SINGLE-TRANSISTOR CHAOTIC OSCILLATORS WITH PREASSIGNED SPECTRUM

Elena V. Efremova, Alexander D. Khilinsky

Institute of RadioEngineering and Electronics, Russian Academy of Sciences,
Mokhovaya St. 11/7, GSP-3, 103907, Moscow, Russia,
E-mail: chaos@mail.cplire.ru

Abstract – **In this report the problem of forming chaotic signals with prescribed spectrum is discussed. An approach to construction of single-transistor chaotic oscillators with preassigned spectrum on the basis of "active component (transistor) – passive quadripole closed in feedback loop" structure is proposed. On example of capacitive three-point oscillator it is shown that in the oscillator with such structure it is possible to obtain chaotic spectrum which envelope is close to the shape of the amplitude-frequency response of feedback loop of the system. The possibility of construction of new transistor oscillators with preassigned spectrum is demonstrated on the example of model of chaotic oscillator with 2.5 degrees of freedom.**

Index terms – **Dynamic chaos, chaotic oscillator, chaotic power spectrum formation.**

I. Introduction

Use of dynamic chaos in radio communications and radiolocation [1 - 4] requires creation of chaotic sources with prescribed spectral and statistical characteristics.

For practical reasons, the circuitry used in chaotic oscillators must include mainly "classical" electronic components. In particular, it is desirable to employ bipolar and field-effect transistors.

Hence, it is reasonable to assume that electronic circuits of chaotic oscillator prototypes should have a minimum number of components and a transistor as a nonlinear element.

Today the theory of construction of regular oscillators [5], including transistor oscillators, is well developed. From the other hand there is a theory of preassigned chaotic power spectrum formation in the class of ring-structure oscillation systems [6]. In this case, the shape of the power spectrum is determined by collective frequency response of frequency-selective elements in the feedback loop of the system. In this paper we try to use present experience in both areas and develop an approach to the transistor oscillator with preassigned spectrum construction.

II. Structure of the oscillator on the basis of active network and passive quadripole

From the theory of oscillations it is known what in the structure composed of an active network with nonlinear I-V characteristic and a passive element closed in feedback loop (Fig. 1) in the case when the conditions of phase and amplitude balance are satisfied regular oscillation are excited. The active network and passive network are the quadripoles.

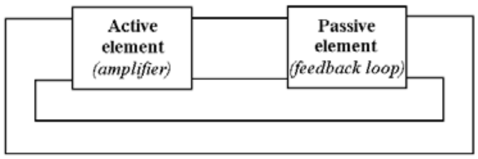


Fig. 1: Structure "active network – passive quadripole closed in feedback loop".

As an active element a bipolar transistor can be used. As passive quadripole any linear quadripole, with which the conditions of exciting and existence of oscillations can be kept, may be used. It is a common model of transistor oscillator. In particular all three-point circuits are special cases of such structure (Fig. 2).

In such a representation of the transistor oscillator structure it can be seen that it has much in common with ring-structure oscillation systems. The difference between ring oscillators and single-transistor oscillators is the use of active quadripole with no dropping characteristic and absence of buffers between circuit elements in transistor oscillator. A possibility of power spectrum envelope control was shown by means of formation of amplitude-frequency response (AFR) of passive element system in feedback loop in the class of ring oscillators. Due to existence of chaotic dynamics in transistor oscillators with 1.5 degrees of freedom an opportunity of chaotic oscillation power spectrum formation in these systems can be expected by means of a choice of element sequence in the feedback loop circuit.

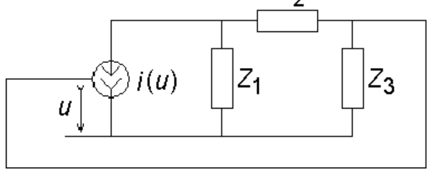


Fig. 2: The structure of three-point circuit

It seems advisable to begin investigations with the simplest oscillators – oscillators based a single transistor and a minimum number of elements which give a possibility of chaotic oscillations (as is known, it is possible in systems with 1.5 degrees of freedom).

Then analyze the process of power spectrum formation in such system after that complicate the system by means of adding new components for creating conditions for more complicated spectrum.

III. Formation of power spectrum in three-point circuit

Let us begin with a well-known oscillator with 1.5 degrees of freedom in which chaotic oscillations exist, i.e., capacitive three-point circuit or Colpitts oscillator [7-8], it is shown in Fig. 3.

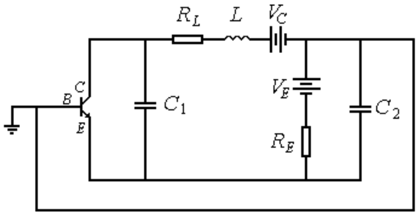


Fig. 3: Schematic diagram of the capacitive three-point circuit.

The oscillator dynamics can be described as follows:

$$
\begin{aligned}
&C_1\dot{V}_{CE} = I_L - I_C,\\
&C_2\dot{V}_{BE} = (V_E - V_{BE})/R_E - I_L - I_B, \qquad (1)\\
&L\dot{I}_L = V_C - V_{CE} - R_L I_L + V_{BE},
\end{aligned}
$$

where V_{CE}, V_{BE} are collector-emitter and base-emitter voltages, respectively; I_L, I_C, I_B are currents through inductance, collector and base, respectively.

$$
\begin{aligned}
&I_B=0, \text{ where } V_{BE} \le V_T;\\
&I_B=(V_{BE} - V_T)/R_{BE}, \text{ when } V_{BE}>V_T; \qquad (2)\\
&I_C=\beta I_B,
\end{aligned}
$$

where $V_T \approx 0.75$, V_T is barrier potential; R_{BE} is base-emitter resistance; and β is transistor gain factor

For analysis we use the parameter set: V_E=2 V, V_C=7 V, L=30 μH, R_L=40 Ohm, R_E=400 Ohm, R_{BE}=200 Ohm, β=300. Let us analyze the signal power spectrum at the passive network output. In Fig. 4 the power spectrum and AFR of the feedback loop are shown. As can be seen in the figure, the power spectrum envelope is close to the AFR form.

Let us fix the value of parameter at L and investigate the spectrum evolution by variation of parameters C_1 and C_2. In search for chaotic modes we use a two-parameter diagram of Lyapunov exponents on the parameter plane (C_1, C_2) (Fig. 5), where the Lyapunov exponent values are denoted by gray scale.

Let us choose two modes with close parameter values and different values of Lyapunov exponent. So in the case C_1=13 nF, C_2=9.7 nF the value of Lyapunov exponent is λ=0.05 and in the case C_1=15 nF, C_2=9.7 nF λ=0.13. In Fig. 4 (a, b) power spectra for these two cases are shown. As can be seen, for two almost identical AFR the power spectra are very different from each other. The more smooth spectra correspond to the larger value of Lyapunov exponent.

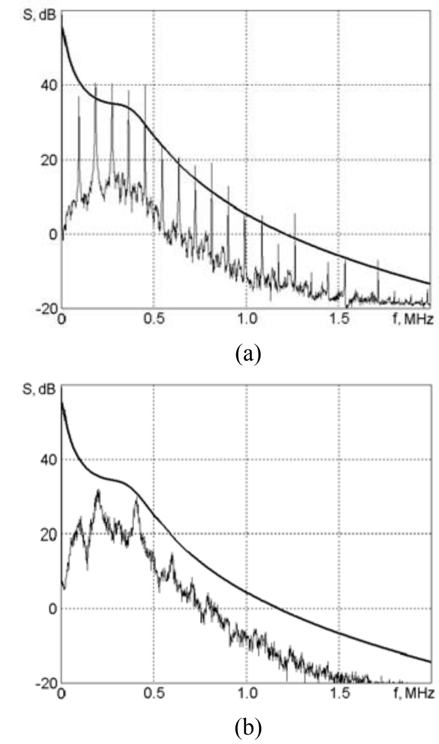


Fig. 4. AFR of feedback loop and signal power spectrum of three-point circuit at the parameters values (a) C_1=13 nF, C_2=9.7 nF and (b) C_1=15 nF, C_2=9.7 nF.

So, these results confirm that in chaotic oscillators of considered structure it is possible to obtain chaotic power spectrum with the envelope close to the form of AFR of feedback loop. Moreover of two modes with close parameter values the more smooth spectrum corresponds to the larger Lyapunov exponent.

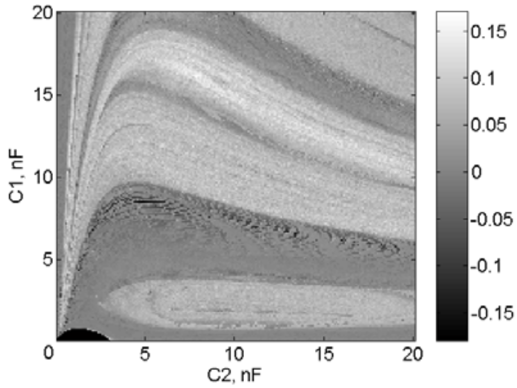


Fig. 5: Two-parameter diagram of Lyapunov exponents in (C_1, C_2) plane.

On the basis of this analysis it is possible to formulate the necessary requirements for forming the preassigned power spectrum of transistor oscillator.

1. The oscillator structure must involve active network (transistor) and passive network closed in the feedback loop.

2. In the oscillator chaotic oscillations must exist as a result of collapse of fixed-frequency or two-frequency regular self-oscillating mode.

3. Frequency-selective circuit in the feedback loop must be adjusted so that the amplitude-frequency response of the open loop (magnitude of complex transfer function of feedback loop) agrees with the desired form of power spectrum envelope.

4. To adjust the power spectrum smoothness it is necessary to provide possibility of changing component nominal values in order to find the mode with larger Lyapunov exponent.

IV. Chaotic oscillator with 2.5 degrees of freedom

Let us show that the described approach to power spectrum formation and analysis is applicable not only in the case of three-point oscillator but allows to make new chaotic oscillators with preassigned spectrum.

For this aim we add an additional *RLC*-link in the feedback loop, that is a low-pass filter. Schematic diagram of this oscillator is shown in Fig. 6.

Dynamic modes of the oscillator are described by the following system of equations:

$$\begin{aligned} C_1\dot{V}_{CE} &= I_L - I_C, \\ C_3\dot{V}_{BE} &= I_{L3} - I_B, \\ L\dot{I}_L &= V_C - V_{CE} - R_L I_L + V_{C2}, \\ C_2\dot{V}_{C2} &= (V_E - V_{C2})/R_E - I_L - I_{L3}, \\ L_3\dot{I}_{L3} &= V_{C2} - V_{BE} - R_3 I_{L3}, \end{aligned}$$

where V_{CE}, V_{BE} are collector-emitter and base-emitter voltages, respectively; V_{C2} is voltage on capacity C_2 and I_L, I_{L3}, I_C, I_B are currents through inductances L, L_3, collector and base, respectively. As in the case of three-point circuit I-V characteristic is a piecewise-linear function (2).

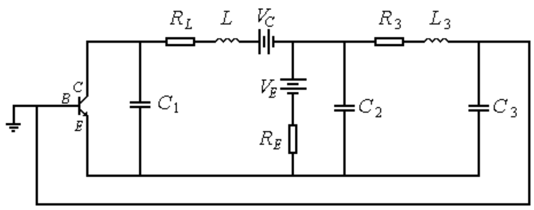
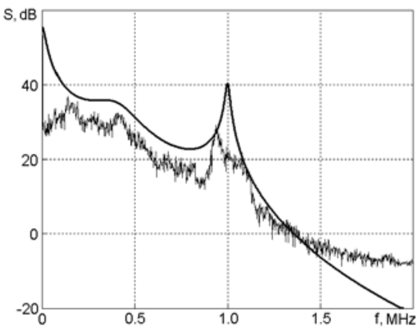


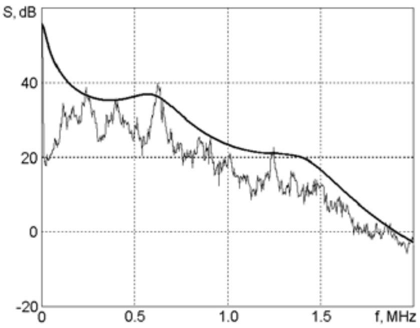
Fig. 6. Schematic diagram of the oscillator with 2.5 degrees of freedom.

For analysis we use the parameter set: V_E=2 V, V_C=7 V, L=30 µH, L_3=30 µH, R_L=40 Ohm, R_E=400 Ohm, R_{BE}=200 Ohm, β=300. Let us investigate the dependence of power spectrum form on AFR of feedback loop.

In Fig. 7 the AFR of feedback loop for different parameter values and corresponding power spectrum are shown. As can be seen the power spectrum envelopes are in good match with AFR of feedback loop both in frequency and in amplitude characteristics.



(a)



(b)

Fig. 7. AFR of feedback loop and signal power spectrum at the parameters values (a) C_1=14 nF, C_2=6.2 nF, C_3=1 nF, R_3=1 Ohm and (b) C_1=14 nF, C_2=1.1 nF, C_3=1 nF, R_3=10 Ohm.

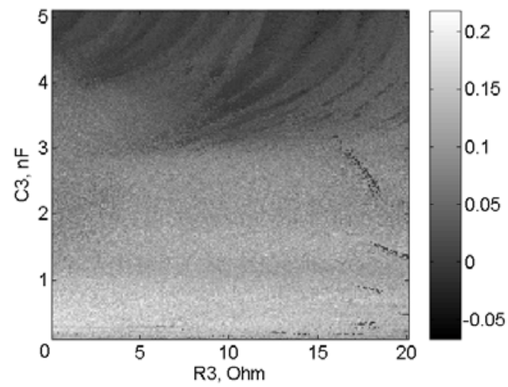
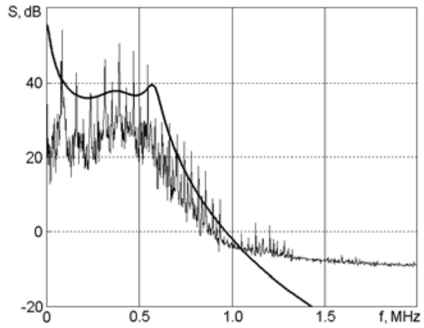
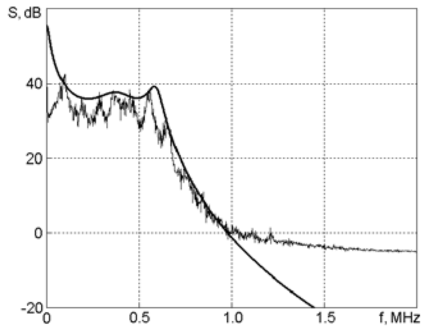


Fig. 8 Two-parameter diagram of Lyapunov exponents in (R_3, C_3) plane on C_1=7.3 nF, C_2=16.4 nF.

To scan for chaotic modes with the most smooth spectrum we use two-parameter diagram of Lyapunov exponent. Two-parameter diagram of Lyapunov exponents in (R_3, C_3) plane is shown in Fig. 8. Let us take two points with different Lyapunov exponent values. Power spectrum of the mode with parameter values C_1=7.3 nF, C_2=16.4 nF, C_3=3.06 nF, R_3=10 Ohm is shown in Fig. 9a. Lyapunov exponent for this mode is λ=0.02. Let us consider a mode with larger Lyapunov exponent value but similar form of the AFR of feedback loop. For the mode with parameters C_1= 7.3 nF, C_2= 16.4 nF, C_3=2.9 nF, R_3=10 Ohm (Fig. 9b) λ=0.08. As can be seen from comparison of Figs. 9a and 9b AFR forms for these two modes are very similar but power spectrum envelope forms are fundamentally different. As in the case of three-point circuit, the value of Lyapunov exponent determines the smoothness of the power spectrum envelope. The larger value of Lyapunov exponent, the more smooth and close to the AFR is the power spectrum envelope of the mode.



(a)



(b)

Fig. 9. AFR of feedback loop and signal power spectrum at the parameters values (a) C_1=7.3 nF, C_2=16.4 nF, C_3=3.06 nF, R_3=10 Ohm and (b) C_1= 7.3 nF, C_2= 16.4 nF, C_3=2.9 nF, R_3=10 Ohm.

Adding new components increases the number of system control parameters. So in such system greater variety of spectrum forms can be expected.

As can be seen in Figs., the agreement of the power spectrum with AFR form is better in the system with 2.5 degrees of freedom than in the three-point circuit.

This shows that increasing degrees of freedom allows to achieve not only complication of AFR form but also more accurate match between AFR form and power spectrum envelope form.

So, on example of oscillator with 2.5 degrees of freedom a possibility of synthesis of transistor chaotic oscillators with preassigned spectrum is confirmed.

V. Conclusions

In this paper the problem of preassigned chaotic power spectrum formation in oscillators based on the structure “transistor – passive quadripole closed in feedback loop” is discussed.

On example of capacitive three-point circuit it is shown that in the oscillators with such structure it is possible to obtain power spectrum close to AFR of feedback loop.

The necessary requirements for forming the preassigned power spectrum in transistor oscillator are formulated.

On example of an oscillator with 2.5 degrees of freedom a possibility of synthesis of transistor chaotic oscillators with preassigned spectrum in agreement with described principles is demonstrated. As is shown, in this oscillator the chaotic power spectrum form can be close to the form of amplitude-frequency response of feedback loop. The spectrum smoothness can be controled by means of varying the system parameter values finding from the two-parameter Lyapunov exponent diagram.

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