Spectral measurements of THz radiation from intrinsic Josephson junction BSCCO stacks; phase locking of the BSCCO oscillators

Valery P. Koshelets, Nickolay V. Kinev, Andrey B. Ermakov, Raphael Wieland,
Eric Dorsch, Olcay Kizilaslan, Dieter Koelle, Reinhold Kleiner, and Huabing Wang

Abstract—**Coherent THz emission from stacks of intrinsic Josephson junctions (IJJs), created naturally in the $Bi_2Sr_2CaCu_2O_{8+x}$ unit cell, was measured by a superconducting integrated receiver (SIR). The noise temperature of the SIR is as low as 120 K and its spectral resolution is better than 0.1 MHz, which exceeds the resolution of modern terahertz-range Fourier spectrometers by several orders of magnitude. In this report, the results of the spectral measurements of THz radiation emitted from intrinsic Josephson junction stacks are summarized. The phase-locked SIR has been used also for the locking of the BSCCO oscillator under the test. About 10 % of the power emitted by the BSCCO oscillator operating at 563 GHz with free-running linewidth of 13.5 MHz has been phase locked. The possibility of mutual locking of two BSCCO oscillators fabricated on one substrate has been investigated by direct measurements of emitted radiation spectra by the SIR.**

Index Terms—**oscillators and spectrometers, phase locking, stacks of intrinsic Josephson junctions, superconducting integrated circuits, terahertz receivers.**

I. INTRODUCTION

In recent years, coherent THz emission has been obtained from stacks of intrinsic Josephson junctions (IJJs), created naturally in the BSCCO unit cell with the CuO layers forming the superconducting electrodes and the BiO and SrO layers forming the barrier layer [1, 2]; a 1- µm-thick crystal consists of about 670 IJJs. Terahertz emission from BSCCO mesa has been obtained both at a low-bias (where the temperature distribution in the stack is almost homogeneous) and a high-bias regime (where an over-heated part and a cold part of the sample coexist) [3, 4].

Coherent emission above 1 THz by intrinsic Josephson BSCCO junction stacks with improved cooling has been demonstrated [5, 6]. Due to the variable size of the hot spot and the temperature rise caused by the self-heating, the emission frequency can be tuned over a wide range of up to 700 GHz [5]. So far, emitted by one device a maximum power of up to 30 µW was obtained [7, 8]. That is already enough for practical implementations, although for most applications good spectral properties are required.

The spectral characteristics of the oscillator were studied using the low-noise SIR with superconducting local oscillator, which was developed at Kotel'nikov IREE [9 - 12] to perform spectral studies of the electromagnetic radiation in the frequency range 450–700 GHz. The SIR was successfully implemented for measuring the profiles of the spectral lines of the gas-molecule radiation and absorption on board of high-altitude balloon [10-12] and can be used for the spectral study of any external terahertz oscillator radiating in the operation frequency range of the receiver. The best noise temperature of the SIR is 120 K and its spectral resolution is better than 0.1 MHz, which exceeds the resolution of modern terahertz-range Fourier spectrometers by several orders of magnitude.

Two configurations for the oscillator and receiver location were used: in the first case the oscillator was located in a cryostat of the SIR in the vicinity of the mixing unit [4, 13]; in the second case the oscillator and the receiver were located in independent cryostats with Mylar quasioptical windows. The SIR operates at temperature of about 4.5 K, whereas the optimal BSCCO-oscillator temperature is 20–50 K. The spectral lines of the oscillator radiation are recorded by the SIR and displayed on the spectrum-analyzer screen in the intermediate-frequency range 4 – 8 GHz. The spectrum analyzer allows one to average the signal, read it by a computer, and perform other necessary digital operations for the spectrum analysis and processing.

Application of the SIR has allowed to measure radiation emitted from intrinsic Josephson junction stacks both at a low-bias and a high-bias regime with spectral resolution better than 1 MHz [4]. While at low bias we found that the linewidth (LW) is not smaller than 500 MHz, at high bias, the emission LW turned out to be in the range 10–100 MHz. We attribute this to the hot spot acting as a synchronizing element; a LW as narrow as 7 MHz has been recorded at high bias [14].

The work was carried out at IREE RAS within the framework of the state task and partially supported by the Russian Foundation for Basic Research, grant No. 17-52-12051, and by the Deutsche Forschungsgemeinschaft, Project KL-930/13-2.

V. P. Koshelets, N V. Kinev, A B. Ermakov are with the Kotel'nikov Institute of Radio Engineering and Electronics RAS, Moscow 125009, Russia. (e-mail: valery@hitech.cplire.ru).

R. Wieland, E. Dorsch, O. Kizilaslan, D. Koelle, R. Kleiner are with Physikalisches Institut and Center for Collective Quantum Phenomena in LISA+, Universität Tübingen, D-72076 Tübingen, Germany (e-mail: kleiner@uni-tuebingen.de).

H. B. Wang is with the Research Institute of Superconductor Electronics, Nanjing University, Nanjing 210093, China (e-mail: hbwang@nju.edu.cn).

It is important to note that the tuning of the BSCCO oscillator frequency is continuous over the range; that was confirmed by fine-tuning of the SIR local oscillator (LO) frequency. Actually for such measurements the lowest frequency is about 450 GHz due to the design of the SIR mixer, while losses in the Nb interconnection lines of the SIR restrict the measurements at frequencies higher than 730 GHz. A combination of the BSCCO mesa and the SIR was used to accurately measure the terahertz absorption spectra of ammonia and water vapor [15, 16]. In this experiment, the bias current through the BSCCO emitter is kept at a constant value, tuned to the respective gas-line frequency, and intermediate-frequency (IF) spectra are taken using the SIR. These are quite encouraging results, although for most practical applications phase-locking of the cryogenic oscillator to a stable reference is required.

II. PHASE-LOCKING OF A BSCCO OSCILLATOR

To check a principle possibility of such locking we used the phase-locked SIR not only for detection of the BSCCO oscillator emission, but also for further locking of the oscillator under test [13, 14]. A simplified block diagram of the experimental setup for phase locking of the BSCCO oscillator is presented in Fig. 1. The BSCCO oscillator signal initially down-converted by the SIR to the IF band 4-8 GHz was then down-converted one more time to a secondary IF band 0.1 – 0.9 GHz. The obtained IF signal is actually a convolution of the BSCCO oscillator signal and stable phase-locked SIR LO. This signal is applied to the room-temperature phase-locking loop (PLL) system, where the phase of the signal is compared with the phase of the stable reference (f_{ref} = 400 MHz). The PLL was equipped by additional Frequency Discriminator (FD), which compares the signal with an internal 400 MHz resonance tank; the FD error signal was applied to the oscillator in parallel with PLL signal and can be adjusted separately. Note that all reference sources used in the experiment (400 MHz, 6 GHz, and tunable 19-21 GHz LO used for the FFO phase-locking) were internally synchronized to the common 10 MHz reference. The error signal is returned back to the BSCCO oscillator to control its phase via an additional 5 Ohm resistor placed in the bias line of the IJJs stack; the resistor is mounted directly on the oscillator holder.

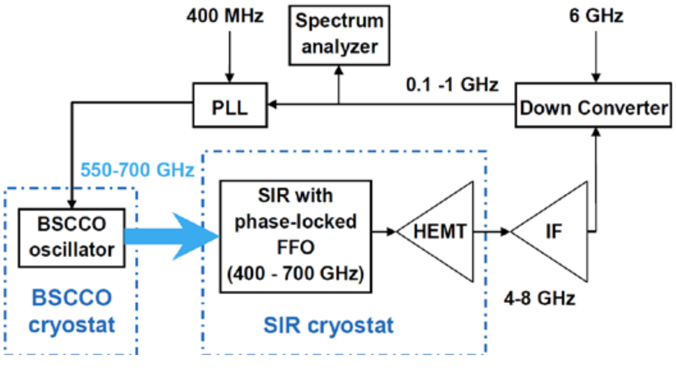

Fig. 1. Simplified block-diagram of the experimental set-up for the phase locking of a BSCCO oscillator by using the SIR with phase-locked FFO.

Results of the BSCCO oscillator frequency and phase locking are presented in Fig 2. The LW of the BSCCO oscillator frequency locked at 563 GHz is 13.5 MHz (Fig. 2a); about 10 % of the oscillator power has been phase locked. The ratio of the phase-locked power to the total power emitted by the oscillator is called a “spectral ratio” (SR); the obtained SR value of about 10 % is reasonably good result taking into account the wide linewidth of the BSCCO oscillator and the long length of the PL loop. At decreasing of the spectrum analyzer resolution bandwidth (RBW) the signal power in the phase-locked peak remains almost unchanged while the unlocked power in the wings is lowering proportionally to the RBW (Fig. 2b, 2c). A signal-to-noise ratio (SNR) of about 47 dB in a bandwidth of 9.1 Hz has been recorded (see Fig. 2c). An even better SNR of about 57 dB was measured for the RBW of 1 Hz [14].

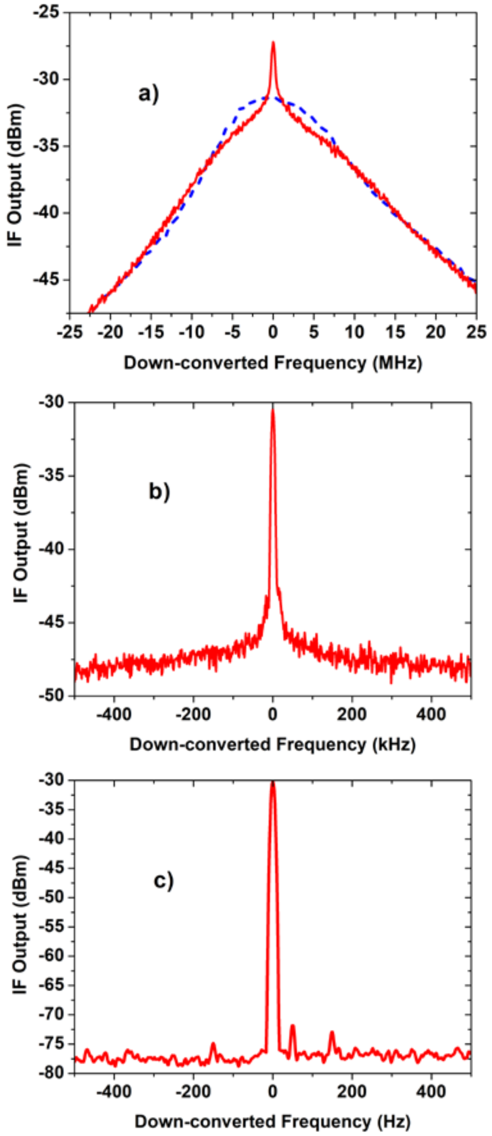

Fig. 2. Spectra of the BSCCO oscillator measured by the SIR relative to the phase-locked at 563 GHz FFO: a) Frequency (dashed) and phase locked (solid), Span 50 MHz, Resolution Bandwidth (RBW) = 470 kHzinewidth = 13.5 MHz; b) Phase locked, Span 1 MHz, RBW = 9.1 kHz; c) Phase locked, Span = 1 kHz, RBW = 9.1 Hz; the signal-to-noise ratio is 47 dB as measured in a bandwidth of 9.1 Hz.

It should be mentioned that such phase-locking was well possible only for some BSCCO oscillators demonstrating a free-running LW of about 10 - 15 MHz (that is comparable to regulations bandwidth of the room-temperature PLL, limited by the length of the cable from the PLL to the BSCCO oscillator). Another important issue is time constants in the PLL regulation loop. To realize a reasonably wide locking range, quite fast variations of the oscillator frequency (voltage on the IJJs stack) are required; the voltage variations should follow the PLL control signal with a delay smaller than 0.1 µs. Such a small delay is quite problematic in the high-bias regime with a large and "inertial" hot-spot region; note that reasonable linewidth values below 20 MHz can be achieved up to now only in the high-bias regime [4, 14].

III. MUTUAL LOCKING OF TWO BSCCO OSCILLATORS

In order to overcome the drawbacks of single JJs and to produce a significant off-chip radiation 1D or 2D arrays of Josephson Junctions (JJs) can be used; the development of such arrays has a long history [17, 18]. To significantly advance the performance of the THz radiation sources one has provide conditions to mutually phase-lock the junctions in the array. For 1D distributed JJ arrays of resistively shunted Nb/AlOx/Nb tunnel junctions, a power exceeding 10 µW was detected on-chip at frequencies from 300 to 500 GHz, the minimum inferred linewidth near 400 GHz, was about 100 kHz [19]. To this end mutual interaction of the IJJs stacks is a rather interesting issue [20]. As a first step in this direction we study the interaction of two BSCCO oscillators fabricated on one substrate.

We perform our measurements on BSCCO stacks embedded between gold electrodes, so-called gold- BSCCO-gold (GBG) structures. These GBG structures were fabricated on a common gold electrode; a sketch of the sample geometry is shown in Fig. 3a. The preparation of the sample is described in detail in Refs. [5, 21]. In brief, a BSCCO single crystal is glued onto a sapphire substrate with epoxy resin. A 100-nm-thick gold film is deposited on the crystal immediately after cleaving. As the third step, the stacks plus contact pads are pre-formed on top of the crystal as the "bone"-shaped structures in Fig. 3a with a total length of 630 µm and a thickness of about 1 µm. The sample is then glued face-down to a second sapphire substrate using epoxy. The base crystal is cleaved away by removing the first sapphire substrate, leaving approximately 0.7-μm-thick BSCCO structures contacted by gold and surrounded by epoxy. The fresh BSCCO surface is immediately covered with a 100-nm-thick gold layer. Photoresist is patterned in a rectangular 200 × 1450 μm^2-wide area using photolithography, and then the whole structure is etched down to the gold layer facing the second substrate by ion milling, resulting in five GBG structures with lateral dimensions of 300 × 50 μm^2 connected by the common gold layer (Fig. 3a). The nominal thickness of the stack corresponds to about 450 IJJs. The sample is fabricated from an as-grown BSCCO single crystal near optimal doping with $T_c \approx 89$ K. Finally, the sapphire substrate is glued onto a hemispheric sapphire lens. Current-voltage characteristics (IVCs) of two IJJs stacks "A" and "B" measured at bath temperature 4.2 K are shown in Fig. 3b and 3c, respectively.

To control the temperature of the sample it was mounted on the lens holder thermally connected to the cryostat bottom only by a copper heat link (cross-section of about 1 mm^2). Both stacks "A" and "B" were biased simultaneously by two independent computer-controlled current sources; the currents were swept from 0 up to 45 mA and then slowly decreased to a value that provides emission at frequencies of about 650 GHz. The SIR LO was phase-locked on this frequency; by tuning currents I_A and I_B it was possible to record emission lines both in low and upper sidebands. All data presented below were measured in the low sideband for frequencies of the oscillators from 642 to 646 GHz. The sample-holder temperature was about 14 K due to self-heating of the stacks; it should be mentioned that sweeping of one current (e.g. I_A) results in a change of the sample temperature and a shift of the stack "B" frequency, which is much smaller than the shift for the stack "A", but still quite considerable (see Fig. 4 and 5).

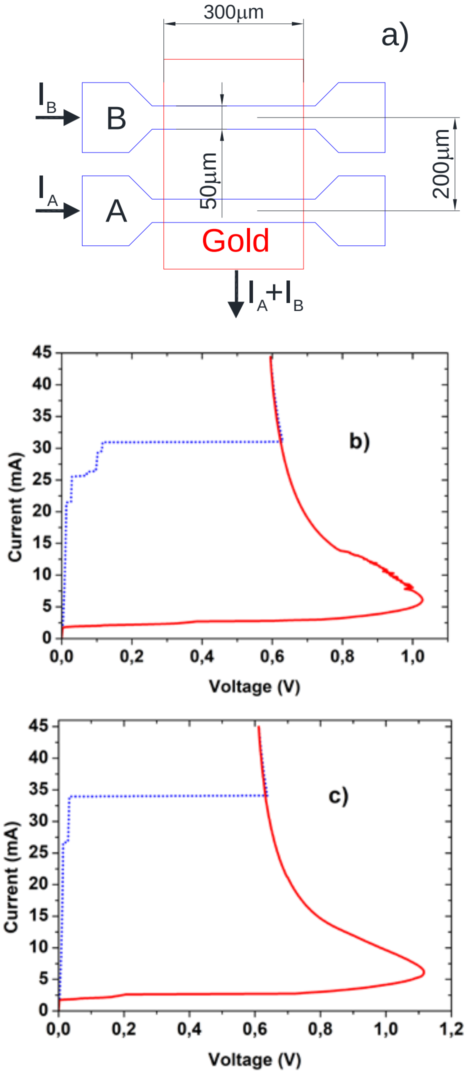

Fig. 3. a) Layout of the BSCCO oscillator circuit based on stacks of intrinsic Josephson junctions (only two stacks used in the experiment are shown); b) and c) Current-voltage characteristics of the IJJs stacks A and B, respectively.

Down-converted spectra emitted by stacks “A” and “B” at a frequency of about 644 GHz are presented in Fig. 4 (log scale for IF output power) and in Fig. 5 (linear scale). The spectra were measured by the SIR with phase-locked LO at frequency 650 GHz at small variation of the bias current for stack “B”, while the current I_A was fixed. When the current I_B increases the voltage (and the frequency) for stack “B” decreases (partially due to extra heating), as a result the down-converted frequency increases (low sideband). The frequency of stack “A” also decreases due to extra heating; that corresponds to a slow motion of the peak “A” on the graphs from left to right. It should be mentioned that amplitudes of all peaks at down-converted frequencies of about 5.9 GHz are suppressed due to standing waves in the IF chain; this unevenness was not corrected for presented data.

When the frequencies of the two stacks coincide (solid curves in Fig. 4) the power in the resulting peak increases; that is even more pronounced in linear scale (Fig. 5). It is important to note that the power emitted by the two stacks exceeds the power of the singe stack more than two times; moreover, the linewidth of the locked stacks is 24 MHz (diamonds and solid line in Fig. 5b) compared to 35 MHz for a single stack (see asterisks and dotted line). Such mutual “locking” of two distantly spaced oscillators might be explained by resonant conditions provided by electromagnetic surrounding.

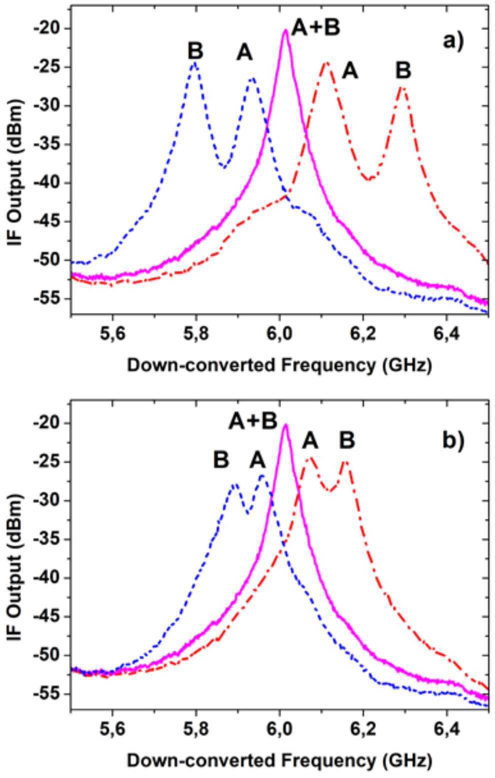

Fig. 4. Down-converted spectra emitted by stacks “A” and “B” at frequency of about 644 GHz for small variation of the bias current for stack “B”; the spectra were measured by the SIR with phase-locked local oscillator at frequency 650 GHz: a) I_A = 24.571 mA, I_B = 32.313 mA (dashed), 32.363 mA (solid) and 32.438 mA (dash-dotted); b) I_A = 24.571 mA, I_B = 32.338 mA (dashed), 32.363 mA (solid) and 32.400 mA (dash-dotted).

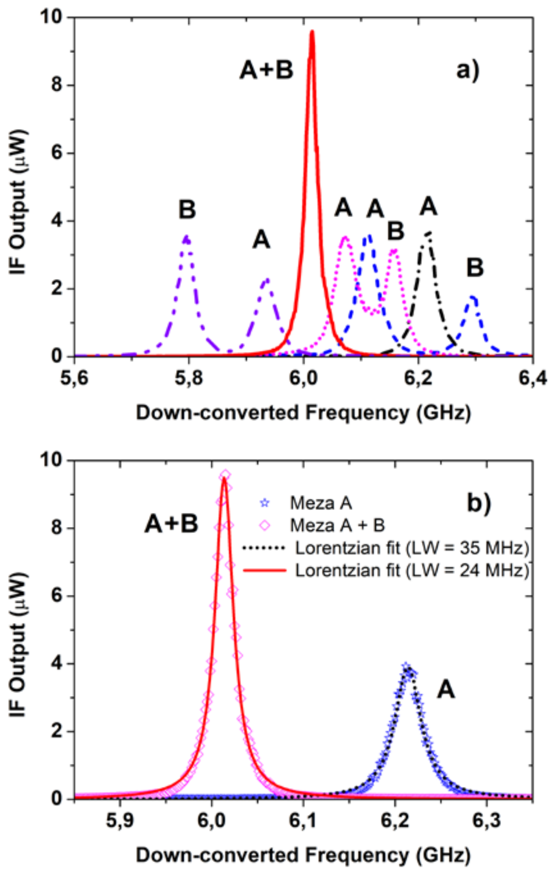

Fig. 5 Down-converted spectra emitted by stacks “A” and “B” (linear scale) at frequency of about 644 GHz for small variation of the bias current for stack “B”; the spectra were measured by the SIR with phase-locked local oscillator at frequency 650 GHz: a) I_A = 24.571 mA, I_B = 32.313 mA (dash-double dotted), 32.363 mA (solid), 32.400 mA (dotted), 32.400 mA (dashed), and 32.438 mA (dash-dotted); b) I_A = 24.571 mA, I_B = 32.363 mA (diamonds), 32.363 mA (asterisks); Lorentz fits to experimental spectra are shown by lines. Note that the power from the locked stacks (A+B) exceeds the value for the most “powerful” stack “A” by 2.6 times (about 4.3 dB); the linewidth of the locked stacks is 24 MHz compared to 35 MHz for a single one.

IV. CONCLUSION

In this report, the results of the spectral measurements of THz radiation emitted from intrinsic Josephson junction stacks are summarized. The SIR was successfully implemented for the spectral measurements of THz radiation emitted from intrinsic BSCCO Josephson junction stacks; a linewidth as narrow as 13.5 MHz has been recorded in the high-bias regime at 563 GHz; about 10 % of the oscillator power has been phase locked by using the phase-locked SIR. That is a very important step towards the development of fully high T_c phase-locked local oscillator, which opens prospects for various practical applications. The possibility of mutual locking of two BSCCO oscillators fabricated on one substrate has been demonstrated by direct measurements of emitted radiation spectra. The resulting linewidth of two IJJs stacks of 24 MHz is noticeably less than 35 MHz measured for each single BSCCO oscillator.

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