# Arrays of Josephson Junctions Coupled by Distributed Circuits

Victor K. Kornev, Nikita A. Shcherbakov, Alexey V. Arzumanov, Peter B. Mozhaev, Alexandre D. Mashtakov, Karen Y. Constantinian, Gennady A. Ovsyannikov

Abstract—Both the linewidth reduction and the gain in output power have been analyzed for Josephson-junction arrays with distributed coupling circuits. Different ways of Josephson junction connection to the distributed circuits, as well as both the nonlinear interaction effect and the crucial role of high frequency losses have been studied. It has been found that the contradictory requirements to the junctions as nonlinear active elements on (i) low wave reflection and (ii) high total power produced can be fulfilled by an increase in number of Josephson junctions with a proper impedance value. The first condition is necessary to provide the most reduction in the linewidth, and the second one is needed to compensate for the power output. An array of the biased in parallel bi-crystal Josephson HTSC junctions coupled by microstrip circuit has been fabricated and preliminary tested.

*Index Terms*\_Arrays, Josephson junctions, distributed parameter systems, oscillators.

## I. INTRODUCTION

The elaboration of promising narrow-linewidth Josephson oscillators for sub-millimeter and near infrared diapasons

is an actual problem. These devices can be used as heterodyne oscillators of high-sensitive integrated receivers for radio astronomy and environmental control systems. Spectral lines of light molecules, which are the subject of the environmental control, as well as atmospheric windows are ranged at frequencies about 500 GHz and higher. There are two most effective ways for the oscillator elaboration.

## A. Flux Flow Oscillators

Josephson flux flow oscillators (FFO) are now in the lead. They show free-running linewidth of a few MHz or even a few hundred kHz at operation frequency ~ 500 GHz [1], [2]. The specially designed feed back circuit (external electronic phase-locking loop) is able to reduce the linewidth of the oscillator down to 1 Hz [3]. Nevertheless there are the drawbacks of the systems as follows: low characteristic resistance, low output power, and the limitation on the oscillation frequency about 600 GHz imposed by the low-Tc-

Manuscript received September 17, 2000. This work was supported in part by INTAS Program of EU under Grant No. INTAS-OPEN-97-1940, Russian State Programs "Condensed Matter" and "Integration", and Education Ministry Program.

V. K. Kornev, N. A. Shcherbakov, A. V. Arzumanov are with the Physics Department, Moscow State University, Moscow, 119899, Russia (telephone: 7-095-939-3000, e-mail: vkorn@cryo.phys.msu.su).

P. B. Mozhaev, A. D. Mashtakov, K. Y. Constantinian, G. A. Ovsyannikov are with Institute of Radio Engineering and Electronics, RAS, Moscow, 103907, Russia (telephone: 7-095-203-0935, e-mail: gena@hitech.cplire.ru). superconductor energy gap [3].

## B. Josephson Junction Array Oscillators

To overcome these problems, the oscillators based on Josephson junction arrays could be employed. The devices are able to provide not only high values of characteristic resistance and output power, but also the operating frequency up to several THz in the case of high- $T_c$  superconductors with high values of energy gap. There is good reason to believe that the external electronic phase-locking loop, which is similar in design to the one used in FFO systems, could also reduce the oscillation linewidth down to a few Hz if the free-running linewidth of the arrays would be provided within a few MHz by the phase-locking phenomenon.

The first theoretical investigations of the mutual phaselocking in the lumped unit cells of Josephson junction arrays were carried out analytically for the case of weak Josephsonjunction coupling and negligibly small intrinsic capacitance [4]. The investigations predicted the oscillation linewidth reduction with number N of junctions to be proportional to N or even to N<sup>2</sup> for some special cases. Numerical simulations of the cells showed the maximum phase-locking effect just for the case of the coupling circuits that provide strong non-local interaction and McCumber parameter value  $\beta$  of about unit or a few units [5]. The further development of the numerical analysis technique yielded in effective ways of Josephson oscillation spectrum calculation [6].

Unfortunately the spectral study for the promising arrays has shown the existing of linewidth reduction with number N only within a coupling radius, i.e. up to characteristic number  $N_c$  of the junctions [7]. It means the existence of fundamental limitation for the linewidth reduction by the mutual phase-locking phenomenon. Therefore it is very unlikely for the real Josephson junction arrays with lumped coupling circuits to provide the linewidth reduction by factor of  $10^2$  or more.

An additional possibility to reduce the oscillation linewidth could be obtained by means of the phase-locking of Josephson junctions caused by the standing electromagnetic waves exited in distributed coupling circuits.

#### **II.** THEORETICAL STUDY

Josephson junction arrays with distributed coupling circuits have been studied by means of numerical simulation technique [6]. The distributed coupling circuits have been modeled by LC-chains [8] (24 elements per wavelength  $\lambda$ ). This allows us to perform numerical simulation of the systems by means of well-known PSCAN program for lumped Josephson junction circuits. A set of resistors connected to capacitors of the chains is used to insert high-frequency losses. Contrary to [8], [10] we use here the loss factor  $\alpha$  which characterizes the losses on wavelength  $\lambda$  (wave amplitude decreases by factor  $e^{\alpha}$  on each wavelength). Josephson junctions have been described by resistively shunted model in the case of small intrinsic capacitance and Werthammer model in the case of tunnel structures.

## A. Distributed System Design

Three different types of the distributed systems with Josephson junction arrays are shown in Fig. 1 and Fig. 2. Josephson junctions in the arrays are biased in parallel. The distributed coupling circuits may practically present sections of ether microstrip or slot or coplanar lines.

"Book stand" like system (Fig. 1a) presents the set of resonators coupled by the shared Josephson junctions. Inphase oscillations of the excited standing waves result in high amplitude of output voltage between points "1" and "2" [8]. The system could be successfully used for antenna excitation as it is schematically shown in Fig. 1b.

Two types of arrays of Josephson junctions separated by sections of distributed line are shown in Fig. 2. Both systems are able to provide reasonably high output power into the connected wave-guide line. The phase locking state of all junctions is provided by interaction with common standing wave excited in the distributed systems. The first array (Fig. 2a) is a parallel array of junctions that interact with the standing wave of voltage. This structure is best suited to the low- $T_c$  tunnel junction technology. So, the experimental results for the system with array of 30 niobium tunnel junctions were presented recently [9], and oscillation linewidth 8 MHz at frequency 566 GHz was reported.

In the second array (Fig. 2b) Josephson junctions interact with the standing wave of current. We have realized just this system on the base of high-Tc superconductor technology (see the whole view of the structure in Fig. 5).





All these different distributed structures are characterized



Fig. 2. Two types of the distributed systems with arrays of Josephson junctions biased in parallel. The junctions separated by sections of distributed line interact with either voltage (a) or current (b) standing wave excited.

by common properties as follows. The lowest oscillation linewidth takes place at reasonably low wave reflection from Josephson junctions. If  $\rho$  is characteristic wave impedance, and Z is impedance of Josephson junction (it is about normal junction resistance  $R_N$  in the case of small junction capacitance), then the requirement of either  $Z >> \rho$  for the voltage-wave-interaction system or  $Z << \rho$  for the current-wave-interaction system should be fulfilled. It means that a united standing wave is excited in the whole system. In this case only the total length of the distributed line with opened or shorted terminals will dictate the standing wave frequency.

In the case of high reflection from Josephson junctions the distributed system will be divided by the junctions on several resonators with the boundary conditions which are nonlinear and conflicting each to other. It yields in strong nonlinear dependence of the excited wave frequency on the wave amplitude and bias current and hence results in low quality factor of this resonator even with small intrinsic loss [10]. In other words, this case is characterized by strong resonance mode interaction, which gives both smooth and abrupt changing in resonance frequency with the bias current.

## B. Junction position

The features of nonlinear interaction between Josephson junctions and the standing current wave result in a restriction on the junction energy contribution in the excited wave [10]. Fig. 3 shows the dependence of the amplitude of the wave excited by Josephson junction on its position in the system shown in Fig. 2b for the case of one junction. Nodes of current in the opened line terminals and maximum in the middle of the line characterize the excited wave mode. In the case of high losses the maximum amplitude of the excited wave takes place at the junction position in the middle. But at low losses the maximum amplitude corresponds to junction position near the node of the current wave. It is seen that the energy contribution even decreases if the amplitude of current through the junction exceeds half the critical current I<sub>c</sub>. Moreover the restriction on the junction energy contribution is accompanied by an increase in the oscillation linewidth (see Fig. 3). The fact seems to be important primarily for the low-temperature superconductor structures especially at operating frequencies about hundred GHz and less.

As for the high-T<sub>c</sub> superconductor systems, experimental



Fig. 3. The dependence of amplitude of the standing wave excited in the  $\lambda/2$  resonator, which is modeled by 12 LC-elements, on Josephson junction position at different values of the loss factor  $\alpha$ : "1" -  $\alpha = 0.01$ , "2" -  $\alpha = 0.05$ , "3" -  $\alpha = 0.3$ . Dashed line is the oscillation linewidth corresponding to the curve "1". The current wave amplitude is normalized on the critical current  $I_C$ , The linewidth is normalized on the linewidth of single Josephson junction  $\Delta f_0$ .

data attest that the losses at the operating frequency f > 600 GHz are rather high [8] and hence the optimum junction positions will correspond to antinodes of the standing current wave.

#### C. Output Power

An existence of output power is equivalent to additional losses in the distributed system. Nevertheless the insertion losses per section of the wave-guide line between Josephson junctions are less then these additional losses by factor N, where N is total number of junctions (and  $\lambda/2$  sections). It means that the contradictory requirement to the system on the high output power and the high quality factor Q, which is determined just by the overall losses per wavelength  $\lambda$ , can be fulfilled by an increase in the number N. In this case the output power for the systems shown in Fig. 2 can ranges up to several tens percent of the incident wave power. In one complete cycle the running wave power decreases by factor  $D = (1-K_{out}) \cdot e^{-2\alpha N} = e^{-(2\alpha N + \gamma)}$ , where  $\gamma = -ln$  (1-K<sub>out</sub>), and K<sub>out</sub> is transmission coefficient. Therefore when  $\gamma/2N$  is much less then  $\alpha$ , the influence of the transmitted wave power is negligibly small. For example, even with transmission coefficient  $K_{out} = 0.6$  the value  $\gamma/2N$  is about 0.01 at N = 50. At the same time, according to [10] the threshold value of the loss factor  $\alpha$  is about 0.2.

## D. Oscillation Linewidth

Most exhaustive characteristic of the resonator systems is quality factor  $Q = W/A_T$ , where W is total energy of the resonator, and  $A_T$  is the loss per oscillation period. Let w be the energy density of the running wave, then we have  $W = 2w\lambda N/2$  and  $A_T = (1-e^{-2\alpha})w\lambda N/2 \approx 2\alpha w\lambda N/2$ . It gives  $Q = \alpha^{-1}$  to be independent on N.

Nevertheless, our calculations for the system presented in Fig. 2b show that oscillation linewidth reduction depends on



Fig. 4. The oscillation linewidth dependence on the total number N of Josephson junctions (and  $\lambda/2$  sections) at different values of loss factor  $\alpha$ : "1" -  $\alpha = 0.05$ , "2" -  $\alpha = 0.1$ , "3" -  $\alpha = 0.2$ . Dashed line is the oscillation linewidth dependence on the loss factor  $\alpha$  at N = 1.

both the loss factor  $\alpha$  and the number of junctions N. It is seen from Fig. 4 that the linewidth decreases just proportionally to N. Most likely this important result can be explained by the phase-locking between Josephson junctions and the excited standing wave.

Therefore one can conclude that in principle it is possible to realize the multi-junction system, which could provide the linewidth reduction by the factor  $k \approx 10^3$  and more even at the losses close to its threshold value ( $\alpha \approx 0.2$ ). If the freerunning linewidth of Josephson junction is about 10 GHz the mentioned above value of k allows to reduce the linewidth down to a few MHz. In this case it would be possible to use an external electronic phase-locking loop for the further reduction of the linewidth down to a few Hz [3].

## III. EXPERIMENTAL STRUCTURE

# A. Design and Fabrication

The multi-junction system, which is shown in Fig. 2b, has been designed and fabricated on the base of high-T<sub>c</sub> Josephson junction technology. Five bi-crystal YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7-x</sub> Josephson junctions are incorporated in the S/I/N microstrip resonator, which is meander-like in shape. The lower electrode of the microstrip line is layer of YBCO containing the junctions. Upper electrode is the platinum filmstrip, which is separated from the YBCO film by CeO2 layer.

General view of the experimental structure layout is shown in Fig. 5. Five Josephson junctions, which are the bridges of 4  $\mu$ m width across a bi-crystal boundary are biased in parallel by dc current I<sub>A</sub>. On microwave frequency these junctions are coupled by a standing wave excited in the meander shaped resonator. The generated microwave power is transmitted by a transfer slot line to the detector Josephson junction (5  $\mu$ m width bridge) biased by dc current I<sub>d</sub>. In the other versions of the experimental structure the transfer line is formed as either microstrip or coplanar line. The detector junction is connected also to an antenna, which is to receive an external microwave signal. The dc bias lines are formed with rf



Fig. 5. Layout of the based on high- $T_c$  Josephson junction technology experimental structure with Josephson junctions array (1), slot transfer line (2), detector junction (3), antenna (4), and rf filters (5). The gray line shows position of the bi-crystal boundary.

rejection filters to provide high impedance at operating frequency (about 500 GHz). All Josephson junctions are situated on the same bi-crystal boundary of a sapphire substrate.

The structure preparation methodic includes several steps as follows. At first, a thin epitaxial CeO<sub>2</sub> buffer layer was deposited on the substrate surface using rf-sputtering technique, to exclude chemical interaction between the substrate and YBCO superconducting film. The YBCO thin film was deposited using the dc-sputtering technique at high pressure. The deposition temperature was 750 °C, oxygen pressure (during deposition) was 2.8 mbar. The YBCO film was patterned by means of photolithography technique and either rf or liquid etching. Thereupon this YBCO electrode was covered with an insulating amorphous CeO2 thin film, and the windows in the insulator film were opened using rfetching technique to provide an electrical contact to the YBCO electrode. Finally the layer of Pt was deposited (RFsputtering technique) and patterned (lift-off procedure) to form the upper electrode of the strip line resonator and the contact pads.

## B. Measurements

Superconducting properties of the structure were tested two times during the structure fabrication - immediately after YBCO thin film preparation and after the patterning of the bottom electrode. The YBCO thin film showed  $T_{c0} > 86$  K; its degradation after the film patterning was less than 2 K. The significant degradation was observed after the CeO<sub>2</sub> insulator layer deposition, which had been resulted from the loss of oxygen in the YBCO film during this preparation step. Patterned in the YBCO electrode Josephson junctions demonstrated IV-curves typical for the resistively shunted junction model with a critical current density of  $10^4$  A/cm<sup>2</sup> and characteristic voltage  $V_c = I_c R_N$  of about 2 mV at 4.2 K.

The detailed experimental study, which includes also the microwave measurements, is now in progress.

#### IV. DISCUSSION

The central problem with the distributed high-T<sub>c</sub>

superconductor systems is to provide the overall losses less then threshold value at frequencies f of about 1 THz and above. It is known from microscopic theory that the surface resistance of superconductors should increase with frequency as  $f^{3/2}$ , and hence the loss factor  $\alpha$  per wavelength should rises as  $f^{1/2}$ . Unfortunately a major contribution to the overall losses is given also by radiation effect and additional surface resistance defined by the low film quality.

It is hoped that the reported experimental structure, which is based on S/I/N microstrip resonator, will be able to decrease the losses caused by surface resistance and radiation effect.

#### REFERENCES

- [1] V. P. Koshelets, S. V. Shitov, A. V. Shchukin, L. V. Filippenko, J. Mygind, A. V. Ustinov, "Self-pumping effects and radiation linewidth of Josephson flux-flow oscillators", *Phys. Rev. B*, vol. 56, pp. 5572-5577, August 1997.
- [2] A. M. Baryshev, A. V. Yulin, V. V. Kurin, V. P. Koshelets, P. N. Dmitriev, L. V. Filippenko, "Forward and backward wave in Cherenkov flux-flow oscillators", *Extended Abstracts of 7th International Superconductive Electronics Conference*, Berkeley, CA USA, June 21-25, 1999, pp. 418-420.
- [3] J. Mygind, V. P. Koshelets, S.V. Shitov, I. V. Filippenko, V. L. Vaks, A. M. Baryshev, W. Luinge, N. Whyborn, "Phase locking of 270-440 GHz Josephson flux flow oscillators", *Extended Abstracts of 7th International Superconductive Electronics Conference*, Berkeley, CA USA, June 21-25, 1999, pp. 83-85.
- [4] A. K. Jain, K. K. Likharev, J. E. Lukens, and J. E. Sauvageau, "Mutual phase-locking in Josephson junction arrays", *Phys. Report*, Amsterdam, vol. 109, pp. 309-426, 1984.
- [5] V. K. Kornev, A. D. Mashtakov, A. V. Arzumanov, and G. A. Ovsyannikov, "Basic cells for phase-locked 1D and 2D Josephsonjunction arrays", *IEEE Trans. Appl. Supercon.*, vol. 5, pp. 3527-3531, December 1995.
- [6] V. K. Kornev and A. V. Arzumanov, "Numerical simulation of Josephson-junction system dynamics in the presence of thermal noise", *Inst. Physics Conf. Ser. No* 158, IOP Publishing Ltd, pp. 627 – 630, 1997.
- [7] V. K. Kornev, A. V. Arzumanov, "Oscillation spectral linewidth for some phase-locked Josephson-junction arrays", *IEEE Trans. Appl. Supercond.*, vol. 9, pp. 4562-4565, June 1999.
- [8] K. Y. Konstantinian, A. D. Mashtakov, G. A. Ovsyannikov, V. K. Kornev, A. V. Arzumanov, N. A. Shcherbakov, M. Darula, J. Mygind, and N. F. Pedersen, "Mm wave Josephson radiation in high-Tc bicristal junction arrays", *Inst. Physics Conf. Ser. No 167*, IOP Publishing Ltd, pp. 771-720, 2000.
- [9] A. Kawakami, Y. Uzawa, and Zh. Wang, "Linewidth measurement of Josephson array oscillators with microstrip resonators", *Inst. Physics Conf. Ser. No* 167, IOP Publishing Ltd, pp. 757-760, 2000.
- [10] V. K. Kornev, A. V. Arzumanov, and N. A. Shcherbakov, "Josephsonjunction arrays with lumped and distributed coupling circuits", *Inst. Physics Conf. Ser. No* 167, IOP Publishing Ltd, pp. 753-756, 2000.