$ of the current ($I(\varphi) = I_1\sin\varphi + I_2\sin(2\varphi) + ...$) vanishes; the sign of I_1 is changed with the variation of θ (or temperature) near θ^* (T^*). Living the detailed analysis of supercurrent temperature dependences for a separate presentation we show here the typical phase diagram of the junction (Fig.2) for not too small exchange
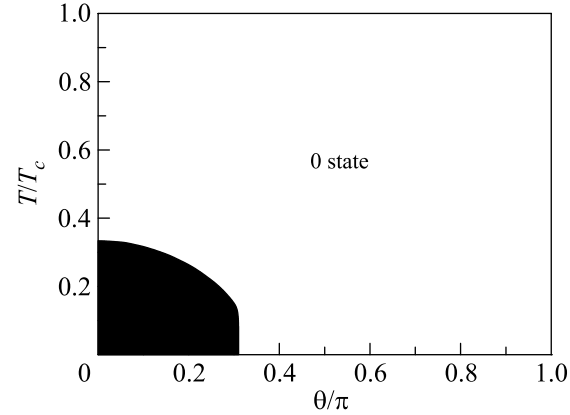

Fig.2. The (T, θ) phase diagram of a SF-I-FS junction with the transparency of the barrier $D = 0.5$ and $h/\Delta = 1.5$, $\varepsilon_b = \Delta$. Black region corresponds to the "π" state of the junction
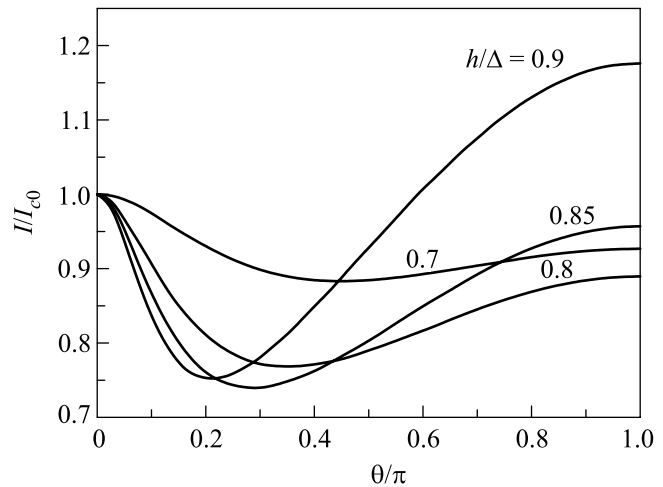

Fig.3. Critical current as a function of the misorientation angle θ normalized to its value at $\theta = 0$ for a SF-I-FS junction with the transparency of the barrier $D = 0.5$; $\varepsilon_b = \Delta$, $T/\Delta = 0.01$; these dependences correspond to "0" state of the junction

fields (in comparison with ε_b). Another interesting property of the supercurrent is nonmonotonic dependences

of the critical current $I_c = \max |I(\varphi)|$ as a function of the angle θ. Note that a similar behavior of the critical current for quantum S-FIF-S junctions with the ballistic transport through the F layers has been analyzed in [9]. We find that in junctions with diffusive F layers the nonmonotonic dependence $I_c(\theta)$ (Fig.3) may occur for realization of both "π" and "0" state for identical orientation of the magnetizations (unlike the case studied in [9]). The results of numerical calculations of the $I_c(h)$ dependences for a low transparency junction at various angle θ are presented in Fig.4. It shows that the ratio
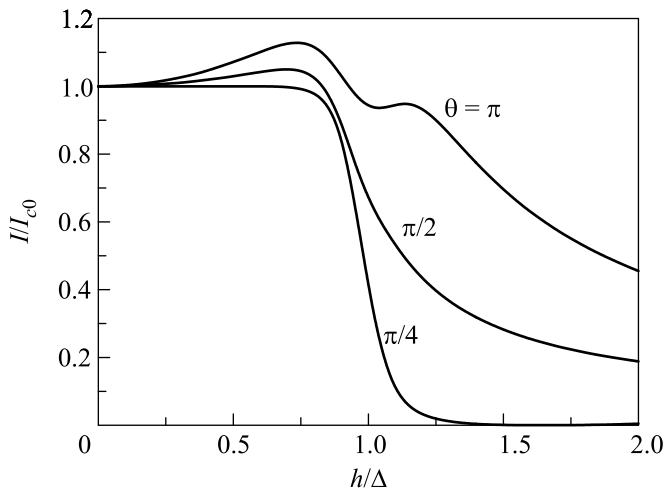

Fig.4. Exchange field dependence of the critical current $I_c(h)$ (normalized to $I_c(0) \equiv I_{c0}$) at various misorientation angle θ for SF-I-FS junction with the transparency of the barrier $D = 0.1$ at $T/\Delta = 0.01$, $\varepsilon_b = \Delta$

$I_c(h)/I_c(0)$ may exceed unity, i.e., the effect of the exchange field induced enhancement of the critical current [3, 4] may occur in some range of h if θ exceeds definite value ($\pi/4 < \theta \le \pi$ for small D).

For the case of SFcFS junctions with a short diffusive constriction the following expression for the current may be obtained with the use of the Green's function technique [11]

$$I(\varphi) = \int_0^1 \rho(D)\mathrm{I}(\varphi, D)dD, \qquad (11)$$

where $\mathrm{I}(\varphi, D)$ is determined by (8) with $R_0 = \pi\hbar/e^2$ being the resistance of a single mode constriction, and the distribution of transparencies is determined by the function $\rho(D) = (R_0/2R_N)/D(1 - D)^{1/2}$ (R_N is the normal state resistance of the constriction) which coincides with the density function found by Dorokhov [19].

For the cases of parallel and antiparallel magnetizations Eqs.(8), (11) reduce to

$$I(\varphi) = \frac{16\pi T \sin\varphi}{eR_N} \sum_{n=0}^{\infty} \begin{cases} \mathrm{Re}\, F^2 W \left(1 - 2f^2 \sin^2\left(\frac{\varphi}{2}\right)\right), & p \\ |F|^2\, W(|g|^2 + |f|^2 \cos\varphi), & a \end{cases}$$

$$(12)$$

where $W(x) = (1 - x^2)^{-1/2} \arctan[(1 - x)/(1 + x)]^{1/2}$. Note that in the case $\theta = 0$ Eq.(12) reduces to the result obtained in Ref. [16]; for $h = 0$ Eqs.(12) coincide and in the limit $\varepsilon_b \gg \Delta$ reproduce the Kulik-Omelyanchuk formula [20] for a diffusive ScS constriction. Numerical calculations show (Fig.5) that the above discussed pe-
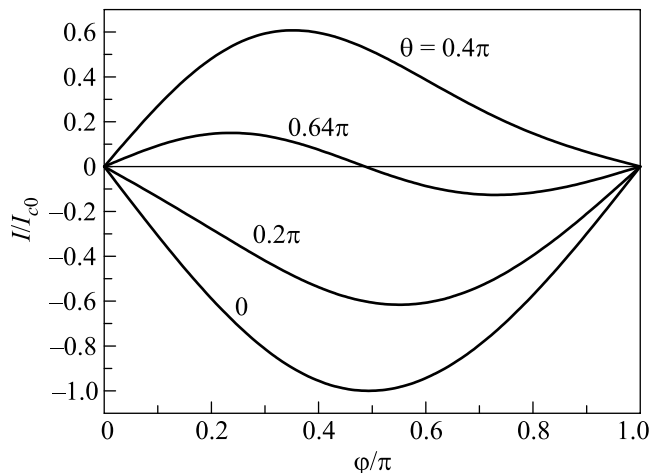

Fig.5. Current-phase relations at various θ for a diffusive SF-c-FS junction at $h/\Delta = 1.5$, $T/T_c = 0.1$, $\varepsilon_b = \Delta$. For $\theta \le \theta_{0\pi} = 0.64\pi$ ($\theta_{0\pi} < \theta \le \pi$) the junction is in the "π" state ("0" state). For θ in the vicinity of $\theta_{0\pi}$ second harmonic $I_2 \sin(2\varphi)$ of the current dominates, first harmonics $I_1 \sin\varphi$ vanishes at $\theta = \theta^* \approx \theta_{0\pi}$

culiar properties manifests itself also in junctions with diffusive constriction.

In conclusion, we have developed a microscopic theory of supercurrent in SFcFS junctions with the arbitrary transparency of the constriction and diffusive electron transport in the F layers for arbitrary angle θ between the in plane exchange fields in the F layers. We showed that the current-phase dependence may be strongly modified with the variation of θ. If the junction is in the "π" state for identical orientation of magnetizations ($\theta = 0$), the variation of θ may result in the transition between the "0" and "π" states and disappearance of the $I_1 \sin\varphi$ harmonic of the supercurrent at definite value of θ. Nonmonotonic dependence of the critical current as a function of θ may occur for both "π" and "0" state of junction corresponding to the case of identical orientation of the magnetizations if the transparency of the junction is not small.

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1. L. N. Bulaevskii, V. V. Kuzii, and A. A. Sobyanin, Pis'ma v ZhETF **25**, 314 (1977) [JETP Lett. **25**, 290 (1977)].

2. A. I. Buzdin, L. N. Bulaevsky, and S. V. Panyukov, Pis'ma v ZhETF **35**, 147 (1982) [JETP Lett. **35**, 178 (1982)].

3. F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Phys. Rev. Lett. **86**, 3140 (2001).

4. F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Rev. Mod. Phys. **77**, 1321 (2005).

5. A. A. Golubov, M. Yu. Kupriyanov, and E. Il'ichev, Rev. Mod. Phys. **76**, 411 (2004).

6. V. V. Ryazanov, V. A. Oboznov, A. S. Prokof'ev et al., J. Low Temp. Phys. **136**, 385 (2004).

7. A. I. Buzdin, Rev. Mod. Phys. **77**, 935 (2005).

8. X. Waintal and P. W. Brouwer, Phys. Rev. B **65**, 054407 (2002).

9. Yu. S. Barash, I. V. Bobkova, and T. Kopp, Phys. Rev. B **66**, 140503(R) (2002).

10. A. V. Zaitsev, Zh. Eksp. Teor. Fiz. **86**, 1742 (1984) [Sov. Phys. JETP **59**, 1015 (1984)].

11. A. V. Zaitsev and D. V. Averin, Phys. Rev. Lett. **80**, 3602 (1998).

12. Yu. V. Nazarov, Superlattices and Microstructures **25**, 1221 (1999).

13. A. F. Volkov, A. V. Zaitsev, and T. M. Klapwijk, Physica C **210**, 21 (1993).

14. K. Maki, in: *Superconductivity*, Ed. R. Park, Marcel Dekker, New York, 1969, p. 1035.

15. J. A. X. Alexander, T. P. Orlando, D. Rainer, and P. M. Tedrow, Phys. Rev. B **31**, 5811 (1985).

16. A. A. Golubov, M. Yu. Kupriyanov, and Ya. V. Fominov, Pisma v ZhETF **75**, 709 (2002) [JETP Lett. **75**, 588 (2002)].

17. Note that in Ref.[18] the Josephson current in $S_F c S_F$ junctions formed by two "ferromagnetic superconductors" (S_F) was studied on the basis of the Green's function approach. The authors of this work arrived at the conclusion about the validity of formula (10) for arbitrary transparency D of the barrier in the constriction. In reality this conclusion is not true if $\mathbf{h}_1 \nparallel \mathbf{h}_2$. The origin of the erronious conclusion is related with that in the process of the derivation of the expression for the current in Ref.[18] the authors used the identity (Eq.(5) of Ref.[18]) which is not applicable for functions determined by the partitioned matrices. It becomes clear from Eqs.(6), (7) which show that the expressions for $[\hat{G}_2, \hat{G}_1]_\pm$, and, as a consequence, the current depend not only on the scalar product $(\mathbf{h}_1 \mathbf{h}_2)$ but also on the vector product $[\mathbf{h}_1 \times \mathbf{h}_2]$.

18. N. M. Chtchelkatchev, W. Belzig, and C. Bruder, Pisma Zh. Eksp. Teor. Phys. **75**, 772 (2002) [JETP Lett. **75**, 646 (2002)].

19. O. N. Dorokhov, Solid State Commun. **51**, 381 (1981).

20. I. O. Kulik and A. N. Omelyanchuk, Zh. Eksp. Teor. Fiz. **68**, 2139 (1975) [Sov. Phys. JETP **41**, 1071 (1975)].