

Spectral properties of phase-locked flux flow oscillator

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In the present paper, the results of theoretical modeling of flux flow oscillator (FFO) included into phase-locked loop (PLL) system and its comparison with experiment are outlined. To describe theoretically the dynamics of phase-locked FFO, we have considered two models: first order PLL system and the PLL system with integral-proportional filter. While the first order PLL system may be treated analytically even in the frame of nonlinear PLL model and gives satisfactory agreement with experimental results, it does not describe all peculiarities of spectral density, which may be described well by a more sophisticated model with the integral-proportional filter. The quantitative prediction of spectral ratio improvement due to increase of PLL regulation bandwidth is given.

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I. INTRODUCTION

The Josephson flux flow oscillator¹ (FFO) has proven to be the most developed superconducting local oscillator for integration with an superconductor-insulator-superconductor (SIS) mixer in a single-chip submillimeter-wave superconducting integrated receiver (SIR).² Such a receiver comprises in one chip a planar antenna and an SIS mixer, pumped by an integrated FFO. The dynamical and fluctuational properties of FFO have been intensively studied both experimentally and theoretically.¹⁻²¹ At the present time, flux flow oscillators have high enough output power, wide operational bandwidth, and rather easy tunability, but they still have a wide linewidth of the emitted radiation from the junction (from 2 to 30 MHz for a Nb–AlO_x–Nb FFO in 400–700 GHz working frequency range, see, e.g., Ref. 20).

In contrast to many other types of oscillators, the FFO fluctuations are mainly caused by *internal* wideband sources, such as thermal and shot noise, that result in a spectral line of emission with nearly perfect Lorentzian shape. The power in the so-called “wings” decreases much slower with frequency offset from the carrier than the Gaussian shaped spectral line obtained when *external* noise sources are dominant. Thus for the FFO, most external noise sources can be almost neglected since they, on one hand, are masked by the internal wideband fluctuations and, on the other hand, can be compensated by frequency locking to reference oscillators, so the main fundamental and technical problem at the present time is the reduction of “natural” FFO linewidth, which determines the final spectral quality of the phase-locked oscillator. In order to obtain the frequency resolution and frequency stability required for practical application of a heterodyne spectrometer (of at least 1 ppm), the integrated local oscillator (LO) must be phase locked to an external reference. To achieve this goal, several different types of ultrawideband phase-locked loop (PLL) systems (the attained regulation

bandwidth is about 10 MHz) has been developed and implemented.¹⁶⁻¹⁸ All these PLL systems are based on analog electronic components that allow them to operate for rather low signal-to-noise ratios of order unity. These achievements enabled the development of a 500–650 GHz integrated receiver for the Terahertz Limb Sounder¹⁹ (TELIS) intended for atmosphere study and scheduled to fly on a balloon in 2008. Here we report some results of theoretical modeling of FFO included into PLL system and its comparison with experiment.

II. STATEMENT OF THE PROBLEM AND MAIN RESULTS

It is known that the flux flow (traveling wave) oscillator is the voltage controlled oscillator (VCO).¹⁹ For a standard VCO, included into the phase-lock loop system, the equation for a phase difference φ between the reference and the phase-locked oscillator has the following form:²²

$$p\varphi = \Delta_0 - \Delta k(p)\sin(\varphi) + \xi(t). \quad (1)$$

Here $p = d/dt$, Δ_0 is the initial detuning of the phase-locked oscillator with respect to the reference, $\Delta = \mu s A A_m / 2$ is the bandwidth of synchronization (detention bandwidth), μ is the coefficient of conversion of the multiplier, s is the slope of the linear part of the characteristics of the control element, A is the amplitude of the phase-locked oscillator, A_m is the amplitude of the reference oscillator, and $\xi(t)$ is the white Gaussian noise with the correlation function $\langle \xi(t)\xi(t+\tau) \rangle = 2D\delta(\tau)$. The synchronization bandwidth is called the area of initial detunings, where the regime of synchronization is possible. The frequency capture bandwidth is called the area of initial detunings, where the regime of capturing is possible. The capture bandwidth is the area of initial detunings, where at any initial conditions the regime of synchronization takes place. In the general case, the synchronization bandwidth is wider than the capture bandwidth, and they coincide for the simplest first order PLL system.

The PLL with the idealized low bandpass filter, when $k(p) = 1$, i.e., the transfer coefficient of the filter in a wide

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band of low frequencies is equal to unity and for high frequencies is equal to zero, is called the idealized PLL system or the first order PLL system. In this case the PLL equation has the form

$$\frac{d\varphi}{dt} = \Delta_0 - \Delta \sin(\varphi) + \xi(t). \quad (2)$$

When the low frequency band filter is the integrating RC circuit, the transfer coefficient is $k(p) = \alpha / (\alpha + p)$, $\alpha = 1/RC$, and the PLL equation has the form

$$\frac{d^2\varphi}{dt^2} + \alpha \frac{d\varphi}{dt} = \alpha[\Delta_0 - \Delta \sin(\varphi)] + \alpha\xi(t). \quad (3)$$

In order to fulfill two contradictory requirements; to follow the fast variations of the signal and to remember the old information, one can use the proportionally integrating filter as the low frequency band filter (as it is in our experimental PLL system): $k(p) = \nu + (1 - \nu)/(1 + T_2 p) = (1 + T_1 p)/(1 + T_2 p)$, $\nu = T_1/T_2 = \alpha T_1$. In this case, the PLL system is described by the system of two first order differential equations:

$$\frac{d\varphi}{dt} = \Omega - \nu\Delta \sin(\varphi) + \xi(t), \quad (4)$$

$$\frac{d\Omega}{dt} = -\alpha\Omega + \alpha\Delta_0 - \alpha\Delta(1 - \nu)\sin(\varphi).$$

In the general statement of the problem, as it is considered in Ref. 23, it is necessary to find the spectral characteristics of the signal

$$z(t) = R_0[1 + r(t)]\cos[\omega_0 t + \varphi(t)], \quad (5)$$

where R_0 and ω_0 are the mean constant values of the amplitude and the frequency. In order to consider the process $z(t)$ to be sinusoidal oscillation with time-varying amplitude and phase, the temporal functions $r(t)$ and $\varphi(t)$ must be slow enough in comparison with $\cos(\omega_0 t)$, which we will consider to be fulfilled. The random function $r(t)$ represents relative fluctuations of the amplitude, while $\varphi(t)$ represents phase fluctuations, which are related to the frequency fluctuations η in the following way:

$$\varphi(t) = \int_{t_0}^t \eta(t) dt. \quad (6)$$

Let us suppose that $\langle r(t) \rangle = \langle \varphi(t) \rangle = \langle \eta(t) \rangle = 0$, and that correlation (or structural) functions $\Phi_r(\tau)$, $\Phi_\eta(\tau)$, $\Phi_{r\eta}(\tau)$ and the corresponding spectral densities $S_r(\omega)$, $S_\eta(\omega)$, $S_{r\eta}^0(\omega)$, $S_{r\eta}^1(\omega)$ are known.

Our task is to find spectral density $S_z(\omega)$ of the signal $z(t)$. In the following we neglect the amplitude fluctuations and will only consider the phase fluctuations $\varphi(t)$, since it is known that only nonstationary phase fluctuations lead to nonzero linewidth of an oscillation.²³ The correlation function of the second kind of the signal $z(t)$ is

$$\Phi_z(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} \langle z(t)z(t + \tau) \rangle dt.$$

We define the power spectral density of the signal $z(t)$ as

$$S_z(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Phi_z(\tau) \cos(\omega\tau) d\tau.$$

It can be demonstrated that, using the condition of slowness of $\varphi(t)$ in comparison with $\cos(\omega_0 t)$, the correlation function of the second kind may be presented in the form

$$\Phi_z(\tau) = A^0(\tau)\cos(\omega_0\tau) - A^1(\tau)\sin(\omega_0\tau), \quad (7)$$

where $A^0(\tau)$ and $A^1(\tau)$ are even and odd functions of the argument τ . In the case when amplitude fluctuations are neglected and the phase increment $\Delta\varphi(t) = \varphi(t + \tau) - \varphi(t)$ is a stationary process, the functions $A^0(\tau)$ and $A^1(\tau)$ have the forms

$$A^0(\tau) = \langle \cos \Delta\varphi \rangle, \quad A^1(\tau) = \langle \sin \Delta\varphi \rangle. \quad (8)$$

The asymmetry of the spectral line takes place for $A^1(\tau) \neq 0$. If fluctuations of amplitude and frequency are statistically independent, then the function $A^1(\tau)$ may be not equal to zero for asymmetric distribution of frequency fluctuations. In practice, however, the distribution of $\eta(t)$ is symmetric, as in the case when the nature of frequency fluctuations is the thermal noise, then the only correlation between $r(t)$ and $\eta(t)$ may lead to $A^1(\tau) \neq 0$ and, correspondingly, to asymmetric form of spectral line. Since we neglected by the amplitude fluctuations, this means that the spectral line is actually symmetric with respect to the carrier, and in the following we, therefore, will focus on an even component:

$$A^0(\tau) = \langle \cos \varphi(t + \tau) \cos \varphi(t) \rangle + \langle \sin \varphi(t + \tau) \sin \varphi(t) \rangle. \quad (9)$$

As it has recently been proven experimentally,^{14,16,17} and, in fact, confirms the assumption of symmetry of frequency fluctuations, the power spectral density of a free-running flux flow oscillator is well described by the Lorentzian shape:

$$W_\nu(\omega) = \frac{R_0^2}{2\pi} \frac{(\Delta f_{\text{FFO}}/2)}{(\Delta f_{\text{FFO}}/2)^2 + \omega^2}, \quad (10)$$

where Δf_{FFO} is the free-running FFO linewidth at 3 dB level. Analytically, the same result may be obtained from Eq. (2) for $\Delta_0 = \Delta = 0$ and noise intensity D equals $\Delta f_{\text{FFO}}/2$, see Ref. 23.

The FFO linewidth was measured in a wide frequency range up to 730 GHz using a novel experimental technique, see Refs. 15–17. A specially designed integrated circuit comprising the FFO, the SIS mixer, and the microwave circuit elements needed for the rf coupling is used for linewidth measurements. Both the SIS and the FFO junctions are fabricated from the same Nb–AlO_x–Nb trilayer. The signal from the FFO is applied to the harmonic mixer (SIS mixer operated in Josephson or quasiparticle mode) along with the signal from a reference frequency synthesizer, $f_{\text{syn}} \approx 20$ GHz. In order to prevent the synthesizer signal (as well as its harmonics) from reaching the FFO, a high-pass microstrip filter with a cut-off frequency of about 200 GHz is inserted between the FFO and the harmonic mixer. The intermediate frequency (IF) signal with frequency $f_{\text{IF}} = \pm(f_{\text{FFO}} - nf_{\text{syn}})$ is boosted first by a cooled amplifier and then by a room temperature amplifier for use in the PLL system. A part

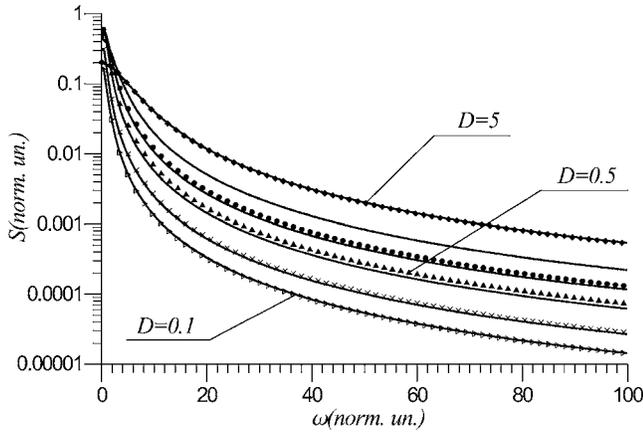


FIG. 1. The spectral density of $\sin \varphi(t)$ for the first order PLL system for $\Delta_0=0.5$, $\Delta=1$, and different values of noise intensity (from bottom to top) $D=0.1, 0.2, 0.5, 1, 2, 5$. Dots are results of computer simulations, solid lines are Lorentzian approximation.

of the signal is applied via the directional coupler to a spectrum analyzer which is also phase locked to the synthesizer using a common reference signal at 10 MHz. Thus, the spectrum obtained at f_{IF} of about 400 MHz, as well as the phase noise evaluated from these data, is the difference between the FFO signal and the n th harmonic of the synthesizer.

In the PLL unit the intermediate frequency is compared in a frequency-phase discriminator with a 400 MHz reference signal. The output signal proportional to the phase difference is returned via the Loop Bandwidth Regulator (maximum bandwidth of about 15 MHz) and then fed back to the FFO via a coaxial cable terminated with cold 50 Ω resistor mounted on the chip bias plate. In order to perform accurate linewidth measurement of a free-running FFO, the IF spectra have to be averaged with a sufficiently narrow video bandwidth. Specially designed frequency-lock (FL) system with narrow bandwidth (<10 kHz) was used for frequency locking of the FFO in order to measure the free-running linewidth Δf of the free-running FFO. In this case it is assumed that drift and only very low frequency noise are eliminated by the narrow-band feedback.

Our aim is to investigate how the shape of spectral density of FFO is changed if the FFO is included into the PLL system. Since in all experimental setups the shape of spectral line of free-running FFO was found to be Lorentzian, if proper shielding and frequency lock were realized, the results, presented below, seem to be rather generic, and theoretically the problem to phase lock the FFO becomes the problem to phase lock an oscillator with Lorentzian line shape and known width.

Let us start our consideration from the analysis of the first order PLL system, Eq. (2). Although this model is ideal and can hardly be realized in practice, it allows to qualitatively understand how the effect of PLL changes the spectral density of the phase-locked oscillator and, in particular, how the variation of the free-running linewidth and the variation of synchronization bandwidth will affect the wings of the spectral density, this is illustrated in Figs. 1 and 2. For the first order PLL system, one can consider not only linear PLL model, which works for the case of strong synchronization since it neglects the phase diffusion, but also the nonlinear

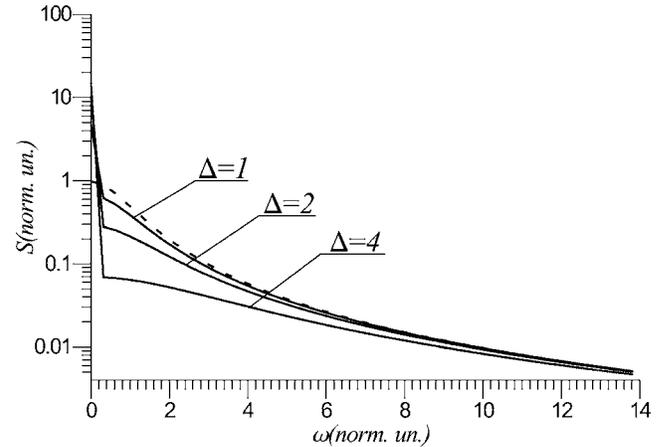


FIG. 2. The spectral density for the first order PLL system for $\Delta_0=0$, $D=1$, and different values of synchronization bandwidths Δ . Dashed line is $\Delta=0$.

one. Using the results of Ref. 24, the characteristic time scale of the correlation functions $\langle \cos \varphi(t+\tau) \cos \varphi(t) \rangle$ and $\langle \sin \varphi(t+\tau) \sin \varphi(t) \rangle$ may be obtained analytically for arbitrary values of noise intensity D , arbitrary initial detuning Δ_0 , and arbitrary synchronization bandwidth Δ . Also, as it has been demonstrated in Ref. 24, if Δ_0 is not so close to Δ , then the temporal behavior of the correlation functions is approximately exponential, and the corresponding spectral densities are approximately Lorentzian.

In Fig. 1 the spectral density of $\sin \varphi(t)$ for the first order PLL system for $\Delta_0=0.5$, $\Delta=1$ is presented as an example of the dependence of spectral wings on different values of noise intensity (free-running linewidth). It is seen that the Lorentzian shape gives a good fit both for small and for large noise intensities (which is rather unusual since the considered system is highly nonlinear), and the largest deviation is observed for intermediate noise intensity, of the order of potential barrier height (or, in other words, when the free-running linewidth is comparable with the synchronization bandwidth).

Now, let us consider how the value of synchronization bandwidth Δ affects the spectral density of the phase-locked oscillator. From Fig. 2, one can see that with increase of the synchronization bandwidth, the power contained in the spectral peak becomes larger, while spectral density around the peak decreases. On the other hand, the wings of the spectral density, located further than approximately 3Δ , are not actually affected by the PLL system. It should be noticed that in the case of the first order PLL system, it is difficult to distinguish visually what the value of Δ is, see Fig. 2. The spectral wings are smooth and there are no specific boundaries, such as climbs or spikes, indicating the synchronization bandwidth.

In spite of the simplicity of the first order PLL model, it nevertheless gives rather good agreement of the spectral ratio (SR) (ratio between the phase-locked power and the total power) with the experimental results, see Fig. 3, dashed line. However, it does not describe all peculiarities of the experimental spectral density even qualitatively (see Fig. 4). To resolve this problem, let us consider a more sophisticated model with the integral-proportional filter of the type that is

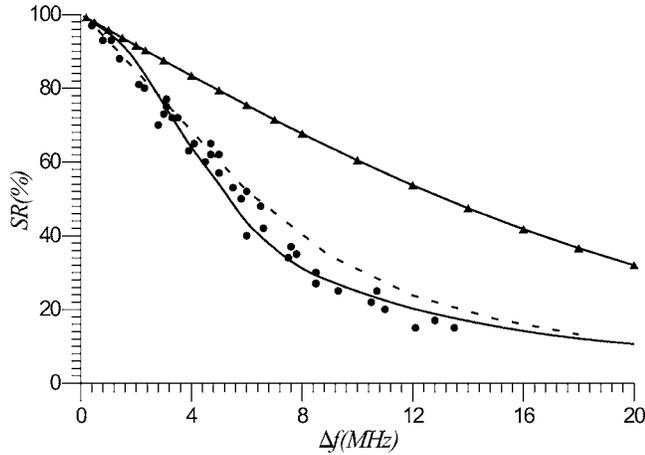


FIG. 3. The spectral ratio vs free-running linewidth Δf . Dots are experimental results, dashed curve is the first order PLL system, solid curve is PLL system with integral-proportional filter (nonlinear theory); curve with triangles is the linear theory [Eq. (11)].

utilized in the experimental PLL system. In contrast with the first order PLL system, in this case one can perform analytical analysis only in the frame of the linear PLL model. Following Ref. 25, the modulo of the transfer function of the PLL with the integral-proportional filter has the form

$$|H_\varphi(\omega)| = \Delta \frac{\sqrt{(\alpha^2 \Delta + [\alpha(\nu - 1) + \Delta \nu^2] \omega^2)^2 + (\alpha \omega - \nu \omega^3)^2}}{\alpha^2 \Delta^2 + [\alpha^2 + 2\alpha \Delta(\nu - 1) + \Delta^2 \nu^2] \omega^2 + \omega^4}. \tag{11}$$

In the frame of linear PLL model, the spectral density of the phase-locked signal may be presented from $|H_\varphi(\omega)|$ as $S_p(\omega) = |1 - H_\varphi(\omega)|^2 S_a(\omega)$, where $S_a(\omega)$ is the spectral density of the free-running oscillator. The transfer function $H_\varphi(\omega)$ [Eq. (11)] qualitatively describes the shape of the spectral density of the phase-locked signal, in particular, it describes the characteristic climb, see Fig. 4; it also gives good quantitative agreement for spectral ratio for the case of large spectral ratios only, see Fig. 3.

Therefore, let us consider the nonlinear PLL model that for the case of the integral-proportional filter may be ana-

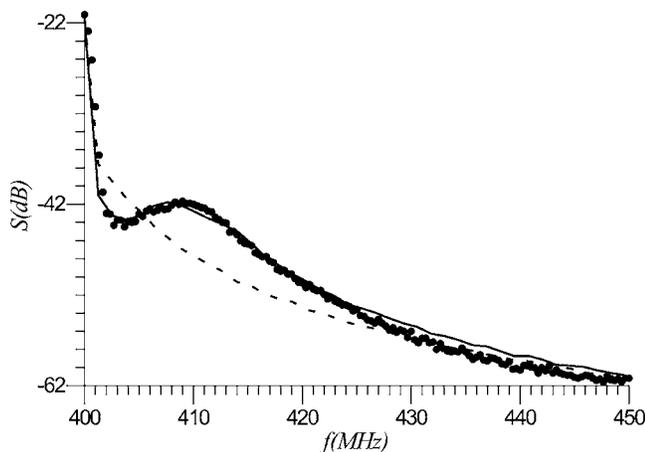


FIG. 4. Comparison of experimental and theoretical spectral densities (only the right half is shown due to symmetry) of the phase-locked FFO. Circles are experimental results, dashed curve is the first order PLL system [Eq. (2)]; solid curve is the PLL with the integral-proportional filter [Eq. (4)].

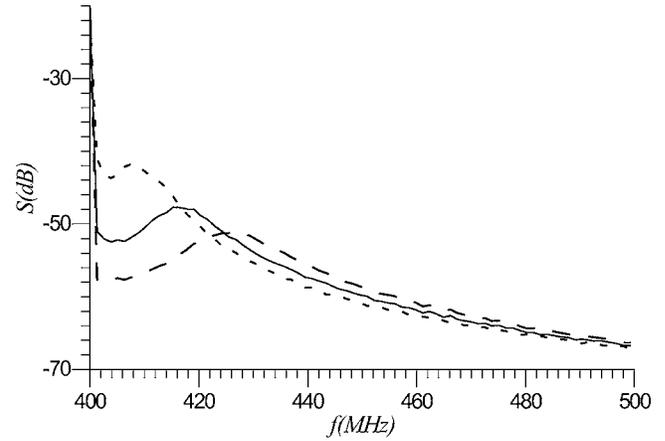


FIG. 5. Comparison of theoretical spectral densities (only the right half is shown due to symmetry) of the phase locked FFO for different regulation bandwidths. Short dashed line is $\Delta_R \approx 10$ MHz; solid line is $\Delta_R \approx 20$ MHz; long dashed line is $\Delta_R \approx 30$ MHz.

lyzed by means of computer simulations of Eq. (4). The results of comparison of experimental and theoretical spectral densities are presented in Fig. 4. It is seen that the experimental curve has a climb, after which it monotonically decreases. The first order PLL system does not describe such a climb. The model, described by Eq. (4), gives good quantitative agreement with the experimental results. The parameters for which good fitting of experimental results has been achieved are $D = \Delta f / 2 = 1.655$ MHz, $\Delta = 25$ MHz, $\Delta_0 = 17.2$ MHz, $\nu = 0.01$, and $\alpha = 7.4$ MHz. The spectral ratio, derived from the numerical simulation of Eq. (4) for $\Delta_R \approx 10$ MHz (where Δ_R is the regulation bandwidth, measured as the distance between the carrier and the spectral climb) is presented in Fig. 3 by the solid curve. It is seen that the agreement with the experimental results is nearly perfect. The results for linear theory [Eq. (11)] are presented by the curve with triangles. Good agreement with experimental results is achieved for spectral ratios above 90%.

Another important issue that should be addressed is the dependence of the spectral ratio on the PLL regulation bandwidth Δ_R . As one may see from Fig. 5, the increase of Δ_R by a factor of 2 or 3 significantly suppresses the spectral density around the carrier. The spectral ratio versus regulation bandwidth Δ_R is presented in Fig. 6. The spectral ratio increases almost linearly with PLL bandwidth; saturation takes place only at Δ_R of about five times larger than the free-running linewidth Δf , ensuring SR value as large as 90%. The effective bandwidth of the existing PLL system used for the FFO phase locking is limited mainly by the delay in the cables between cryogenic FFO and room temperature PLL electronics. To overcome this limitation, the cryogenic phase detector²⁶ is under development.

Equally important is decrease of free-running linewidth of an oscillator (compare curves for different Δf in Fig. 6). In Fig. 7 the spectral ratio versus free-running linewidth Δf is presented for three values of the regulation bandwidth Δ_R of 10, 20 and 30 MHz (see also Fig. 5). Optimization of the FFO layout and its parameters is underway (see Refs. 20, 21, and 27), but even for the present Nb–AlO_x–Nb FFOs and existing PLL systems, SR more than 50% can be obtained in

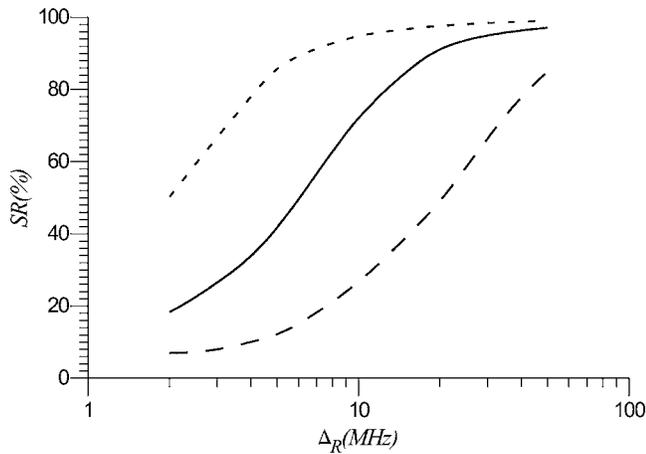


FIG. 6. The spectral ratio vs regulation bandwidth Δ_R for PLL system with integral-proportional filter (nonlinear theory). Short dashed line is $\Delta f = 1$ MHz; solid line is $\Delta f = 3$ MHz; long dashed line is $\Delta f = 10$ MHz.

the range of 550–700 GHz.²⁸ This made possible the development of the single-chip superconducting integrated spectrometer for the TELIS balloon project²⁸ intended to measure a variety of stratosphere trace gases.

III. CONCLUSIONS

In conclusion, the spectral properties of the phase-locked flux flow oscillator have been studied. It has been demonstrated that the spectral density of the phase-locked FFO may be described well by the PLL model with integral-proportional filter. The simplest first order PLL model does not describe the specific climb of the spectral density but gives rather good agreement with the experimentally observed spectral ratio. As it follows from the performed analysis, for PLL with 10 MHz regulation bandwidth, the free-running FFO linewidth must not exceed 5–6 MHz to phase lock at least 50% FFO power and 3 MHz to phase lock 70% FFO power. The obtained results allow us to predict a possible spectral ratio for a given PLL system at experimentally measured free-running FFO linewidth.

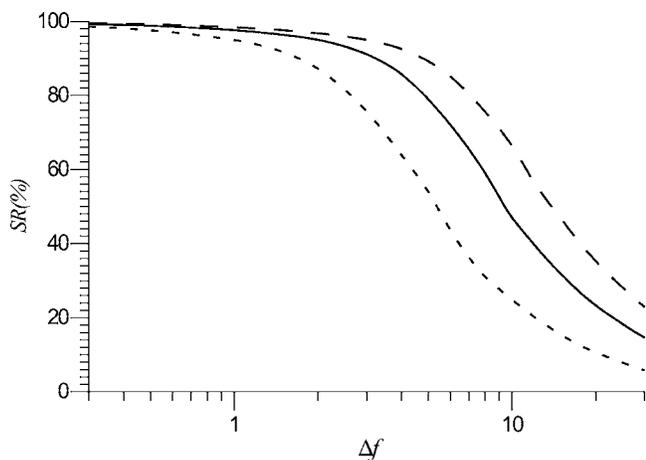


FIG. 7. The spectral ratio vs free-running linewidth Δf for PLL system with integral-proportional filter (nonlinear theory). Short dashed line is $\Delta_R = 10$ MHz; solid line is $\Delta_R = 20$ MHz; long dashed line is $\Delta_R = 30$ MHz.

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