

# Self-Mixing Spectra of Terahertz Emitters Based on $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Intrinsic Josephson-Junction Stacks

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Josephson junctions can serve as mixers for electromagnetic radiation, producing difference frequencies  $|mf_s - nf_{\text{LO}}|$  of the signal frequency  $f_s$  and the local oscillator frequency  $f_{\text{LO}}$ , where the latter can be provided by ac Josephson currents, and  $m$  and  $n$  are natural numbers. In order to obtain a better understanding of the purity of the terahertz radiation generated by stacks of intrinsic Josephson junctions (IJJs), we study self-mixing—i.e.,  $f_s$  is also produced by Josephson currents *inside* the stacks—in the difference-frequency range between 0.1 and 3.0 GHz. Simultaneously, we perform off-chip terahertz emission detection and transport measurements. We find that at high-bias currents, when a hot spot has formed in the stack, the power level of self-mixing can be low and sometimes is even absent at the terahertz emission peak, pointing to a good phase locking among all IJJs. By contrast, at low-bias currents where no hot spot exists, the self-mixing products are pronounced even if the terahertz emission peaks are strong. The mixing products at high operation temperature, at which the temperature variation within the stack is moderate, are minor, indicating that the low junction resistance, perhaps in combination with the lowered Josephson critical current density, may play a similar role for synchronization as the hot spot does at low temperature. While these observations are helpful for the task to synchronize thousands of IJJs, the observation of self-mixing in general may offer a simple method in evaluating the coherence of terahertz radiation produced by the IJJ stacks.

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## I. INTRODUCTION

The possibility to generate coherent terahertz emission using stacks of intrinsic Josephson junctions (IJJs) in the high-transition-temperature (high- $T_c$ ) superconductor  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  (BSCCO) has led to intensive research, both in terms of experiment [1–37] and theory [38–62]; for recent reviews, see Refs. [63–65]. A unit cell of BSCCO consists of superconducting  $\text{CuO}_2$  layers and insulating  $\text{BiO}$  and  $\text{SrO}$  layers, resulting in natural (intrinsic) Josephson junctions along the  $c$  axis [66]. A single crystal of 1.5- $\mu\text{m}$  thickness forms a stack of  $N \sim 1000$  IJJs. If the voltage across all IJJs is equal, the junctions oscillate at a frequency  $f_J = V/N\Phi_0$ , where  $\Phi_0$  is the flux quantum and  $V$  is the voltage across the whole stack of IJJs. This makes BSCCO stacks interesting candidates for tunable

(by voltage) terahertz oscillators, provided that the IJJs can be made to oscillate in phase over a certain range of voltages. Coherent off-chip terahertz emission was first demonstrated for 1- $\mu\text{m}$ -thick BSCCO stacks, with an extrapolated output power of up to 0.5  $\mu\text{W}$  for frequencies between 0.5 and 0.85 THz [1]. The emission frequency  $f_e$  turned out to be inversely proportional to the width of the stack. Because of that, it has been proposed that resonant modes utilizing the stack as a cavity and oscillating along the width of the stack play an important role for synchronization. A variety of cavity resonances have indeed been found in subsequent experiments [1–4,6,7,9,10]. The nonuniform part of the ac Josephson currents synchronizes with the cavity modes enhancing emission [6,61,62]. Also, efforts to improve the terahertz emission properties have increased the emitted power to up to tens of microwatts [19,21,22,28] and the emission frequency range from 0.3 to 2.4 THz [25,27,28]. Li *et al.* [14] utilized a Nb/AlN/NbN integrated receiver for

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measuring the linewidth  $\Delta f_e$  of terahertz radiation from IJJ stacks, and found  $\Delta f_e \sim 500$  MHz and 23 MHz at low-bias and high-bias regimes, respectively. For a theoretical discussion, see Ref. [54]. At high-bias currents, Joule heating of the IJJ stacks becomes severe. The BSCCO *c*-axis resistance decreases with increasing temperature. As a consequence, current voltage characteristics (IVCs) exhibit a positive differential resistance at low currents, but start to backbend at larger currents, exhibiting a negative differential resistance. Above some current in the backbending regime the current and temperature distribution in the stack becomes strongly nonuniform—a hot spot, i.e., a region with a temperature above  $T_c$ , forms [3,7,15,20,48,53]. The hot spot grows in size with increasing current. At not-too-large currents the hot spot coexists with regions that are still superconducting and produce terahertz radiation. The hot spot affects the properties of the terahertz radiation, as evidenced by the strongly different values for  $\Delta f_e$  under, respectively, high-bias and low-bias conditions. In particular, ac currents generated by the Josephson effect may lead to additional ac currents flowing through the hot area. These currents, in turn, may lead to an improved synchronization [54], in addition to synchronization effects mediated by cavity resonances.

The exact role of the hot spot for terahertz generation and phase synchronization is discussed controversially [14,23,24,54]. To better understand the underlying mechanisms, it is interesting to study the gradual evolution of the terahertz emission properties with changing bias current and relate them to transport properties. To attain this goal, we develop a technique to study the purity of the emitted terahertz radiation via self-mixing and combine this with simultaneous off-chip terahertz emission detection and transport measurements. The idea of self-mixing is that, due to the nonlinearity of the Josephson elements, two or more nearby terahertz emission peaks produce mixing signals at the difference frequencies (and multiples thereof) which can be detected at MHz and GHz frequencies. By contrast, if all IJJs oscillate at the same frequency, there will be no output signal due to self-mixing.

## II. SAMPLES AND MEASUREMENT TECHNIQUES

We perform measurements on stand-alone BSCCO stacks embedded between gold electrodes, a gold-BSCCO-gold (GBG) structure, which is shown schematically together with the experimental setup in Fig. 1. Data for two different samples are presented [67]. Sample 1 is fabricated from a BSCCO single crystal with  $T_c = 89$  K. Both the top and the bottom gold layers are 100 nm thick. The square stand-alone stack is  $240 \mu\text{m}$  long,  $240 \mu\text{m}$  wide, and about  $1.8 \mu\text{m}$  thick. The rectangular sample 2, fabricated from a single crystal with  $T_c = 84$  K, is  $320 \mu\text{m}$  long,  $60 \mu\text{m}$  wide, and about  $1.2 \mu\text{m}$  thick. The stacks are glued onto sapphire (sample 1) and MgO (sample 2) substrates and are mounted onto a hemispherical silicon

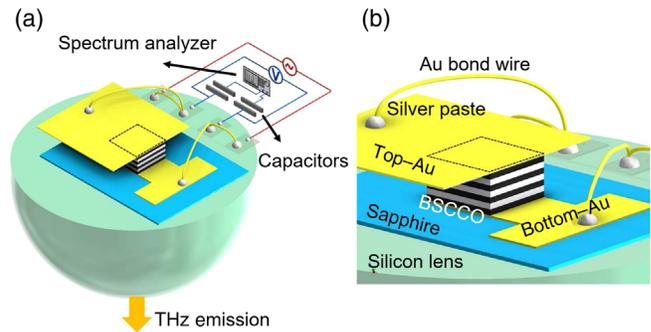


FIG. 1. (a) Schematic view of the experimental setup (not to scale), and (b) its closeup showing the BSCCO intrinsic Josephson-junction stack in GBG geometry.

lens (6 mm in diameter). Sample 1 is set in a helium flow cryostat and sample 2 is placed into a Gifford-McMahon cryocooler. IVCs and the terahertz emission power  $P_e$  are measured simultaneously, utilizing either a Golay cell (sample 1) or a  $\text{YBa}_2\text{Cu}_3\text{O}_7$  grain-boundary junction [19] (sample 2) as a detector. In the latter case we quote the height  $h_{\text{step}}$  of the radiation-induced Shapiro step as a measure for the emitted terahertz power ( $h_{\text{step}}$  follows the Bessel function  $J_1$  as a function of radiation amplitude; here, we work in the linear regime, where approximately  $h_{\text{step}} \propto P_e^{1/2}$ ). For some bias points, terahertz emission spectra are taken using a homemade Fourier spectrometer. For the (simultaneous) detection of self-mixing the voltage leads are connected to semirigid cables through two 470-pF capacitors as shown in Fig. 1(a). The mixing signal is amplified by a low noise amplifier with a gain of 35 dB (sample 1) or 50 dB (sample 2), in the frequency range from 100 MHz to 3 GHz, and then subsequently monitored by a spectrum analyzer.

## III. RESULTS

For sample 1, Figs. 2(a) to 2(c) show (a) the IVC and (b),(c)  $P_e$  vs, respectively,  $I$  and  $V$  at a bath temperature  $T_b = 40$  K. When the current  $I$  through the IJJ stack is increased from zero, the stack is in its zero-voltage state up to about 20 mA. When increasing  $I$  further, first some junctions and, at  $I = 60$  mA, basically all IJJs have switched to their resistive state. Lowering the current in the resistive state, the IVC exhibits a negative differential resistance down to about 28.5 mA. For lower currents the IVC exhibits a positive differential resistance and, for currents below 10 mA, groups of IJJs switch back to the zero-voltage state. Terahertz emission, cf. Figs. 2(b) and 2(c), is on a high power level in the current range from 20 to 7 mA, which is in a regime where there is no hot spot in the sample. The maximum emission power is found at  $I = 10.49$  mA, where the voltage across the stack is 1.26 V. The emission power  $P_e$  is  $2.98 \mu\text{W}$  at this bias. Emission can be also found in the high-bias regime—i.e., in

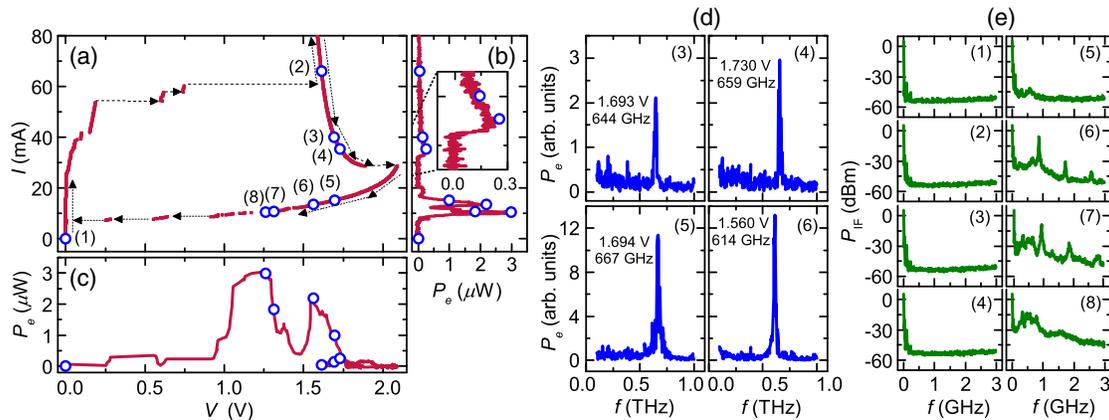


FIG. 2. For sample 1: IVC (a) and terahertz emission power vs (b) bias current and (c) voltage at  $T_b = 40$  K. Graphs (d) show Fourier spectra of emitted radiation taken at bias points (3) to (6) indicated in (a). Graphs (e) show IF spectra measured at bias points (1) to (8) indicated in (a). The inset in (b) is an enlargement of the emitting region at high bias, between 25 and 45 mA.

the presence of a hot spot—in a narrow current interval. Overall, the emission power of this sample at high bias is considerably lower than that which is detected at low bias.

Figure 2(d) shows spectra of the emitted terahertz radiation measured at the bias points (3) to (6) indicated in Fig. 2(a). These bias points are located on the outermost branch of the IVC where all IJJs in the stack are resistive [at bias points (7) and (8) some of the IJJs may have switched back to the zero-voltage state]. There is only one emission peak in each spectrum, occurring at the frequencies and voltages indicated in the graphs. From the emission frequencies  $f_e$  and the corresponding voltages across the IJJs stack, using  $V = Nf_e\Phi_0$ , we can estimate the number of IJJs involved in radiation as  $N = 1250 \pm 20$ , which is consistent with the thickness of the IJJ stack ( $1.8 \mu\text{m}$ ). At the points of strongest emission, it is likely that cavity resonances are excited. The case of a square IJJ stack is discussed in Refs. [62,65]. The resonances occur at frequencies  $f_r = (c_1/2L)\sqrt{k^2 + l^2}$ , where  $k$  and  $l$  are integers and  $c_1$  is the mode velocity of the in-phase resonance depending on  $T_b$  and the thickness of the stack [3].  $L = 240 \mu\text{m}$  is the length of the square. At temperatures well below  $T_c$  and a stack thickness well above the in-plane London penetration depth  $c_1 \approx (6-7) \times 10^7$  m/s, making the (4,2) and (2,4) modes occurring in an estimated frequency range between 580 and 670 GHz the most suitable candidates for the low-bias emission peak in  $P_e$  vs  $V$ , cf. Fig. 2(b). If points (5) and (6) represent different resonances, the (3,3) mode is an alternative candidate for point (5). For the high-bias emission peak containing points (3) and (4),  $c_1$  is likely to be lowered further. In addition, only part of the stack is superconducting, making it hard to estimate mode indices.

Figure 2(e) shows intermediate frequency (IF) spectra, as measured at the bias points (1) to (8) indicated in Fig. 2(a). The IF spectrum in the zero-voltage state at point (1) shows the noise background of the system. The IF spectrum in the

resistive state at point (2), where the whole sample is hot and without terahertz emission, looks similar to the noise background. For points (3) and (4), a hot spot is present in the stack and a weak terahertz emission signal appears. At these bias points the IF signal resembles the background spectrum of point (1). By contrast, at point (5), taken in the low-bias regime, the IF signal is stronger than the background noise. For slightly lower bias currents the terahertz emission gets gradually stronger, cf. Fig. 2(b), and the IF spectra exhibit different profiles; an example is shown for point (6) where one can identify three peaks at, respectively, 0.86, 1.72, and 2.58 GHz. When the stack is biased at point (7), the IF spectrum shows several peaks, the three main ones occurring at 0.93, 1.86, and 2.79 GHz. At point (8), which is at the highest terahertz emission peak in the low-bias regime, there is again a substantial increase in IF noise power, however, with no prominent peaks.

For sample 1 the terahertz emission power in the high-bias regime is poor, and potential additional signals in the IF spectra at points (3) and (4) may be hidden in the background. Thus, for comparison, Fig. 3 shows data for sample 2, taken at  $T_b = 18$  K. Figure 3(a) displays the outermost resistive branch of the IVC and 3(b) and 3(c) show  $h_{\text{step}}$  vs  $I$  and  $V$ , respectively. In the high-bias regime terahertz emission is on a high power level in the current range from 18.6 to 12 mA. The maximum emission power is found at  $I = 17.3$  mA, corresponding to a voltage across the stack of 0.76 V. At this bias the emission power is  $25 \mu\text{W}$  and  $f_e$ , determined from the voltage position of the Shapiro step induced in the detector junction, is 0.496 THz. In the whole emitting region  $f_e$  varied between 0.48 and 0.58 THz. From  $f_e$  and the corresponding voltage across the IJJs stack we estimate the number of IJJs involved in radiation as  $N \sim 740$ , which is somewhat less than expected from the thickness of the IJJs stack of  $1.2 \mu\text{m}$  ( $N \sim 800$ ). At low bias, a small emission peak is detected near the return current to the superconducting state. Here,  $f_e$  is about

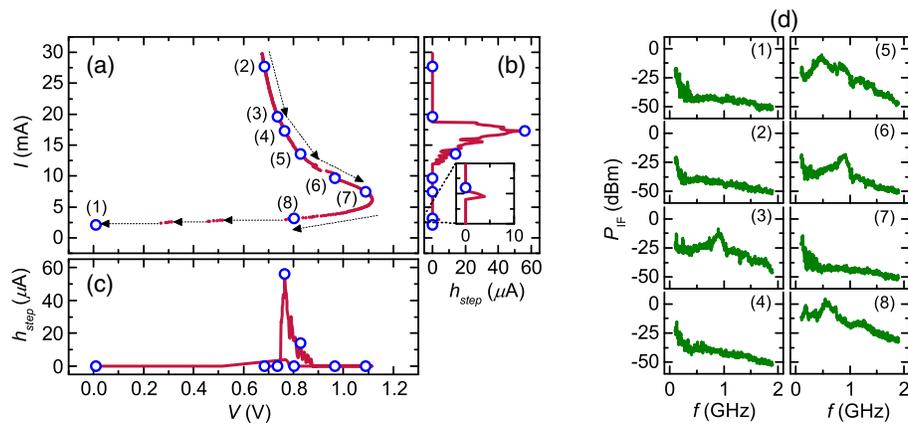


FIG. 3. For sample 2: IVC (a) and height  $h_{\text{step}}$  of the Shapiro step induced in the detector junction vs (b) bias current and (c) voltage at  $T_b = 18$  K. Graphs (d) show eight IF spectra, measured at bias points (1) to (8) marked in (a). The inset in (b) is an enlargement of the emitting region at low bias, between 2.5 and 3.5 mA.

0.57 THz, and we estimate  $N \sim 650$ , i.e., some junctions seem to have switched back to the zero-voltage state. When attributing the high-bias emission peak in  $h_{\text{step}}$  vs  $V$  to a single resonance mode one finds that, for a mode velocity  $c_1 = 6 \times 10^7$  m/s, a (1,0) mode having one half wave along the short side of the stack is compatible with the observed value of  $f_e$ . There also could be (0,  $l$ ) modes, with  $l$  counting half waves along the long side. The maximum emission power is in fact very high, favoring the (1,0) mode. For the low-bias emission near point (8) both the (1,0) and the (0,5) mode are candidates.

IF spectra are shown in Fig. 3(d) for the bias points indicated in (a). Spectrum (1) is taken at zero bias, yielding the noise background. Spectrum (2), taken at very high bias, resembles the noise background while at point (3), taken close to the onset of terahertz emission, the IF power has increased. At bias point (4) where terahertz emission is strongest, the IF spectrum is again close to the noise background while at point (5), where the terahertz emission power is still significant, a peak is visible in the IF spectrum. At bias point (6) the terahertz emission power is low but detectable. The corresponding IF spectrum is clearly above the background, in contrast to point (7) where neither at terahertz nor at the IF frequencies an emission power above background is detectable. Finally, the low-bias IF spectrum (8) is again above background and resembles the one of bias point (5). Note that at low bias the IF spectra could have contributions at higher frequencies than seen by our setup. High-resolution measurements of terahertz emission using the superconducting receiver [14] revealed low-bias linewidths that can exceed 5 GHz, implying that  $P_{\text{IF}}$  could have contributions up to such frequencies.

To understand the overall behavior of the IF spectra of Figs. 2 and 3, we suggest that the enhanced IF power (relative to the background) be due to self-mixing, occurring when the terahertz emission spectra consist of two or more nearby lines, with mixing signals and their harmonics appearing in the frequency window of the IF spectrum. By contrast, if the terahertz spectrum is pure or if no terahertz emission is present, we expect no additional IF signal. This

would imply that in the low-bias regime phase lock is incomplete, in agreement with the observations of Ref. [14] and also in agreement with self-mixing data obtained for other IJJ stacks not included here. In the high-bias regime there seems to be good phase lock near the point of strongest emission, however, particularly at the end points (in bias current) of the emitting region phase lock is again incomplete. For bias point (5) in Fig. 3(d), although inside the emitting region, the IF spectrum is well above background; thus also here groups of IJJs seem to radiate at different frequencies. All emitting regions are at least compatible with the previous findings that cavity modes have been excited. However, the observed self-mixing signals indicate that for some bias conditions there is either a strong mode competition or additional synchronization via the hot spot is truly necessary to achieve complete phase lock. The latter conclusion is particularly supported by the differences observed between the high-bias and low-bias regimes.

Sample 1 also emitted at very high temperatures, and Fig. 4 shows data taken at  $T_b = 80$  K. Figure 4(a) displays the IVC, and Figs. 4(b) and 4(c) show  $P_e$  vs  $I$  and  $V$ , respectively. Graphs (d) present Fourier spectra of emitted radiation at bias points (3) and (4) indicated in (a), and graphs (e) show IF spectra, as recorded at bias points (1) to (4). The IVC exhibits a positive differential resistance in the resistive state, for currents above 24 mA. At such elevated values of  $T_b$  the temperature in the stack, at least according to simulations, varies only modestly (a few K); a hot spot providing large temperature gradients is absent. Terahertz emission, cf. Figs. 4(b) and 4(c), is on a high power level in the current range from 33.9 to 24.6 mA. The maximum emission power of  $1.69 \mu\text{W}$  is found at  $I = 28.7$  mA, where the voltage across the stack is 0.528 V. The corresponding terahertz emission spectrum is shown in Fig. 4(d), bias point (3), with an emission frequency of 210 GHz, which is an unusual small value for IJJ stacks and presumably related to the fact that the Josephson plasma frequency  $f_{\text{pl}}(T) \propto \sqrt{j_c(T)}$  depends on temperature, saturating at low temperatures and approaching zero

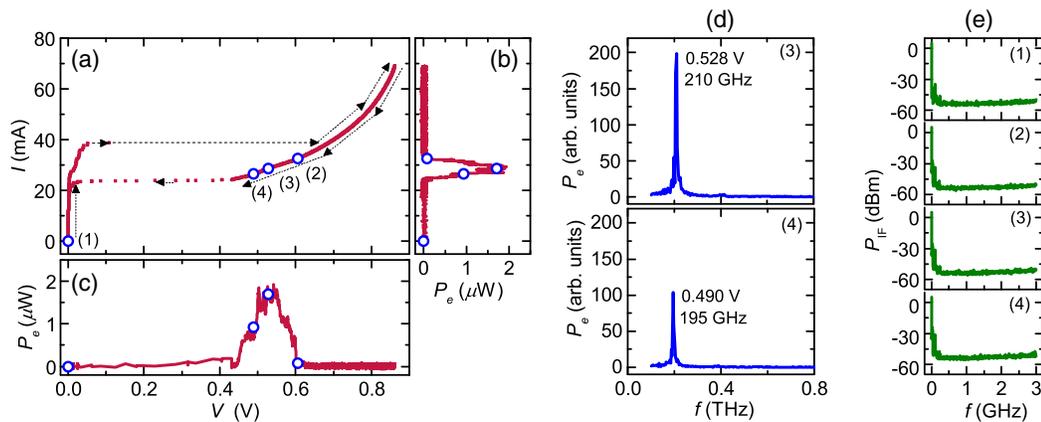


FIG. 4. For sample 1: IVC (a) and terahertz emission power vs (b) bias current and (c) voltage at  $T_b = 80$  K. Fourier spectra of emitted radiation taken at point (3) and point (4) are shown in (d) and four IF spectra, measured at bias points (1) to (4), are shown in (e).

for  $T \rightarrow T_c$  [68]. Using  $V = Nf_e\Phi_0$ , we can estimate the number of IJJs involved in radiation at this bias point as  $N \approx 1215$ , which is again consistent with the thickness of the IJJ stack ( $1.8 \mu\text{m}$ ). Given that close to  $T_c$  the mode velocity  $c_1$  very strongly depends on the sample temperature, we do not attempt to assign cavity-mode indices to possible resonances excited by ac Josephson currents.

The IF spectrum at point (1) gives the noise background of the system in the zero-voltage state. At bias point (2), which is at a current slightly above the onset of terahertz emission, the IF spectrum is at the background level. In fact, because of the high bath temperature, we cannot rule out that the temperature in the whole stack is above  $T_c$  at this bias. More importantly, the IF spectra are at the background level also at points (3) and (4), although strong terahertz emission occurs here. This indicates that good phase lock is achieved throughout the emitting region at this high bath temperature. This sounds counterintuitive but is in accordance with previous observations that the linewidth of radiation decreases with increasing bath temperature [14,54], suggesting that conditions for phase locking are fulfilled better at high bath temperatures, despite an increased thermal noise.

It is well known that the ability of serial arrays of Josephson junctions to phase lock strongly depends on the electrical parameters of the system [72]; in particular, the Stewart-McCumber parameter  $\beta_c = 2\pi I_c R^2 C / \Phi_0$ , where  $I_c$ ,  $R$ , and  $C$  are, respectively, the junction critical current, resistance, and capacitance, should be not much larger than unity. Indeed,  $\beta_c$  decreases with increasing bath temperatures, which may explain our observations.

#### IV. CONCLUSIONS

In summary, in order to obtain a better understanding of the purity of the terahertz emission signal generated by BSCCO intrinsic junction stacks, we developed a technique to study the emitted terahertz radiation via self-mixing and combined this technique with simultaneous off-chip terahertz emission detection and transport measurements. We find that, at low bath temperatures, in the low-bias regime

the terahertz emission (although showing only a single peak) is accompanied with noisy IF spectra, sometimes exhibiting several peaks. This indicates that groups of junctions oscillate at different frequencies. By contrast, in the high-bias regime the IF spectra can be near the background level. This points to a better ability to phase lock all IJJs in the presence of a hot spot, in agreement with the previous observation that in the high-bias regime the linewidth  $\Delta f_e$  of terahertz radiation is much lower than at low bias [14]. Our data also indicate good phase lock at  $T_b = 80$  K, which is important for potential applications using cryocoolers or liquid nitrogen as coolant.

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