Linewidth of Josephson Flux Flow Oscillators

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Abstract. A combination of wide-band electronic tuneability and moderate free-running linewidth makes the Josephson Flux Flow Oscillator (FFO) a perfect on-chip local oscillator for integrated submm-wave SIS receivers. The possibility of FFO phase locking at all frequencies of interest has to be proven before one initiates real FFO applications. To achieve this goal a comprehensive set of linewidth measurements of the FFO operating in different regimes has been performed. FFO's with tapered shape have been successfully implemented in order to avoid the superfine resonant structure with voltage spacing of about 20 nV and extremely low differential resistance, recently observed in the IVC of the standard rectangular geometry. The obtained results have been compared with existing theories and FFO models in order to understand and possibly eliminate excess noise in the FFO. The intrinsic linewidth increases considerably at voltages above the boundary voltage because of the abrupt increase of the internal damping due to Josephson self-coupling. The influence of FFO parameters, in particular the differential resistances associated both with the bias current and with the applied magnetic field on the radiation linewidth, has been studied. Possible means of decreasing the free-running FFO linewidth will be discussed.

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I. Introduction

The Superconducting Integrated Receivers (SIR) [1, 2] with low power consumption are very attractive for both radio-astronomical research and remote monitoring of the Earth atmosphere in the submm wave range. A lightweight and compact ultra-sensitive submm SIR is a single-chip device, which comprises an SIS-mixer with a quasioptical antenna and a superconducting local oscillator. Flux flow oscillators (FFOs) [3] are presently being developed for integration with SIS-mixers for use in submillimeter wave all-superconductor integrated receivers [1, 2]. The FFO is a *long* Josephson tunnel junction in which an applied dc magnetic field and a dc bias current, I_B, drive a unidirectional flow of fluxons, each containing one magnetic flux quantum, $\Phi_0 = h/2e \approx 2\ 10^{-15}$ Wb. Symbol h is Planck's constant and e is the elementary charge. An external coil or an integrated control line with current, I_{CL}, can be used to generate the dc magnetic field applied to the FFO. According to the Josephson relation the junction biased at voltage V oscillates with a frequency $f = (1/\Phi_0)*V$ (about 483.6 GHz/mV). The velocity and density of the fluxons and thus the power and frequency of the emitted mm-wave signal may be adjusted independently by both the bias current and the magnetic field. A noise temperature below 100 K (DSB) has been achieved at 500 GHz for a single chip SIR [1,2].

The frequency resolution of a receiver (along with its noise temperature and the antenna beam pattern) is one of the major parameters in spectral radio astronomy. In order to obtain the required frequency resolution the local oscillator must be phase locked to an external reference. Up to now phase locking was demonstrated for a FFO biased on resonant Fiske steps in the frequency range 270 – 440 GHz [4]. In this case the initial free-running FFO linewidth is only about 1 MHz as significantly decreased by geometric Fiske resonances. Evidently, these resonances make continuous frequency tuning difficult.

Moreover, a superfine structure of closely spaced "resonances" with a frequency separation of $\approx$10 MHz has been observed [5] for an FFO of standard rectangular design. The fine structure manifests itself as a highly nonlinear relation between the measured FFO frequency and the bias or/and control-line currents. The FFO IVC consists of a series of separate steps with extremely low differential resistance. Since the frequency separation between adjacent resonances is comparable to the maximum bandwidth of the PLL system, switching between adjacent resonances creates considerable difficulties for phase locking of the FFO. Complete phase locking can be realized only in certain frequency ranges near the center of resonance. Even a small external interference may cause switching between adjacent resonances and thus prevents phase locking. In order to avoid this resonant structure we have developed a new design of the FFO [6] (see Fig.1). In this design the FFO is tapered from both sides in such a way that its width is decreased from 6 μm to 1.5 μm over a distance of 20 μm. As a result of this modification the resonant structure was almost totally suppressed, and hence the FFO frequency can be continuously adjusted [6].

2. Reasons for FFO linewidth broadening, new FFO design, FFO phase locking

Another phenomenon that considerably complicates phase locking of the FFO is the increase of the intrinsic linewidth at voltages $V > V_{JSC} = 1/3*V_{gap}$. It occurs due to an abrupt increase of the internal damping [7] caused by Josephson self-coupling (JSC); V_{gap} is the superconducting gap voltage. Moreover, the measured values of the FFO linewidth (FWHP, full width, half power) at $V > V_{JSC}$ [7] are considerably larger that calculated using existing theoretical models [8, 9] for the lumped tunnel junction:

$$\delta f = (2\pi/\Phi_0)^2 \ (R_d^B)^2 \ S_i(0), \tag{1}$$

where $S_i(0)$ is the power density of the low frequency current fluctuations, $R_d^B = \partial V/\partial I_B$ is the dc differential resistance which transforms current fluctuations to voltage (and phase) noise:

$$S_i(0) = (e/2\pi) \ \{I_{qp} \coth(v) + 2 \ I_s \coth(2v)\}, \quad \text{with} \quad v = (eV_{dc})/(2 \ k_B \ T_{eff}), \tag{2}$$

The radiation linewidth is determined by the wide-band noise spectral density of the bias current, $S_I(0)$, at low frequencies, $0 < f < \delta f_{AUT}$, where δf_{AUT} is the linewidth of the free-running junction [9, 10]. This value is a nonlinear superposition of thermal and shot noise down-converted by the Josephson junction to low frequencies.

Recently an additional noise term, which accounts for the influence of the white noise in the bias current introduced via the magnetic field channel, has been introduced [6, 11] in the phenomenological FFO model. The existence of this term was proven by a detailed analysis of the profile of the free-running FFO spectra, which have been measured using the integrated harmonic mixer technique [12]. A frequency locking system with a relatively low loop gain has been used to stabilize the FFO frequency. In this case it is assumed that only very low-frequency noise and drift are eliminated by the narrow-band feedback. Thus the intrinsic linewidth, determined by much faster fluctuations, which are assumed to be the "natural" ones, can be carefully measured. The shape of the FFO spectrum provides us with important information about the relationship between internal and external fluctuations as well as the spectral distribution of these fluctuations. According to theory [9, 10] the shape is Lorentzian for wide band fluctuations, whereas the profile will be Gaussian for narrow-band electromagnetic interference (EMI with frequencies smaller than the intrinsic FFO linewidth δf_{AUT}).

The measured FFO line shape is very well approximated by the Lorentzian expression [6, 11] both on the Flux Flow Step (FFS) and on the steep Fiske Step (FS) with extremely low differential resistance $R_d^B < 0.01\ \Omega$. The last fact is very important for linewidth calculations. It gives us experimental evidence that the wide-band fluctuations are present in the magnetic field, because according to [13] at small R_d^B the FFO linewidth is mainly determined by fluctuations of the magnetic field (the term proportional to $(R_d^B)^2$ becomes negligibly small at $R_d^B < 0.01\ \Omega$). Since an influence of the wide-band fluctuations in the biasing resistors feeding the control line (CL) can be neglected [13], it means that there must be a channel for transfer of the wide-band fluctuations from the bias current I_B to the FFO via the magnetic field. This has never been taken

into account in the theoretical considerations. Partially, it can be ascribed to self-field effect - a part of the bias current is flowing along the FFO electrodes creating a magnetic field proportional to I_B. Under these circumstances the layout of the FFO electrodes as well as a value of the differential resistance of the CL, $R_d^{CL} = \partial V_{FFO}/\partial I_{CL}$, become very important.

In order to reduce both the bias and control line differential resistances a new design of the FFO electrodes has been made. A photo of the central part of the FFO of a new design is shown in Fig. 1. The control line is realized as a separate strip in an additional superconducting layer. The details of the new FFO layout and the obtained results will be presented elsewhere, here we just notice that both differential resistances can be considerably reduced.

As a result of all these modifications we find a free-running FFO linewidth $\leq$ 10 MHz also in the flux flow regime at $V > V_{JSC}$: from 490 to 710 GHz, limited only by the gap frequency of the Nb-AlO$_x$-Nb FFO [14]. A free-running FFO linewidth of about 1 MHz has been measured on steep Fiske steps from 250 to 450 GHz. This provides the possibility of *phase locking* the FFO in the whole operational range from 250 to 710 GHz, and in particular for $V > V_{JSC}$ where continuous frequency tuning is possible. The FFO (as any Josephson junction) is a perfect voltage-controlled oscillator and hence its frequency can be stabilized and the FFO linewidth can be decreased by phase locking to an external reference oscillator using a phase-lock loop (PLL) system with bandwidth larger than δf_{AUT} (see Fig. 2). Actually, a PLL system will effectively suppress the influence from the external low frequency fluctuations and alter the differential resistances R_d^B and R_d^{CL} in the bias point. We have developed a special PLL unit utilizing the integrated SIS harmonic mixer to down-convert the FFO signal to a 400 MHz IF frequency.

A PLL system, of course, cannot affect wide-band thermal and shot noise fluctuations, but can decrease both differential resistances (defined as long time averages) in Eq. (1) to zero, e.g., that $R_d^B = 0$ is seen as a constant-voltage step in the FFO I-V curve. In principle, this results in zero residual radiation linewidth measured relative to the reference oscillator. The 1 Hz linewidth displayed at 449.6 GHz is due to the limited resolution bandwidth of the spectrum

analyzer. At lower FFO frequencies on Fiske steps, where the free-running linewidth δf_{AUT} is considerably smaller, an even better residual phase noise has been realized (see Fig. 2b). One can see that residual phase noise close to the carrier is as low as 90 dB below the carrier.

3. Intrinsic FFO linewidth: theory and experiment.

On the base of the assumption concerning the influence of wide-band fluctuations via magnetic filed the linewidth of FFO has calculated using the perturbed Sine-Gordon model [15] with normalized bias current, η, and magnetic field, Γ, fluctuations: $\eta = \eta_0 + \eta_F(x, t)$, $\Gamma = \Gamma_0 + \Gamma_F(x, t)$. These fluctuations are supposed to be wideband noise and are small, i.e. for $\eta_0 \neq 0$ and $\Gamma_0 \neq 0$ the variances of $\eta_F(x, t)/\eta_0$, $\Gamma_F(x, t)/\Gamma_0$ are much smaller than unity. Following [9], we suppose that $\eta_F(x, t)$ has a Gaussian distribution with zero mean value $<\eta_F(x, t)> = 0$ and that its spectral density is so wide that $\eta_F(x, t)$ for a one dimensional junction may be approximated by white noise with the correlation function:

$$< \eta_F(x,t)\eta_F(x',t') >= \frac{2k_B T \omega_p L}{R_N I_c^2 \lambda_J} \delta(x-x')\delta(t-t') \tag{3}$$

Here $<\,>$ denotes ensemble average, k_B is the Boltzmann's constant, T - temperature, ω_P - plasma frequency, R_N - normal state resistance, λ_J - Josephson penetration length and I_c and J_c are the critical current and the critical current density, respectively. Also we suppose, that the internal control line current fluctuations are small and that all the magnetic field fluctuations are induced by the fluctuating bias current: $\Gamma_F(x, t) = \sigma \eta_F(x, t)$, where the factor σ may be obtained from experiment. In the present designs σ usuallyis of the order of unity. We note, that σ will be different for different types of FFOs and depends on the junction geometry and the current distribution in the base electrode.

In the so-called flux-flow regime the effective nonlinearity of the system is drastically reduced due to fulfilling the following conditions: $\eta/\alpha >> 1$ and (or) $\Gamma >> 1$, where α is the damping coefficient. In order to derive the linearized equation for the slow component of the

phase $\phi(x, t)$ one can use the Poincare method: obtain the solution as a series expansion in the small parameter $\varepsilon = (\alpha/\eta)^2 << 1$. From the linearized equation for slow-phase dynamics, one can obtain the correlation function $K_v[\tau]$ for frequency fluctuations. Using the standard technique of frequency modulation theory [10], one can get the expression for the dimensionless linewidth in the case of stationary Gaussian frequency fluctuations [15]

$$\Delta\Omega = \frac{\pi}{\int\limits_0^\infty \exp[-\chi(\tau)]d\tau}, \qquad \text{where } \chi(\tau) = \frac{1}{2}\int\limits_{-\tau}^{+\tau}(\tau - |\xi|)K_v[\xi]d\xi \qquad (4)$$

is a statistical structural function, which takes into account the finite length of the FFO and the parametric broadening of the linewidth due to the effect of the second and higher harmonics. However, it has been demonstrated [15] that the FWHP linewidth of the FFO may be well described by the following approximate expression

$$\delta f_{FFO} = \frac{1}{\pi}\left(\frac{2\pi}{\Phi_0}\right)^2 (R_d^B + \sigma R_d^{CL})^2 \frac{k_B T}{R_N}, \qquad (5)$$

where the current-field noise conversion factor σ is of the order unity. For $\sigma = 0$ Eq. (5) is identical to Eq. (1) for the lumped junction (see also [16]). As with the Thomson oscillator we can neglect higher harmonics for the practical range of FFO parameters.

The calculated dependence of the linewidth on R_d for the case of wide-band fluctuations only via I_B ($\sigma = 0$) is shown in Fig. 3 by the dashed line 1). The solid line is calculated for each experimental point taking into account a measured R_d^{CL}. One can see that the calculations coincide with the linewidth data measured for a FFO with the separated CL over the whole range of experimental parameters using only one value of $\sigma = 1$ both on the Fiske steps and on the flux flow step. The dependence for fixed values of $R_d^{CL} = 0.003\ \Omega$ and $R_d^{CL} = 0.04\ \Omega$ are presented by the dotted and the dash-doted lines 2) and 3), respectively. These lines confine the range of possible linewidth variations. Even better agreement with experimental data has been found [11] if we use the exact expression Eq. (2) for $S_i(0)$, which taking into account JSC effect. The best fit

was obtained for $\sigma=2.9$, that exactly corresponds to the value of the ratio between the currents, which flowing in the separate CL and the base electrode and producing the same magnetic field. In the case when the base electrode was used as a control line the value of σ is about unity.

Conclusion

New design of the FFO enables us to suppress the superfine resonance structure and considerably reduce the differential resistance related to both magnetic field and bias current. As a result of all these modifications we find a free-running FFO linewidth ≤ 10 MHz in the flux flow regime at $V > V_{JSC}$: from 490 to 710 GHz, limited only by the gap frequency of Nb. A free-running FFO linewidth of about 1 MHz has been measured on steep Fiske steps from 250 to 450 GHz. This provides the possibility of phase locking the FFO in the whole operational range from 250 to 710 GHz, in particular, for $V > V_{JSC}$ where continuous frequency tuning is possible. Calculations based on a theory, which takes into account an additional noise term, are in the excellent agreement with the experimentally measured FFO linewidth. This enables quantitative calculations of the FFO linewidth.

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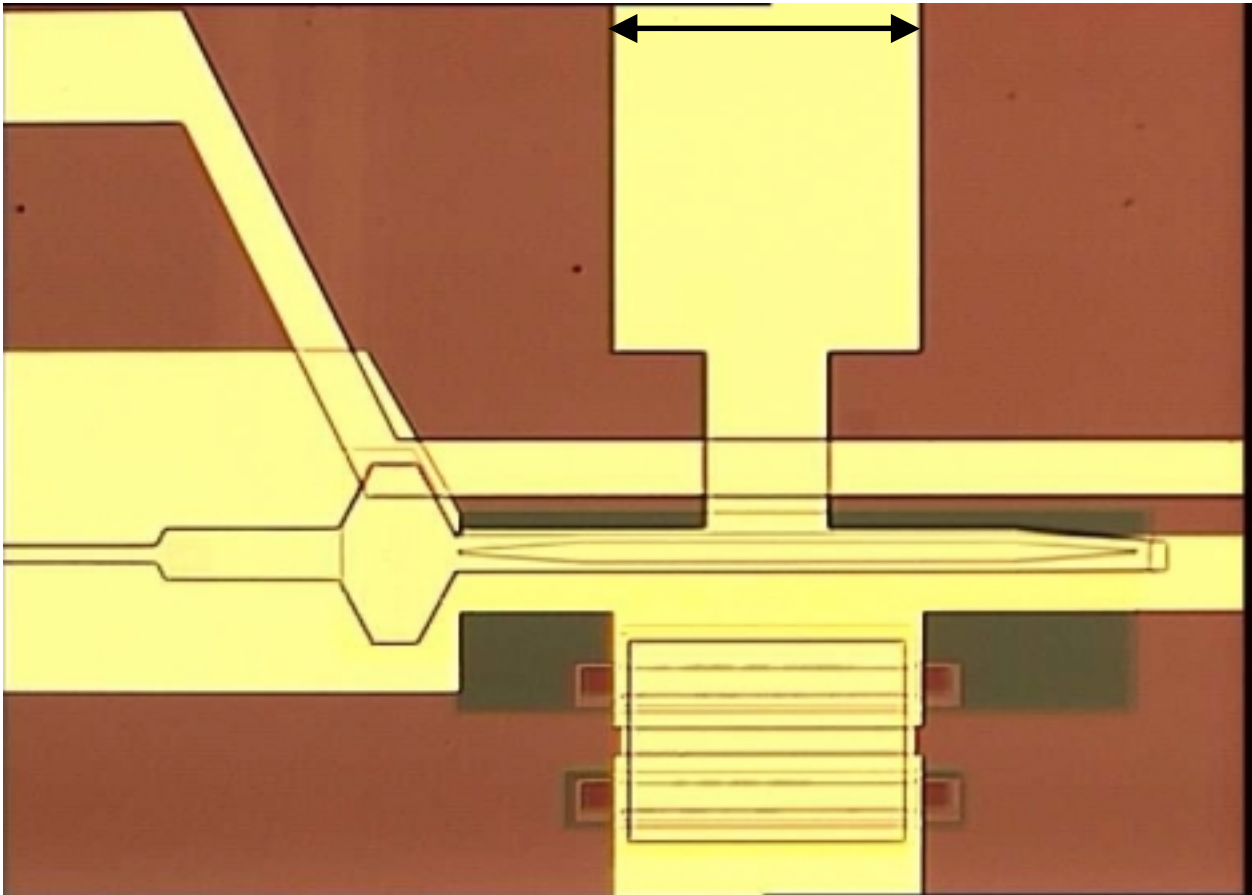
Figure Captions

Fig. 1. Photo of the central part of the superconducting integrated circuit used for linewidth measurements, showing the layout of the FFO of a new design. One can see a tapered FFO with the bias resistor placed below the FFO, the separate control line (on top of the FFO), and the impedance transformer on the left side.

Fig. 2. Residual spectra of the FFO operating at 459.6 GHz. **(a)** Spectrum analyzer span - 10 MHz, the FFO is frequency locked (A) and phase locked FFO (B); the free-running linewidth is δf_{AUT} = 520 kHz. **(b)** Down-converted spectra, span 100 Hz. Note the 1 Hz linewidth and the 90 dB signal to noise ratio.
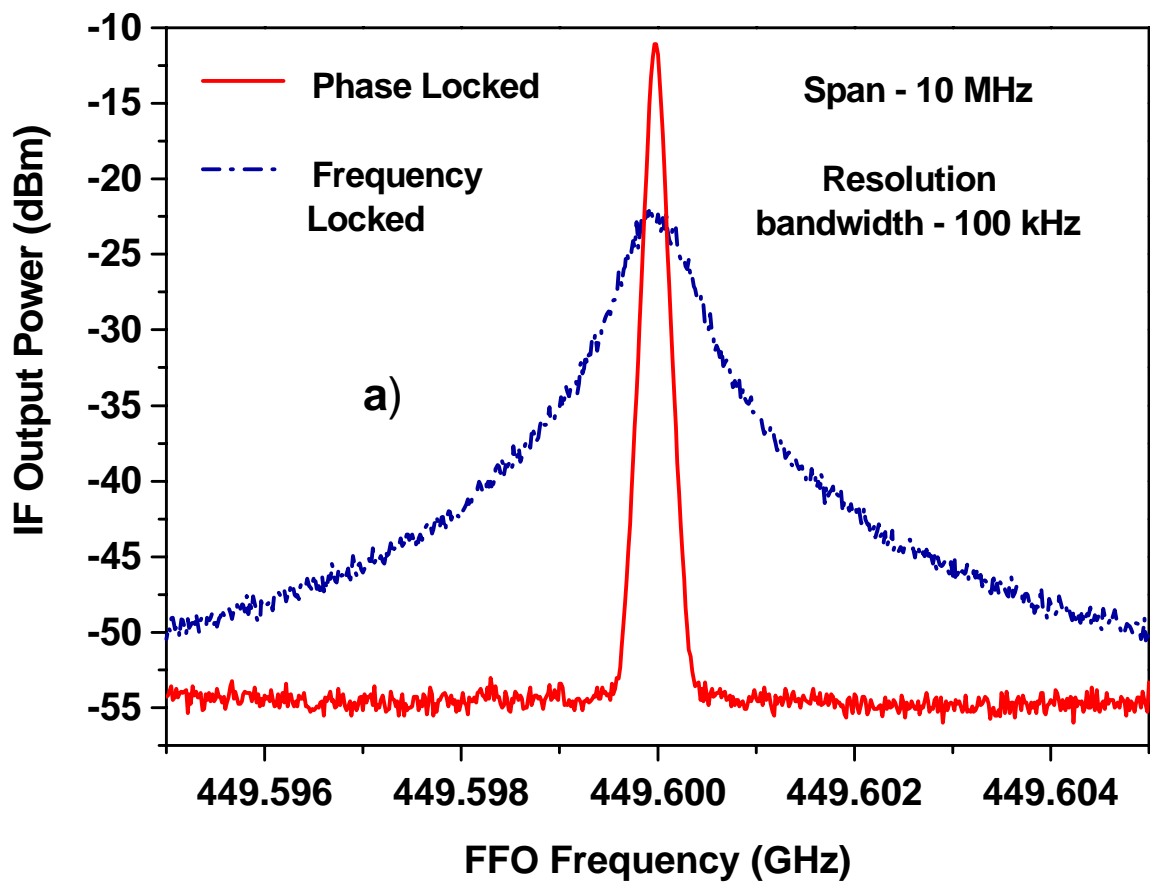
Fig. 3. Dependence of the FFO radiation linewidth on the differential resistance R_d (see text). All theoretical curves are calculated using the experimental values, R_N = 0.036 Ω, T_{eff} = 4.2 K.
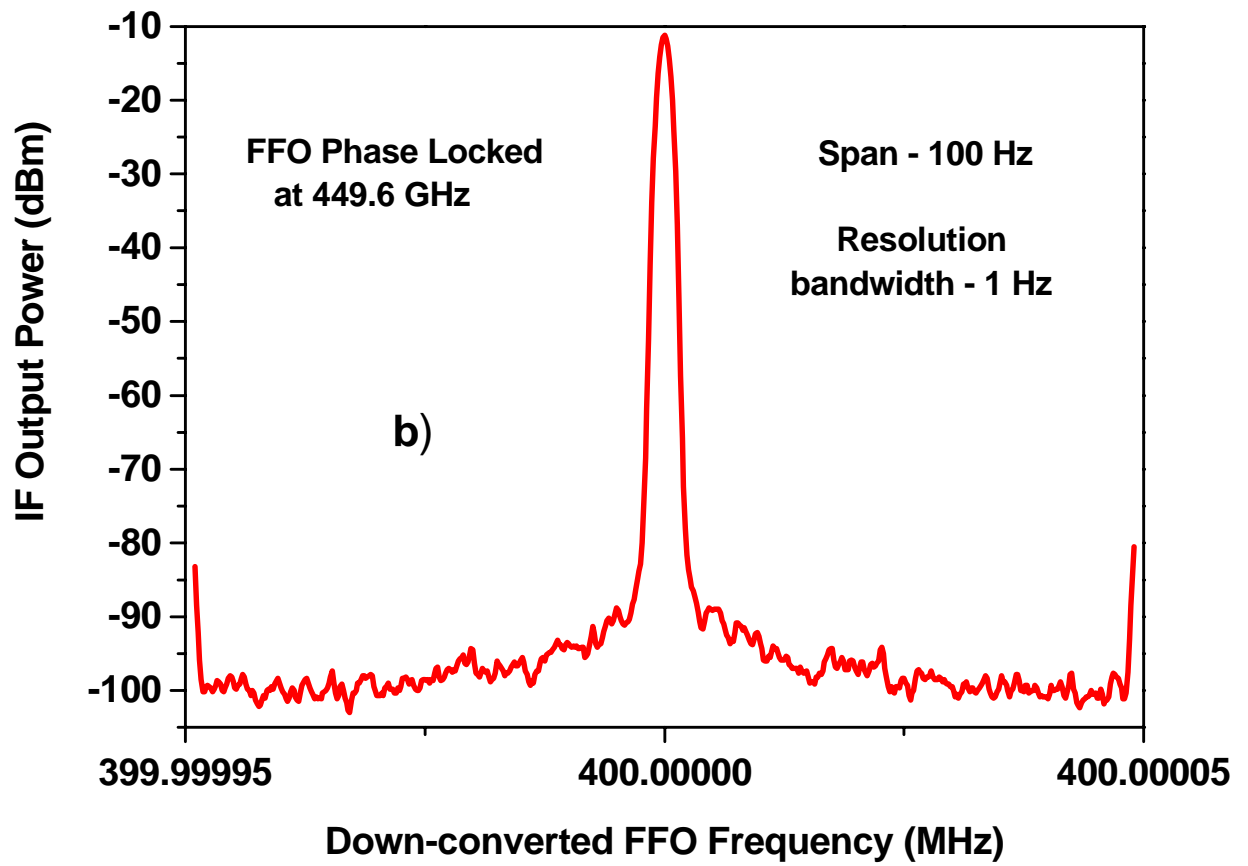
100 μm

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a)

Span - 10 MHz

Resolution
bandwidth - 100 kHz

Phase Locked

Frequency
Locked

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FFO Phase Locked at 449.6 GHz

Span - 100 Hz

Resolution bandwidth - 1 Hz

b)
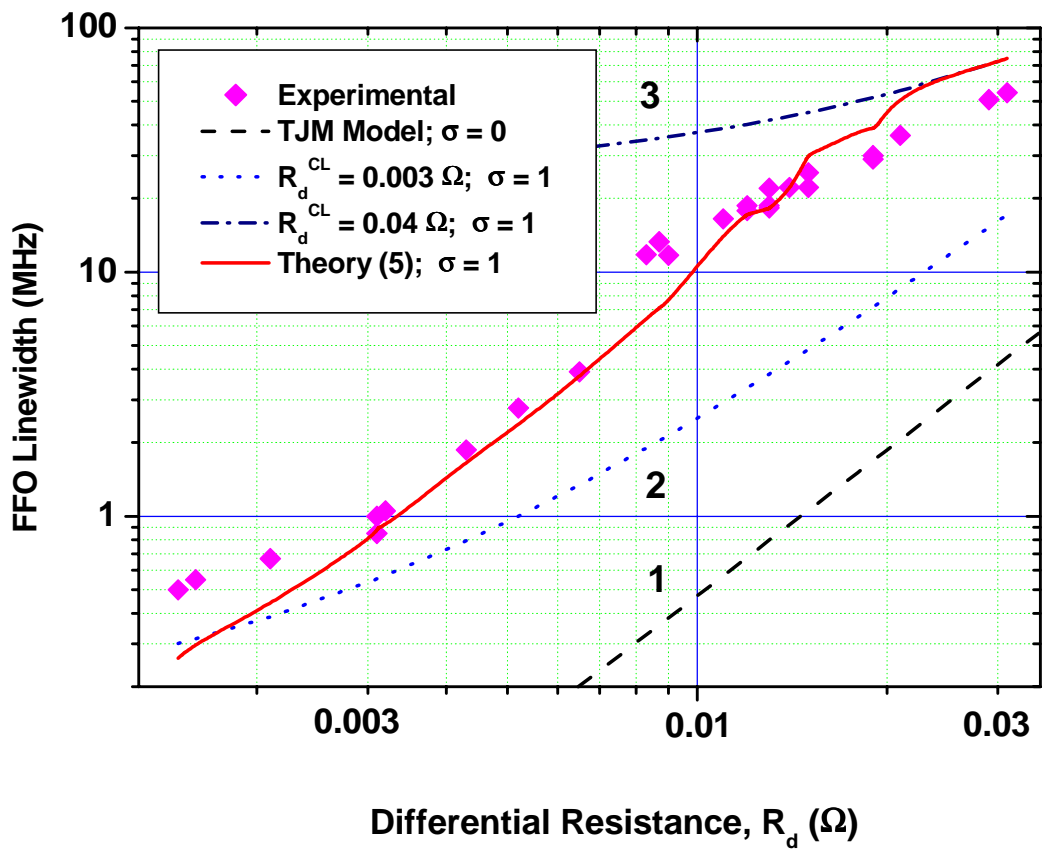
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V. Koshelets, et al.; **FIG. 3** (EUCAS-2001)