SOFT DECISION ERROR CORRECTION IN HIGH-SPEED DIRECT CHAOTIC COMMUNICATIONS

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Abstract – **The problem of noise resistance in telecommunication systems based on wideband and ultrawideband direct chaotic signals is discussed. Error probabilities for a model of a receiver with an envelope detector are calculated. Different soft decision error correction methods for such a receiver are discussed. Estimations of soft decision convolutional codes performance are made for various receiver schemas.**

Index terms – **Dynamic chaos, high-speed communications, direct chaotic communication, error correction, convolutional codes.**

I. Introduction

Direct chaotic communication (DCC) systems are systems in which the information-carrying chaotic signal is generated directly in RF or microwave band [1-3]. Information is put into the chaotic signal by means of modulating either the chaotic source parameters or the chaotic signal after it is generated by the source. Consequently, information is retrieved from the chaotic signal without intermediate heterodyning.

DCC systems are suitable for wide- and ultrawideband operation. By estimations, DCC systems can deliver bandwidth from tens of megabits up to one gigabit per second.

The simplest DCC system is the one with the direct modulation of a chaotic sequence with an information signal. Transmission of bit "1" is coded as presence of signal and otherwise bit "0" is coded as absence of signal (fig. 1).

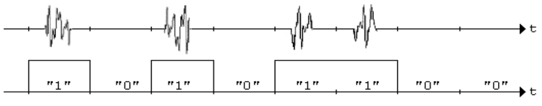

Fig. 1: Modulated chaotic signal and information sequence.

There are some factors that affect performance of DCC system besides SNR. The first is the signal base $B = 2\Delta F\tau$, where ΔF is the frequency band of a chaotic sequence and τ is the duration of a chaotic radio pulse. The second factor is a receiver class – coherent or non-coherent. Coherent receiver stores exact copies of all the signals a transmitter can send. A non-coherent receiver does not use waveforms in its operation.

A non-coherent receiver, which incorporates an envelope detector, works as follows. Carrier signal is being integrated in time interval of chaotic radio pulse. Then the integral of the received energy is compared with some threshold value. If it is greater than this threshold, a bit "1" is considered to be transmitted, otherwise decision is made on a bit "0".

In the following chapters possibilities of errors in noisy environment are estimated and error correction with convolutional codes discussed in connection with usage of chaotic carrier. The chaotic generator was modelled with discerete chaotic samples obtained with the Tent Map.

II. DCC bit error rate estimation

Let x_i be a transmitted signal, ξ_i – additive white Gaussian noise (AWGN) with dispersion σ_0^2. The variate x_i has uniform distribution from -α to α, and $\left\langle x_i^2 \right\rangle = \dfrac{E_1}{N}$, where N is a number of samples per bit, E_1 is a mean energy of bit 1 in a DCC system.

We can estimate an output of the receiver. In the case of bit 0 an integrated energy is $OUT_{E0} = \sum_1^N \xi_i^2$, from which

$$\left\langle OUT_{E0} \right\rangle = N\sigma_0^2 . \qquad (1)$$

The dispersion of this signal is equal to $D[OUT_{E0}] = N \cdot D[\xi^2] = N \cdot (\langle \xi^4 \rangle - \langle \xi^2 \rangle^2)$. Since $\left\langle \xi^2 \right\rangle = \sigma_0^2$ and $\left\langle \xi^4 \right\rangle = \dfrac{1}{\sqrt{2\pi}\sigma_0} \int_{-\infty}^{+\infty} \xi^4 \exp\left(-\dfrac{\xi^2}{2\sigma_0^2} \right) d\xi = 3\sigma_0^4$:

$$D[OUT_{E0}] = N \cdot 2\sigma_0^4 . \qquad (2)$$

In the case of bit 1:

$$OUT_{E1} = \sum_1^N (\xi_i + x_i)^2 = N\left\langle (\xi + x)^2 \right\rangle = \\ = N\left(\left\langle x^2 \right\rangle + \left\langle \xi^2 \right\rangle \right) = N\left(\frac{E_1}{N} + \sigma_0^2 \right) = E_1 + N\sigma_0^2 . \qquad (3)$$

Using the fact that variate x has a uniform distribution: $\langle x^4 \rangle = \frac{1}{2\alpha}\int_{-\alpha}^{\alpha} x^4 dx = \frac{\alpha^4}{5}$, $\langle x^2 \rangle = \frac{\alpha^2}{3} = \frac{E_1}{N}$, wherefrom $\langle x^4 \rangle = \frac{9E_1^2}{5N^2}$, and $D[x^2] = \frac{4E_1^2}{5N^2}$.

$$\begin{aligned} D[OUT_{E1}] &= N \cdot D[(x+\xi)^2] = \\ &= N \cdot \left(D[x^2] + 4D[x\xi] + D[\xi^2]\right) = \\ &= \frac{4E_1^2}{5N} + 4E_1\sigma_0^2 + 2N\sigma_0^4 \end{aligned} \quad (4)$$

Thus, distributions of zero and nonzero signals in the output of energy-based receiver are deduced. To calculate optimal threshold for binary symmetric channel, we take the intersection of probability density characteristics of 1 and 0 bits signals (fig. 2):

$$\frac{1}{\sqrt{2\pi D_0}} e^{-\frac{(x-M_0)^2}{2D_0}} = \frac{1}{\sqrt{2\pi D_1}} e^{-\frac{(x-M_1)^2}{2D_1}}, \quad (5)$$

where D and M are dispersion and average of distribution for 1 and 0 bits accordingly. By finding logarithm of both parts of equation and solving resulting quadratic equation we came to

$$x_{opt} = \frac{D_1 M_0 - D_0 M_1 \pm \sqrt{D_0 D_1 \left[(M_1 - M_0)^2 + (D_1 - D_0)\ln \frac{D_1}{D_0} \right]}}{D_1 - D_0}. \quad (6)$$

Using the fact that one of the roots is always negative:

$$x_{opt} = \frac{D_1 M_0 - D_0 M_1 + \sqrt{D_0 D_1 \left[(M_1 - M_0)^2 + (D_1 - D_0)\ln \frac{D_1}{D_0} \right]}}{D_1 - D_0} \quad (7)$$

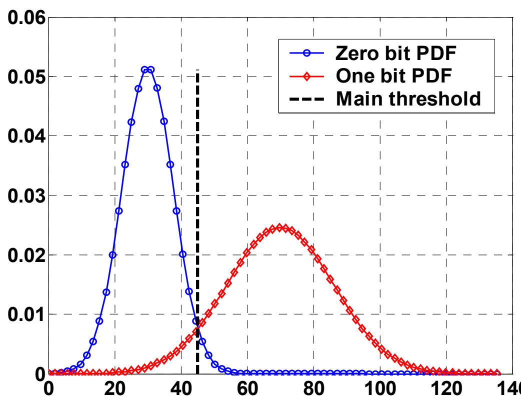

Fig. 2: A threshold for hard decision decoding.

Using (7) we could derive the following equation for bit error rate (BER):

$$\begin{aligned} BER &= \frac{1}{2}\left[p(1|0) + p(0|1)\right] = \\ &= \frac{1}{2}\left[\frac{1}{\sqrt{2\pi D_0}} \int_{x_{opt}}^{+\infty} \exp\left(-\frac{(x-M_0)^2}{2D_0} \right) dx + \frac{1}{\sqrt{2\pi D_0}} \int_{-\infty}^{x_{opt}} \exp\left(-\frac{(x-M_1)^2}{2D_1} \right) \right] = \\ &= \frac{1}{4}\left[erfc\left(\frac{x_{opt} - M_0}{\sqrt{2D_0}} \right) + 2 - erfc\left(\frac{x_{opt} - M_1}{\sqrt{2D_1}} \right) \right] \end{aligned} \quad (8)$$

Since probability of 1 and 0 bits in information stream are equal to ½, the average energy per bit is $E_b = \frac{E_1}{2}$. From SNR definition $SNR = \frac{E_b}{N_0}$ follows $SNR = \frac{E_1}{4\sigma_0{}^2}$ and

$$E_1 = SNR \cdot 4\sigma_0^2. \quad (9)$$

Using derived equations and assigning σ_0 to 1, we could calculate *BER* for arbitrary *N* and *SNR*. Let us examine DCC system with 100 Mbit/s bit rate and carrier frequency about 3÷5 GHz. The BER to SNR per bit dependency is shown in fig. 3. Acceptable BER for wireless computer system is less than 10^{-8}. According to fig. 3, SNR needed to achieve BER 10^{-8} is more than 22 dB, which is not acceptable. A more usable SNR level of 15 dB gives BER of 10^{-3}. Thus error correction algorithms are to be used to achieve acceptable error rate in DCC systems.

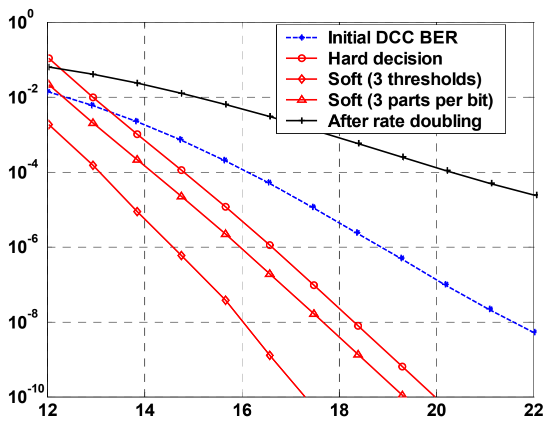

Fig. 3: Convolutional code with K=3 and r=1/2 BER performance evaluation (for DCC of 100 Mbit/s, band 3-5 GHz).

III. Convolutional coding in DCC systems

Now we introduce convolutional coding to a DCC system in order to reduce bit error probability. A code with the constraint length *K*=3 and the rate *r*=1/2 is enough to estimate BER performance for different decoding approaches.

The BER performance of convolutional codes with Viterbi decoding can be found by calculating a union upper bound:

$$BER = \sum_{d=d_{free}}^{\infty} \beta_d \cdot P_2(d), \qquad (10)$$

where d_{free} and β_d values depend on the code parameters and are well tabulated [4] for the most codes. The pairwise error probability $P_2(d)$ depends on demodulation method for a DCC channel.

For hard decision decoding, which implies strict decision on the value of a channel bit (0 or 1), $P_2(d)$ can be computed as follows [5]:

$$P_2(d) = \begin{cases} \sum_{k=(d+1)/2}^{d} C_k^d p^k (1-p)^{d-k}, \text{ for odd } d \\ \sum_{k=d/2+1}^{d} C_k^d p^k (1-p)^{d-k} + \frac{1}{2} C_{d/2}^d p^{d/2} (1-p)^{d/2}, \text{ for even } d \end{cases}. \qquad (11)$$

In this equation the value of p is equal to the value of BER in a channel without error correction coding. The value of BER for a DCC can be evaluated with (1)-(9). Here we must note that the mean energy E_c of a channel bit and E_c/N_0 ratio will reduce by a factor of $1/r$, where r is a code rate, since we reduce signal base in order to preserve channel rate. This affects on the value of p in (11). Results are shown on the fig. 3.

We also can consider an ability of soft decision decoding in a DCC system. Soft decision implies that demodulator does not make strict decision on the value of a channel bit, instead of this it assigns to the channel bit a value lying in between 0 and 1. This gives to the Viterbi decoder additional information about the channel symbols, which, in turn, leads to a better BER performance of the decoder.

The receiver for soft decision uses 3 thresholds instead of 1 to determine the value of a bit. To calculate optimum positions for two additional thresholds we use a method similar to the one for evaluating the main threshold presented in the previous section. We subdivide probability distribution picture on two parts. One part is above and another is below main threshold. Then we normalize all four parts of the power density function (PDF), to make integral probability equal to one (fig. 4-6). After that we have to find cross point of the PDF on both parts. These two points will give as position of two additional thresholds. Doing computations, we get:

$$x_{lower} = \frac{D_1 M_0 - D_0 M_1 + \sqrt{D_0 D_1 \left[(M_1 - M_0)^2 + (D_1 - D_0) \ln \frac{D_1 \cdot p(0|1)^2}{D_0 \cdot p(0|0)^2} \right]}}{D_1 - D_0}$$

$$x_{upper} = \frac{D_1 M_0 - D_0 M_1 + \sqrt{D_0 D_1 \left[(M_1 - M_0)^2 + (D_1 - D_0) \ln \frac{D_1 \cdot p(1|1)^2}{D_0 \cdot p(1|0)^2} \right]}}{D_1 - D_0}$$

(12)

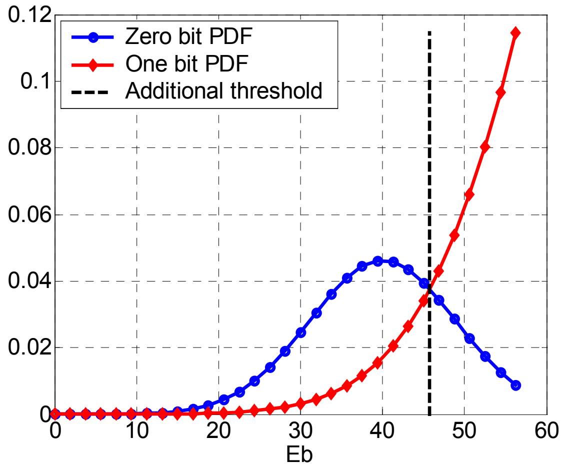

Fig. 4: Lower threshold computation for soft decision decoding.

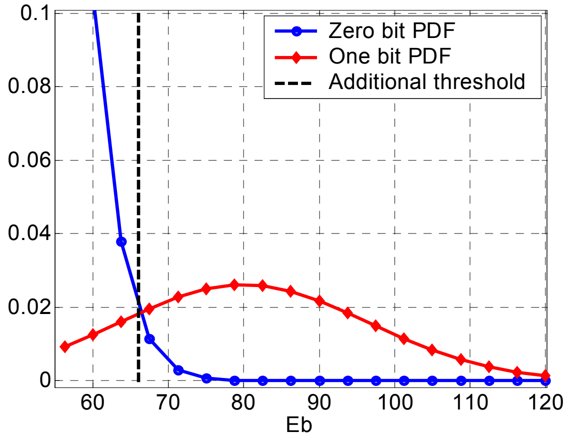

Fig. 5: Upper threshold computation for soft decision decoding.

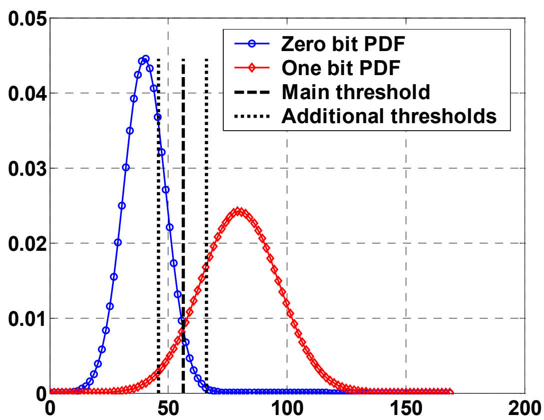

Fig. 5: Thresholds for soft decision decoding.

The Viterbi decoding algorithm for convolutional codes is a maximum likelihood method. This means that this method maximizes a likelihood function or a logarithmic likelihood function:

$$L(\bar{r},\bar{v}) = \ln p(\bar{r} \mid \bar{v}) = \sum_{i=1}^{N} \ln p(r_i \mid v_i), \quad (13)$$

where $\bar{r}$ is a received sequence of symbols, $\bar{v}$ is one of the possible transmitted sequence. A Vitebi decoder chooses such a sequence $\bar{v}'$, for which $p(\bar{r} \mid \bar{v}') = \max_{\bar{v}} p(\bar{r} \mid \bar{v})$.

In our case an output symbol takes 4 values, which we denote as Y_0, Y_1, Y_2 and Y_3. To use the Viterbi decoding algorithm and maximize a likelihood function we must compute the values of $\ln[p(Y_0 \mid 0)]$, $\ln[p(Y_1 \mid 0)]$, ..., $\ln[p(Y_3 \mid 1)]$. The values of $p(Y_0 \mid 0)$, $p(Y_1 \mid 0)$, ..., $p(Y_3 \mid 1)$ can be obtained in the way, which was used for the formula (8). The pairwise error probability $P_2(d)$ is computed as follows [5]:

$$P_2(d) = \sum_{0 \le K_0 \le K_1 \le K_2 \le d} p(Y_0 \mid 0)^{K_0} p(Y_1 \mid 0)^{K_1} p(Y_2 \mid 0)^{K_2} p(Y_3 \mid 0)^{d-K_0-K_1-K_2} \times \frac{d!}{K_0! K_1! K_2! (d-K_0-K_1-K_2)!}, \quad (14)$$

where

$$K_0 \ln\left[\frac{p(Y_0 \mid 1)}{p(Y_0 \mid 0)}\right] + K_1 \ln\left[\frac{p(Y_1 \mid 1)}{p(Y_1 \mid 0)}\right] + K_2 \ln\left[\frac{p(Y_2 \mid 1)}{p(Y_2 \mid 0)}\right] + (d-K_0-K_1-K_2) \ln\left[\frac{p(Y_3 \mid 1)}{p(Y_3 \mid 0)}\right] > 0 \quad (15)$$

From results on the fig. 3 we see that the coding gain at BER of 10^{-8} for soft decision is better by 2 dB than that one for hard decision.

The method above assumes use of three hardware comparators instead of one in a receiver. This leads to undesirable complication of a receiving device. As a result of this we must also consider a reception method, which still uses only one comparator, but with higher performance.

Let us divide a channel bit to 3 parts. The energy of each part is integrated separately and is compared with a threshold calculated with (1)-(9). In this case a demodulator output for each bit can be presented as a number of parts, which energy is above the threshold calculated. Let $\bar{r}$ be a received sequence of symbols, $\bar{v}$ is one of the possible transmitted sequence. The distance between these sequences for three-part bit division can be calculated with the following equation:

$$D_3(\bar{r},\bar{v}) = \sum_i d_3(r_i, v_i) = \sum_i (3v_i + (1-2v_i) r_i). \quad (16)$$

This distance function is similar to Hamming distance for bit sequences. It easy to show that minimizing of the distance (16) maximizes the logarithmic likelihood function (13), thus this function can be use as a metric in the Viterbi decoding algorithm. A decoder with such a metric is simple to implement, and it does not cause losses in performance.

BER performance of such a system can be estimated with the equation

$$BER = \sum_{d=d_{free}}^{\infty} \beta_d \cdot P_2(3d), \quad (17)$$

where $P_2(d)$ is taken from (11). Probability p in (11) is computed for one of three parts of a channel bit.

The fig. 3 shows, that this method brings a slightly better coding gain (by 0.5 dB at BER of 10^{-8}) in comparison with hard decision and is less effective (by 1.5 dB) than the three-threshold method.

This result is not surprising, because the three-part method does not take into account any channel properties, including nature of channel errors. This approach can be used for any noisy channel, for which an error probability on each bit part is calculated or measured, while the three-threshold method was adapted to an AWGN channel. A fact that we neglect a character of errors in a channel does not allow gaining a total effect of maximum likelihood decoding. Nevertheless, the nature of errors in a real channel can radically differ for those one in an AWGN channel, thus we must use variety of approaches to estimate a real effect of soft decision decoding.

IV. Conclusions

Convolutional coding with Viterbi decoding is shown to reduce effectively the probability of errors in high-speed direct-chaotic channels. A simple and not expensive method based on 3 thresholds in a receiver is used to increase noticeable BER performance of soft decision decoding. The methods of soft decision implementation and BER performance estimation can be used for more complicated DCC systems in a future.

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