Investigation of Spectral Properties of Phase-Locked Flux-Flow Oscillator

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Abstract—**We present the results of theoretical modeling of FFO included into PLL system and its comparison with experiment. To theoretically describe the dynamics of phase-locked FFO we have considered two models: the first order PLL system and the PLL system with integral-proportional filter. While the first order PLL system may be treated analytically, and gives good coincidence of the spectral ratio with experimental results, it does not describe all peculiarities of spectral density, that may be well described by more sophisticated model with the integral-proportional filter.**

I. Introduction

The Josephson Flux Flow Oscillator (FFO) [1] has proven to be the most developed superconducting local oscillator for integration with an SIS mixer in a single-chip submm-wave Superconducting Integrated Receiver (SIR) [2]. Such a receiver comprises in one chip a planar antenna and an SIS mixer, pumped by an integrated FFO. In order to obtain the frequency resolution and frequency stability required for practical application of a heterodyne spectrometer (of at least one part per million) the integrated local oscillator (LO) must be phase-locked to an external reference. To achieve this goal several different types of ultra wideband phase-locked loop systems (PLL) (the achieved regulation bandwidth is about 10 MHz) has been developed and implemented [3]-[5]. All these PLL systems are based on analog electronic components, that allow them to operate for rather low signal-to-noise ratios of order unity. These achievements enabled the development of a 550 - 650 GHz integrated receiver for the Terahertz Limb Sounder (TELIS) [6] intended for atmosphere study and scheduled to fly on a balloon in 2006. Here we report some results of theoretical modeling of FFO included into PLL system and its comparison with experiment.

II. Model

It is known, that the Flux Flow (Traveling Wave) Oscillator is the voltage controlled oscillator (VCO). For a standard VCO, included into the phase lock loop system, the equation for a phase difference between the reference and the phase locked oscillator has the following form [7]:

$$p\varphi = \Delta_0 - \Delta k(p)\sin(\varphi) + \xi(t) \qquad (1)$$

Here $p=d/dt$, Δ_0 - is the initial detuning of the phase locked oscillator with respect to the reference, $\Delta=\mu s A A_m/2$ is the bandwidth of retention (synchronization), μ is the coefficient of conversion of the multiplier, *s* is the slope of linear part of the characteristics of the control element, *A* is the amplitude of the phase locked oscillator, A_m is the amplitude of the reference oscillator, $\xi(t)$ is the white Gaussian noise with the correlation function $\langle\xi(t)\xi(t+\tau)\rangle=2D\delta(\tau)$. The PLL with the idealized low bandpass filter, when $k(p)=1$, i.e., the transfer coefficient of the filter in a wide band of low frequencies is equal to unity and for high frequencies is equal to zero, is called the idealized PLL system or the first order PLL system. If one has to fulfill two contradictory requirements: to follow the fast variations of the signal and to remember the old information, one can use as the low frequency band filter the proportionally-integrating filter (as it is in our experimental PLL system), then $k(p)=\upsilon+(1-\upsilon)/(1+T_2 p)=(1+T_1 p)/(1+T_2 p)$, $\upsilon= T_1/T_2=\alpha T_1$. In this case the PLL system is described by the system of two first order differential equations:

$$\frac{d\varphi}{dt}=\Omega-\nu\Delta\sin(\varphi)+\xi(t), \qquad (2)$$

$$\frac{d\Omega}{dt}=-\alpha\Omega-\alpha\Delta_0-\alpha\Delta(1-\nu)\sin(\varphi)$$

III. Results

Our aim is to investigate how the shape of spectral density of FFO is changing if the FFO is included into the PLL system. In order to get the required spectral density one has to compute the Fourier transform of the following correlation function: $\langle\cos\varphi(t)\cos\varphi(t+\tau)+\sin\varphi(t)\sin\varphi(t+\tau)\rangle$. Let us start our consideration from the analysis of the first order PLL system, $k(p)=1$, whose nonlinear model may be analyzed both analytically and numerically. Let us consider how the value of synchronization bandwidth affects the spectral density of the phase locked oscillator. From Fig. 1 one can see that with increase of the synchronization bandwidth the power, contained in the spectral peak becomes larger, while spectral density around the peak decreases. On the other hand, the wings of spectral density, located further than, approximately, 3Δ, are not, actually, affected by the PLL system. It should be noted, that it is difficult to distinguish, what is the value of Δ, looking at the plot (see Fig. 1), there are no specific boundaries indicating it. As it is seen from Fig. 2, the first

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order PLL model does not describe all peculiarities of the experimental spectral density even qualitatively. To resolve this problem, let us consider more sophisticated model with the integral-proportional filter (2). In the case of a "strong" synchronization, when the phase diffusion may be neglected (and, therefore, sinφ may be linearized) one can perform analytical analysis in the frame of linear PLL model. Following [8], the modulo of the transfer function of the PLL with the integral-proportional filter has the form:

$$|H_{\varphi}(\omega)| = \Delta\sqrt{\frac{(\alpha^2\Delta+[\alpha(\nu-1)+\Delta\nu^2]\omega^2)^2+(\alpha\omega-\nu\omega^3)^2}{\alpha^2\Delta^2+(\alpha^2+2\alpha\Delta(\nu-1)+\Delta^2\nu^2)\omega^2+\omega^4}}$$

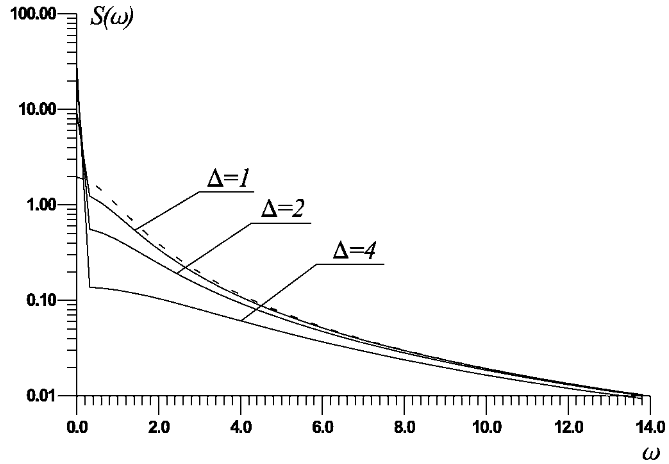

Fig. 1. The spectral density for the first order PLL system for Δ_0=0, D=1 and different values of synchronization bandwidths Δ. Dashed line - Δ =0 – spectral line of an autonomous oscillator.

The spectral density of the phase-locked signal may be presented from $|H_{\varphi}(\omega)|$ as: $S_p(\omega)=|1- H_{\varphi}(\omega)|^2 S_a(\omega)$, where $S_a(\omega)$ is the spectral density of the autonomous oscillator. The transfer function $H_{\varphi}(\omega)$ qualitatively describes the shape of the spectral density of the phase-locked signal, in particular, it describes the characteristic climb, but it gives good quantitative coincidence for spectral ratio (ratio between the phase-locked power and the total power) for the case of large spectral ratios only. Therefore, let us consider the nonlinear PLL model, which for the case of the integral-proportional filter may be analyzed by means of computer simulations of Eq. (2). The results of comparison of experimental and theoretical spectral densities are presented in Fig. 2. It is seen that the experimental curve has a climb, after which it monotonically decreases. The first order PLL system does not describe such a climb. The model, described by Eq. (2) gives good qualitative coincidence with the experimental results. The quantitative coincidence is, however, not perfect, that can be explained by the fact, that in the experimental setup the PLL circuit contains not one integral-proportional filter, but three of them. The spectral ratio, computed from numerical simulations of Eq. (2) is presented in Fig. 3 by solid line with crosses. It is seen, that the coincidence with the experimental results is nearly perfect. The results for linear theory are presented by the solid line with triangles. We note, that in spite of the simplicity of the first order PLL model, it nevertheless gives rather good coincidence of the spectral ratio with the experimental results, see Fig. 3.

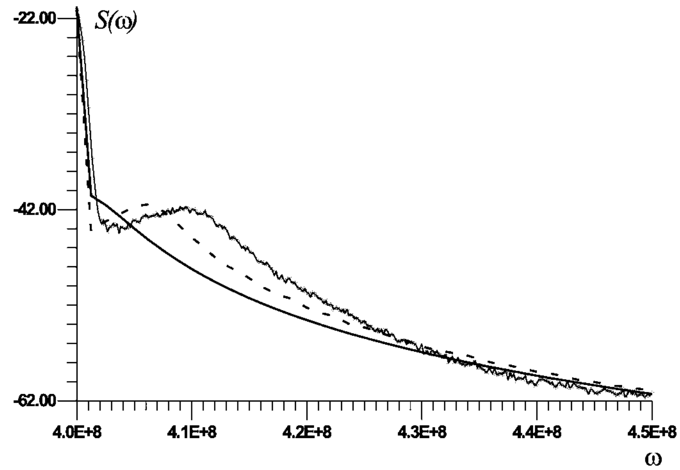

Fig. 2. The power spectral density. Solid line with dots - experimental results, solid line - first order PLL system, dashed line - PLL system with integral-proportional filter, nonlinear theory.

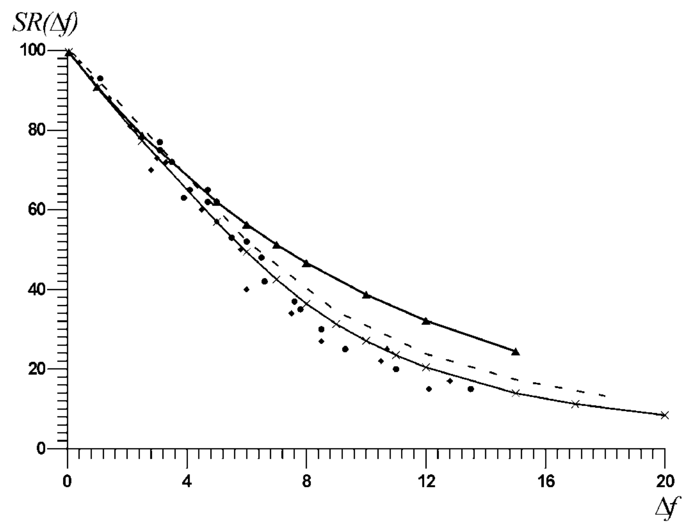

Fig. 3. The spectral ratio versus autonomous linewidth. Dots - experimental results, dashed line - first order PLL system, solid line with crosses - PLL system with integral-proportional filter, nonlinear theory, solid line with triangles - linear theory.

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