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Dynamics of underdamped Josephson junctions with non-sinusoidal current-phase relation

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Abstract

Results on analytical and computational investigations of high-frequency dynamics of Josephson junctions, characterized by non-zero capacitance and the second harmonic in the current-phase relation are presented. These attributes each have influence on the behaviour of integer Shapiro steps and lead to the formation of non-integer Shapiro steps. Analytic theory of the integer and non-integer Shapiro steps has been developed for the so-called high-frequency limit. The analytical and numerical results are compared with experimental data for hybrid heterostructures YBCO/Au/Nb. Detector response for the case of high fluctuation level has been considered as well.

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Keywords: Josephson junction; Current-phase relation

1. Introduction

When rf signal is applied to Josephson junction, its *I–V*-curve shows a set of Shapiro steps resulting from phase-locking of Josephson oscillations. Analytical description of the Shapiro step dependence on the signal amplitude was obtained only for a high-frequency limit in the frame of resistively shunted junction (RSJ) model [1] describing an overdamped junction with McCumber parameter $\beta = 2\pi I_C R_N^2 C/\Phi_0 \ll 1$. At the same time, many types of Josephson junctions do not meet the model. Most of all, this concerns to the junctions on the base of high-T_c d-wave superconductors. Such junctions are usually characterized by $\beta > 1$ and some digression from sinusoidal current-phase relation assumed in RSJ model. Both the factors can cause origin of the sub-harmonic steps unavailable in the frame of RSJ model. Among the junctions, one should mention s-wave superconductor/normal metal/d-wave superconductor (SND) Josephson junctions [2,3].

In this work we deliver results of analytical theory for dependence of the harmonic and sub-harmonic Shapiro step amplitude on amplitude of the applied rf signal taking into account the impact of both factors: β and second harmonic in the current-phase relation. The theory is developed for the so-called high-frequency limit, when at least one of the three following conditions is fulfilled:

$$\omega \gg 1 \quad \text{or} \quad \beta\omega^2 \gg 1 \quad \text{or} \quad a \gg 1 \tag{1}$$

(frequency ω and the rf signal amplitude a are normalized by characteristic Josephson frequency Ω_c and voltage V_c, correspondingly). The analytical results are compared with data of numerical simulation and experimental data for S/N/D junctions.

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2. Analytical theory approach

The analytical consideration of Josephson junction dynamics is performed using the following master equation:

$$\beta\ddot{\varphi} + \dot{\varphi} + \sin\varphi + q\sin 2\varphi = i + a\sin(\omega t) + i_f, \tag{2}$$

where the bias current i and fluctuation current i_f are normalized by critical current I_c, and factor q describes the second harmonic contribution. The term ($\sin\varphi + \sin 2\varphi$) is a small parameter in the extreme case (1), therefore Josephson-junction phase φ and constant component of the current i can be presented as expansions in the order of vanishing:

$$\varphi = \varphi_0 + \varphi_1 + \varphi_2 + \cdots, \quad \bar{i} = \bar{i}_0 + \bar{i}_1 + \bar{i}_2 + \cdots, \tag{3}$$

and Eq. (2) can be reduced to the set of equations as follows:

$$\beta\ddot{\varphi}_0 + \dot{\varphi}_0 = \bar{i}_0 + a\sin(\omega t) + i_f, \tag{4}$$

$$\beta\ddot{\varphi}_1 + \dot{\varphi}_1 = \bar{i}_1 - \sin(\varphi_0) - q\sin(2\varphi_0), \tag{5}$$

$$\beta\ddot{\varphi}_2 + \dot{\varphi}_2 = \bar{i}_2 - \varphi_1\cos(\varphi_0) - 2q\varphi_1\cos(2\varphi_0). \tag{6}$$

The 0-order approximation (solution of Eq. (4)) describes autonomous I–V curve. In the case of negligible fluctuations ($i_f = 0$), the first- and second-order approximations that can be found from (5) and (6) describe accordingly harmonic and sub-harmonic Shapiro steps. The opposite case of $i_f \neq 0$ corresponds to large-scale fluctuations inasmuch as the term i_f is put in Eq. (4) for 0-order approximation. In such a case the first- and second-order approximations that can be found from (5) and (6) describe detector response at high fluctuation level.

3. Negligible fluctuations

3.1. The case q = 0

At $q = 0$, the amplitudes of harmonic Shapiro steps result from Eq. (5). The step amplitudes are described by the following expressions:

$$\Delta i_n = 2|J_n(x)|, \tag{7}$$

$$x = a/\omega\sqrt{(\omega\beta)^2 + 1}. \tag{8}$$

If $\beta = 0$, formulas (7) and (8) coincide with the well known ones for RSJ model [1].

Amplitudes of the sub-harmonic Shapiro steps result from Eq. (6). The sub-harmonic step amplitudes are described by the following sum:

$$\Delta i_{(2n+1)/2} = 2\beta\left|\sum_{m>n} J_{(2n+1)-m}(x)J_m(x) \Big/ \left((\omega\beta)^2((2n+1)/2-m)^2+1\right)\right|. \tag{9}$$

Keeping only the major term, one can reduce the sum as follows:

$$\Delta i_{(2n+1)/2} = 2\beta\left|J_{n+1}(x)J_n(x) \Big/ \left[(\omega\beta)^2/4 + 1\right]\right|. \tag{10}$$

3.2. The case q ≠ 0

Eq. (5) gives the following formula for the harmonic Shapiro step amplitudes:

$$\Delta i_n = 2\max_{\Theta}\left[J_n(x)\sin(\Theta) + qJ_{2n}(2x)\sin(2\Theta)\right], \tag{11}$$

where x is defined by (8). This formula can be extended for the case of several harmonics in the junction current-phase relation as follows:

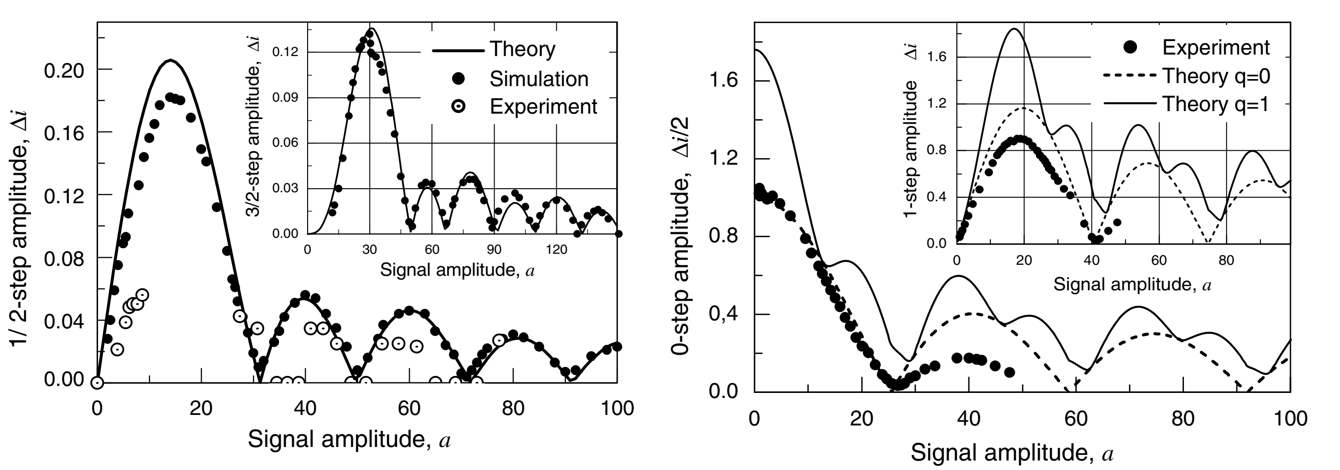
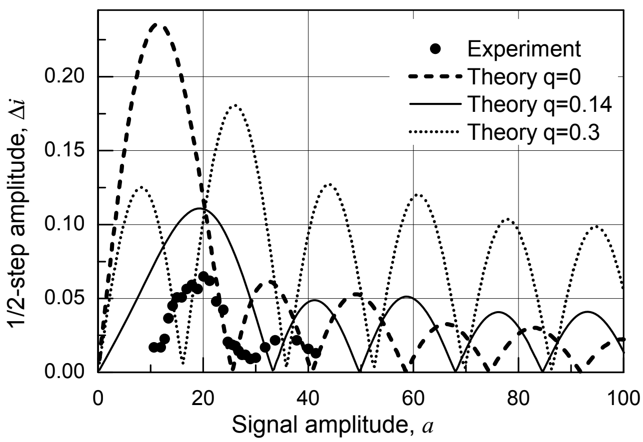


Fig. 1. Left side: Dependences of the 1/2- and 3/2-step amplitudes on the applied signal amplitude a at frequency $\omega = 0.611$, $\beta = 35$ and $q = 0$. Solid line corresponds to formula (10); filled dots, numerical simulation; and empty dots, experimental results for the c-oriented Nb/Au/YBCO junctions. Right side: Dependences of the critical current amplitude $\Delta i/2$ (0-step) and the 1-step amplitude Δi (in inset) on the applied signal amplitude a at frequency $\omega = 1.62$ and $\beta = 4$. Dashed and solid lines correspond to formula (11) at $q = 0$ and $q = 1$ correspondingly, the filled dots correspond to experimental results for the c-tilted Nb/Au/YBCO junctions.



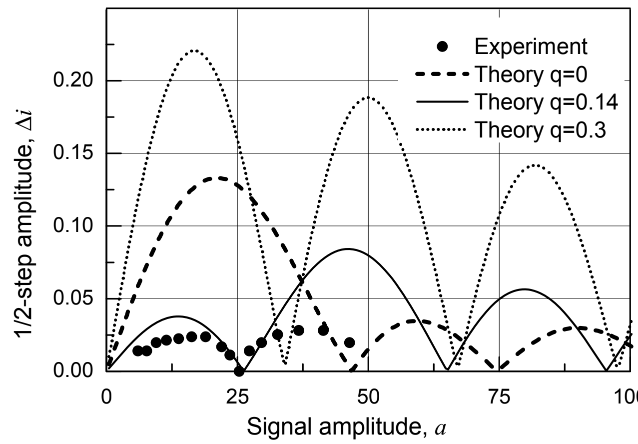


Fig. 2. Dependence of the 1/2-step amplitude Δi on the applied signal amplitude a at $\beta = 4$ for frequencies $\omega = 1.62$ (left side) and $\omega = 2.2$ (right side). Dashed, solid and dotted lines correspond to the step behaviour given by formula (13) accordingly at $q = 0$, $q = 0.14$ and $q = 0.3$. The filled dots are experimental data for the c-tilted Nb/Au/YBCO junction.

$$\Delta i_n = 2\max_{\Theta}\left\{\sum_k q_k J_{kn}(kx)\sin(k\Theta)\right\}. \tag{12}$$

And finally, the sub-harmonic Shapiro step amplitudes resulting from Eq. (6), are given by the following expression:

$$\Delta i_{1/2} = 2\max_{\Theta}\left[\sin(\Theta)\left\{qJ_1(2x) + \beta\frac{J_1(x)J_0(x)}{(\beta\omega)^2/4+1} + 4q^2\beta\frac{J_2(2x)J_0(2x)}{(\beta\omega)^2+1}\cos(\Theta)\right\}\right], \tag{13}$$

where x is defined by (8) as well.

Figs. 1 and 2 present the analytical results, as well as experimental data for both the c-oriented and c-tilted Nb/Au/YBCO junctions formed on NdGaO substrates (junction areas ranged from $10\times10\ \mu m^2$ to $30\times30\ \mu m^2$) and measured at 4.2 K under electromagnetic irradiation at frequency 36–120 GHz [2,3]. In the latter case, the S/N/D heterojunctions based on single-domain films of (1 1 2 0) YBCO have been prepared on specially oriented (7 1 0 2) NdGaO substrates, yielding an inclined growth of epitaxial YBCO. The c-oriented junction parameters were estimated as $q = 0$ and $\beta = 35$, while the parameters for the c-tilted junctions are $q = 0.14$ and $\beta = 4$.

4. Detector response

Detector response $\text{resp} = i(v) - i_a(v)$ is the difference between the I–V curve under rf signal impact and the autonomous one. As a rule, it is more convenient to use the frequency difference $\delta_n = n\omega - v$ instead of normalized voltage $v = V/V_c$, where V_c is characteristic voltage of the junction.

4.1. The case of negligible fluctuations

In the case of negligible fluctuations, the set of Eqs. (4)–(6) yields the harmonic detector response for arbitrary β as follows:

$$\text{resp} = \begin{cases} |J_n(x)| & \text{if } \delta_n = 0 \\ J_n(x)^2/\delta_n\sqrt{\delta_n^2\beta^2+1} & \text{if } \delta_n \neq 0, \end{cases} \tag{14}$$

4.2. Large-scale fluctuations

We have considered the impact of the large-scale δ-correlated fluctuations on detector response in the high-frequency limit. In this case, when noise-factor $\gamma = I_f/I_c$ (in case of thermal fluctuations, $I_f = 2ekT/\hbar$) it is much more than 1 and therefore the term i_f is put in Eq. (4), the set (4)–(6) allows us to analyse detector response at arbitrary values of β and q. In practice this case may correspond to the junctions with especially low critical current.

When $q = 0$ and $\beta = 0$, the harmonic detector response is described by the simple expression:

$$\text{resp} = \frac{1}{2}J_n^2(x)\left[\frac{\delta_n}{\delta_n^2+\gamma^2}\right]. \tag{15}$$

At arbitrary value of β and $q = 0$, more complicated expression takes place:

$$\text{resp} = \frac{1}{2}J_n^2(x)\left[\frac{\delta_n}{\delta_n^2+\gamma^2} - \frac{\delta_n}{\delta_n^2+(\gamma+1/\beta)^2}\right]. \tag{16}$$

In the general case of arbitrary values of β and q the harmonic detector response is as follows:

$$\text{resp} = \frac{1}{2}J_n^2(x)\left[\frac{\delta_n}{\delta_n^2+\gamma^2} - \frac{\delta_n}{\delta_n^2+(\gamma+1/\beta)^2}\right] + \frac{1}{2}q^2J_{2n}^2(2x)\left[\frac{\delta_n}{\delta_n^2+\gamma^2} - \frac{\delta_n}{\delta_n^2+(\gamma+2/\beta)^2}\right]. \tag{17}$$

The second harmonic in current-phase relation yields also sub-harmonic detector response ($\bar{v} \approx n\omega/2$):

$$\text{resp} = q^2J_n^2(2x)\left[\frac{\delta'_n}{\delta'^2_n+\gamma^2} - \frac{\delta'_n}{\delta'^2_n+(\gamma+2/\beta)^2}\right], \tag{18}$$

where $\delta'_n = 2\bar{v} - n\omega$. In all the expressions (14)–(18) argument x is given by (8).

5. Conclusion

Generalizing formulas for both harmonic and sub-harmonic Shapiro steps in the presence of non-zero junction capacitance and second harmonic in current-phase relation are obtained. The analytical theory generalizes the well-known high-frequency-limit consideration developed earlier for RSJ model [1] to the stated departures from RSJ model. The formulas are verified by numerical simulation and mainly by experimental results for YBCO/Au/Nb heterostructures. Some quantitative disagreement of the experimental data, which takes place mostly for sub-harmonic steps shown in Fig. 2, follows from distributed character of the junctions with the size of order of characteristic Josephson length λ_J.

At relatively small signal amplitude a, harmonic detector response is proportional to a^{2n} i.e. linear in respect to the signal power P at $n = 1$, and proportional to P^n at $n > 1$. One should emphasize that the consideration of second harmonic in the junction current-phase relation gives the second-order contribution to the harmonic responses, and the main contribution proportional to power P to the sub-harmonic responses at $\bar{v} \approx n\omega/2$. It means that observation of the sub-harmonic response enables mostly in a sensitive way to detect second harmonic in current-phase relation.

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Further reading

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